



Problem Set III – Estimation and Inference (III) – Solution

- 1. Allows the population effect on log earnings of being married to depend on gender (B).
- 2. -2.10 (C)
- 3. The OLS estimator for all the coefficients is not identifiable due to perfect collinearity. (A)
- 4. A male baby weights more in average 3.2% than a female (holding all the other factor fixed). (D)

## EVIEW'S OUTPUT

Dependent Variable: COURSEEVAL Method: Least Squares Date: 11/14/16 Time: 22:52 Sample: 1 463 Included observations: 463

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C BEAUTY OPTIONAL MINORITY NNENGLISH FEMALE BEAUTY*FEMALE	4.074543 0.230706 0.655822 -0.134972 -0.267469 -0.172871 -0.140596	0.032848 0.041837 0.106038 0.076501 0.105121 0.048898 0.062287	124.0406 5.514442 6.184780 -1.764314 -2.544397 -3.535339 -2.257223	0.0000 0.0000 0.0783 0.0113 0.0004 0.0245
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood F-statistic Prob(F-statistic)	0.163912 0.152910 0.510684 118.9241 -342.3041 14.89947 0.000000	Mean dependent var S.D. dependent var Akaike info criterion Schwarz criterion Hannan-Quinn criter. Durbin-Watson stat		3.998272 0.554866 1.508873 1.571430 1.533500 1.515216

**Estimated Equation:** 

CourseEval = 4.074543 + 0.230706Beauty + 0.6558220ptiona - 0.134972Minority- 0.267469NNEnglish - 0.172871Female - 0.140596Beauty.Female

a)  $\beta_2 = 0.655822$ . Regarding, all other factors fixed (*ceteris paribus*), a course that is optional contributes, on average, more 0.655822 points to the teaching evaluation score when compared to a course that is not optional. It is possible to conclude that the difference in the type of course (Optional/not Optional) is statistically important do describe the model. The sign of this coefficient makes sense since when a course is optional, students choose the course according to their interests and likes. This could mean that the course is attended by motivated students and therefore better final marks, which can also imply a better score in the teaching evaluation.

<sup>5.</sup> 

**b)** Not a not native English speaker  $\rightarrow NNEnglish = 0$  (which corresponds to the base group);

Teaches an Optional course  $\rightarrow Optional = 1$ .

The difference in the estimated teaching evaluation score of an instructor with this conditions and giving that all other factors remain fixed is given by:

$$\begin{split} &\Delta CourseEval = \\ &\hat{E}[courseEval|Beauty, Optional = 1, Minority, NNenglish = 0, Female] - \\ &\hat{E}[courseEval|Beauty, Optional = 0, Minority, NNenglish = 0, Female] = \widehat{\beta_0} + \\ &\widehat{\beta_1}Beauty + \widehat{\beta_2} + \widehat{\beta_3}Minority + \widehat{\beta_4} \times 0 + \widehat{\beta_5}Female + \widehat{\beta_6}Beauty. Female - (\widehat{\beta_0} + \\ &\widehat{\beta_1}Beauty + \widehat{\beta_2} \times 0 + \widehat{\beta_3}Minority + \widehat{\beta_4} \times 0 + \widehat{\beta_5}Female + \widehat{\beta_6}Beauty. Female) = \widehat{\beta_2} \end{split}$$

Therefore,

$$\Delta CourseEval = \widehat{\beta_2} = 0.655822$$

Hence, regarding that all other factors remain fixed, an instructor that is not a not native English speaker and that is teaching an optional course has a teaching evaluation score, on average, higher 0.655822 points than an instructor that is a not a not native English speaker and that is teaching a not optional course.

c)  $\widehat{\beta_1} = 0.230706$  and  $\widehat{\beta_6} = -0.140596$ .

 $\widehat{\beta_1}$  is the slope coefficient for a man related do the variable *Beauty*.

 $\widehat{\beta_6}$  is the coefficient of the interaction term between *Beauty* and *Female*, ie, it gives the difference of the effect of the variable *Beauty* in the teaching evaluation score between female and male.

Regarding, all other factors fixed (*ceteris paribus*), the partial effect of the variable *Beauty* on *CourseEval* is given by  $\frac{\Delta CourseEval}{\Delta Beauty} = \widehat{\beta_1} + \widehat{\beta_6} \times female = 0.230706 - 0.140596 \times female$ .

For a male means that in each unit increased on the variable *Beauty* the variable *CourseEval* increases, on average, 0.230706 percentage points.

For a female this means that in each unit increased on the variable *Beauty* the variable *CourseEval* increases, on average, 0.230706 - 0.140596 = 0.09011 percentage points. Hence, for a female, the return of *Beauty* on the teaching evaluated score, *CourseEval*, is, on average, 0.140596 points less than for a male.

