GAME THEORY

Exercise list 1

Exercise 1

Solve the following games using (iterative) elimination of (weakly) domied strategies.

1.

	L	*CL*	*CR*	*R*
U	5, 10	0, 11	1, 10	10, 20
MU	4, 0	1, 0	2, 0	20, 1
MD	3, 2	0, 4	4, 3	50, 1
D	2, 93	0, 92	0, 91	100, 90

2.

	L	*C*	*R*
U	3, 3	0, 3	0, 0
M	3, 0	2, 2	0, 2
D	0, 0	2, 0	1, 1

Exercise 2

Consider a second-price sealed-bid auction with two bidders denoted by i = 1, 2, with valuations $v_1 > v_2$. Valuations are common knowledge. Formalize this auction as a startegic-form game and find the equilibrium in weakly dominant strategies.

Exercise 3

oPlayers 1 and 2 simultaneously choose a positive integer smaller or equal to *K*. If both players choose the same number, player 2 pays 1€ to player 1; otherwise no payment occurs. Determine the unique Nash equilibrium of this game.

Exercise 4

Determine the set of Nash equilibria of the following games:

1.

	L	*R*
U	0, 1	0, 2
D	2, 2	0, 1

2.

	L	*R*
U	6, 0	0, 6
D	3, 2	6, 0

3.

	M1			*M2*		
		L	*R*		*L*	*R*
	U	1, 1, 1	0, 0, 0	*U*	0, 0, 0	0, 0, 0
	D	0, 0, 0	0, 0, 0	*D*	0, 0, 0	2, 2, 2

Exercise 5

Consider a first-price sealed-bid auction with two bidders denoted by i = 1, 2, with valuations $v_1 > v_2$. Valuations are common knowledge. Formalize this auction as a startegic-form game and determine the set of Nash equilibria.

Exercise 6

Consider the Cournot model with n firms, which simultaneously choose how much to produce. Let q_i be the quantity produced by firm i and let $Q = q_1 + ... + q_n$ be total quantity produced. Let p be the equilibrium price and assume that the inverse market demand is: $p(Q) = \max\{0, a-Q\}$. Total cost of producing q_i by firm i is $c_i(q_i) = c_i\, q_i$, with $c_i < a$ for all $i=1,..., n$. All of this is common knowledge.

i. Assume $c_i = c$ for all i=1,..., n. Determine, as a function of n, the quantities produced, the price, and the profits in Nash equilibrium (Cournot equilibrium).

ii. Determine the limits of the functions obtained in i. when n goes to infinity. Explain.

iii. Assume n =2. Determine the Nash equilibrium when $0 < c_i < \frac{a}{2}$ for each firm? What if $c_1 < c_2 < a$, but $2c_2 > a + c_1$?

Exercise 7

Find the Nash equilibrium of a Bertrand duopoly where the two firms in the market have the same cost structure.