Make sense that both coefficients have positive signs since a good appearance of an individual is always more appellative.

It was concluded that the effect of Beauty is less for females than for males.

## d)

The estimated equation for a female is:

*CourseEval* = 3.927025 + 0.105327*Beauty* + 0.3514690*ptional* - 0.248262*Minority* - 0.140033*NNEnglish* 

Dependent Variable: COURSEEVAL Method: Least Squares Date: 11/15/16 Time: 14:56 Sample: 1 463 IF FEMALE=1 Included observations: 195

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C BEAUTY OPTIONAL MINORITY	3.927025 0.105327 0.351469 -0.248262	0.042767 0.047094 0.180008 0.100350	91.82421 2.236523 1.952523 -2.473960	0.0000 0.0265 0.0523 0.0142
NNENGLISH	-0.140033	0.158647	-0.882668	0.3785
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood F-statistic Prob(F-statistic)	0.070347 0.050776 0.524945 52.35784 -148.4905 3.594357 0.007516	Mean dependent var S.D. dependent var Akaike info criterion Schwarz criterion Hannan-Quinn criter. Durbin-Watson stat		3.901026 0.538803 1.574261 1.658184 1.608241 1.541989

The estimated equation for a male is:

 $\widehat{CourseEval} = 4.062822 + 0.246844Beauty + 0.7392160ptional + 0.020286Minority - 0.413829NNEnglish$ 

Dependent Variable: COURSEEVAL

Method: Least Squares Date: 11/15/16 Time: 16:40

Sample: 1 463 IF FEMALE=0 Included observations: 268

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	4.062822	0.032673	124.3487	0.0000
BEAUTY	0.246844	0.040927	6.031338	0.0000
OPTIONAL	0.739216	0.137714	5.367768	0.0000
MINORITY	0.020286	0.125113	0.162144	0.8713
NNENGLISH	-0.413829	0.147970	-2.796699	0.0055
R-squared	0.221561	Mean dependent var		4.069030
Adjusted R-squared	0.209722	S.D. dependent var		0.556652
S.E. of regression	0.494850	Akaike info criterion		1.449357
Sum squared resid	64.40255	Schwarz criterion		1.516353
Log likelihood	-189.2138	Hannan-Quinn criter.		1.476265
F-statistic	18.71392	Durbin-Watson stat		1.600990
Prob(F-statistic)	0.000000			

We can see that the coefficients related to all variables are different for male and female. Looking to the Eviews's output it seems that there is a difference in the regressors depending on the gender. For example, the coefficients of the variable *Minority* are very different,  $\hat{\beta}_{3female} = -0.248262$  and  $\hat{\beta}_{3male} = 0.020286$ . (Meaning that, regarding all other factors fixed, for a non-white female the teaching evaluation score is 0.248262 points lower than for a white female. For the male this difference between white or non-white is almost zero).

## e)

In the previous exercise we suspected that there is a significance difference in the return of the regressors on the teaching evaluation score according to the gender. Therefore we will perform a Chow test.

Restricted model: (for all the observations)  $CourseEval_i = \beta_0 + \beta_1Beauty_i + \beta_2Optional_i + \beta_3Minority_i + \beta_4NNEnglish_i + u_i$ i = 1, ..., 463

Unrestricted specification:

Model for females:

 $\begin{aligned} CourseEval_{i} &= \beta_{10} + \beta_{11}Beauty_{i} + \beta_{12}Optional_{i} + \beta_{13}Minority_{i} + \beta_{14}NNEnglish_{i} \\ &+ u_{i} \quad if \ i \ is \ a \ female \end{aligned}$ 

• Model for males:

 $\begin{aligned} CourseEval_{i} &= \beta_{20} + \beta_{21}Beauty_{i} + \beta_{22}Optional_{i} + \beta_{23}Minority_{i} + \beta_{24}NNEnglish_{i} \\ &+ u_{i} \quad if \ i \ is \ a \ male \end{aligned}$ 

Estimated equation of the restricted model on Eviews:

Dependent Variable: COURSEEVAL Method: Least Squares Date: 11/15/16 Time: 17:02 Sample: 1 463 Included observations: 463

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	4.001095	0.026552	150.6916	0.0000
BEAUTY	0.153427	0.030801	4.981174	0.0000
OPTIONAL	0.667038	0.107401	6.210709	0.0000
MINORITY	-0.199628	0.075992	-2.626983	0.0089
NNENGLISH	-0.233592	0.106477	-2.193819	0.0288
R-squared	0.131303	Mean dependent var		3.998272
Adjusted R-squared	0.123716	S.D. dependent var		0.554866
S.E. of regression	0.519410	Akaike info criterion		1.538494
Sum squared resid	123.5623	Schwarz criterion		1.583178
Log likelihood	-351.1614	Hannan-Quinn criter.		1.556085
F-statistic	17.30653	Durbin-Watson stat		1.484155
Prob(F-statistic)	0.000000			

Test of hypothesis:

$$H_0: \beta_{1j} = \beta_{2j} \ (j = 0, 1, 2, 3, 4) \ vs. \ H_1: H_0 \ false$$

Test statistic (Chow Statistic):

 $F = \frac{(SSR_r - (SSR_1 + SSR_2))}{SSR_1 + SSR_2} \times \frac{n - 2(k+1)}{k+1} \sim F_{(k+1,n-2(k+1))}$  (Under  $H_0$ ), in this case  $F \sim F_{(5,453)}$ 

Observed value of the Test Statistic:

$$F_{obs} = \frac{(123.5623 - (52.35784 + 64.40255))}{52.35784 + 64.40255} \times \frac{463 - 2(4+1)}{4+1} = 5.27792$$

**Rejection Rule** 

Reject  $H_0$  if  $F_{obs} > c$ , where c is the critical value.  $\alpha = 1\% \Rightarrow c = 4.977432; \ \alpha = 5\% \Rightarrow c = 3.150411; \ \alpha = 10\% \Rightarrow c = 2.393255$ 

Also, p-value= P ( $F > F_{obs}$ ) =0.0001

Conclusion:

For  $\alpha = 1\%, 5\%, 10\%$   $F_{obs} > c \Rightarrow \text{Reject } H_0$ Alternatively  $pvalue = 0.0001 < \alpha \Rightarrow \text{Reject } H_0$ .

Hence there is enough evidence to assume that there are differences in the regression functions across the two groups (females and males) at a level of significance of 1%, 5% and 10%. This means that including iteration terms with gender is important to describe the model, ie, as it was suspected the returns of the regressors are different according to gender.

Equivalently, we can compare the restricted model with the unrestricted model.

Unrestricted model:

 $\begin{aligned} CourseEval_i = \beta_0 + \beta_1 Beauty_i + \beta_2 Optional_i + \beta_3 Minority_i + \beta_4 NNEnglish_i + \\ \alpha_0 Female_i + \alpha_1 Female_i. Beauty_i + \alpha_2 Female_i. Optional_i + \alpha_3 Female_i. Minority_i + \\ \alpha_4 Female_i. NNEnglish_i + u_i \quad i = 1, \dots, 463 \end{aligned}$ 

Estimated equation of the unrestricted model on Eviews:

Dependent Variable: COURSEEVAL Method: Least Squares Sample: 1 463 Included observations: 463

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	4.062822	0.033521	121.2038	0.0000
BEAUTY	0.246844	0.041989	5.878799	0.0000
OPTIONAL	0.739216	0.141287	5.232012	0.0000
MINORITY	0.020286	0.128359	0.158043	0.8745
NNENGLISH	-0.413829	0.151810	-2.725967	0.0067
FEMALE	-0.135797	0.053239	-2.550712	0.0111
FEMALE*BEAUTY	-0.141517	0.061947	-2.284469	0.0228
FEMALE*OPTIONAL	-0.387747	0.224209	-1.729398	0.0844
FEMALE*MINORITY	-0.268548	0.160919	-1.668838	0.0958
FEMALE*NNENGLISH	0.273796	0.215842	1.268501	0.2053
R-squared	0.179123	Mean dependent var		3.998272
Adjusted R-squared	0.162814	S.D. dependent var		0.554866
S.E. of regression	0.507690	Akaike info criterion		1.503470
Sum squared resid	116.7604	Schwarz criterion		1.592838
Log likelihood	-338.0534	Hannan-Quinn criter.		1.538652
F-statistic	10.98321	Durbin-Watson stat		1.553276
Prob(F-statistic)	0.000000			

Test of hypothesis:

$$H_0: \alpha_0 = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0 \text{ vs. } H_1: \exists \alpha_j \neq 0, (j = 0, 1, 2, 3, 4)$$

$$F = \frac{(SSR_r - SSR_{ur})}{SSR_{ur}} \times \frac{n - 2(k+1)}{k+1} \sim F_{(k+1,n-2(k+1))}$$
 (Under  $H_0$ ), in this case  $F \sim F_{(5,453)}$ 

<u>Observed value of the Test Statistic:</u>  $F_{obs} = \frac{123.5623 - 116.7604}{116.7604} \times \frac{463 - 2(4+1)}{4+1} = 5.27792$ 

The conclusion is the same.