1. **C.** A random walk process has a constant mean.
2. **B.** $Cov(y_t, y_{t+1}) = 0 \text{ } and \text{ } Cov(y_t, y_{t+2}) = -0.75\sigma^2$.
3. **D.** A weakly dependent process shows a mean reversion behavior over time.
4.

a) Breusch-Godfrey test. The test has the aim to test for second order serial correlation

$$u_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \rho_1 u_{t-1} + \rho_2 u_{t-2}$$
$$\widehat{\hat{u}_t} = -0.021 + 0.007x_t - 0.001x_{t-1} + 0.214\hat{u}_{t-1} - 0.110\hat{u}_{t-2}$$

Test of hypothesis:

$H_0: \rho_1 = \rho_2 = 0 \;\; vs \;\; H_1: \exists \rho_1, \rho_2 \neq 0$

It is possible to compute the F test or to use the Lagrange multiplier LM form of the statistic (Breusch-Godfrey test).

Test Statistic:

$LM = mR^2{}_{\hat{u}_t} \sim X^2_{(q)}$ (Under H_0)

Observed value of the test statistic:

$R^2{}_{\hat{u}_t} = 0.282$

$LM_{obs} = 75 \times 0.282 = 21.15$

Critical value:

$\alpha = 1\% \;\Rightarrow X^2_{(2);0.01} = 9.21034;$

$\alpha = 5\% \;\Rightarrow X^2_{(2);0.05} = 5.991465;$

$\alpha = 10\% \;\Rightarrow X^2_{(2);0.10} = 4.60517;$

Rejection Rule:

Reject H_0 if $LM_{obs} > c$, where c is the critical value.

Conclusion:

For $\alpha = 1\%, 5\% \text{ } and \text{ } 10\%,\; LM_{obs} > c \;\Rightarrow$ Reject H_0.

Hence, for any level of significance α, there is enough evidence to conclude that the errors are serially correlated.

b) No, a dynamically complete model means that there is no serial correlation.

In question a) was concluded that the errors are serially correlated, therefore the model is not dynamically complete.

5.

a)

$$\log(C02_t) = \beta_0 + \beta_1 t + \beta_2 \log(GDP_t) + u_t$$

$$\widehat{\log(C02_t)} = -34.02813 - 0.02360t + 2.042060 \log(GDP_t)$$

Eview's Output:

Dependent Variable: LCO2
Method: Least Squares
Date: 12/09/16 Time: 00:58
Sample: 1980 2012
Included observations: 33

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-34.02813	3.225419	-10.54999	0.0000
T	-0.023260	0.004757	-4.889732	0.0000
LGDP	2.042060	0.176390	11.57698	0.0000

R-squared	0.953674	Mean dependent var	3.813881
Adjusted R-squared	0.950585	S.D. dependent var	0.330839
S.E. of regression	0.073544	Akaike info criterion	-2.295365
Sum squared resid	0.162260	Schwarz criterion	-2.159319
Log likelihood	40.87353	Hannan-Quinn criter.	-2.249590
F-statistic	308.7889	Durbin-Watson stat	1.184836
Prob(F-statistic)	0.000000		

$\widehat{\beta_0} = -0.3402813$, is the intercept of the model.

$\widehat{\beta_1} = -0.023260$ is the coefficient of the time trend. Holding all other factors fixed, β_1 measures the average proportionate change (growth rate) in the variable CO2 per period. Therefore, $C02_t$ decreases on average 2.326% per period.

$\widehat{\beta_2} = 2.04206$. If there is a 1% increase in the variable GDP_t, $C02_t$ increases on average 2.042060% around its trend.

b)

$u_t = \alpha_0 + \alpha_1 t + \alpha_2 \log(GDP_t) + \rho_1 u_{t-1}$

$\widehat{u_t} = 1.788624 + 0.001872t - 0.096974 \log(GDP_t) + 0.381154\widehat{u_{t-1}}$

Eview's Output:

Dependent Variable: RES
Method: Least Squares
Date: 12/09/16 Time: 01:15
Sample (adjusted): 1981 2012
Included observations: 32 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.788624	2.987098	0.598783	0.5541
T	0.001872	0.004392	0.426242	0.6732
LGDP	-0.096974	0.163324	-0.593748	0.5574
RES(-1)	0.381154	0.169624	2.247058	0.0327

R-squared	0.161386	Mean dependent var	0.003706
Adjusted R-squared	0.071535	S.D. dependent var	0.069040
S.E. of regression	0.066524	Akaike info criterion	-2.466028
Sum squared resid	0.123914	Schwarz criterion	-2.282811
Log likelihood	43.45645	Hannan-Quinn criter.	-2.405297
F-statistic	1.796146	Durbin-Watson stat	1.988532
Prob(F-statistic)	0.170806		

Test of hypothesis:

$H_0: \rho_1 = 0 \ vs \ H_1: \rho_1 \neq 0$

Test statistic:

$t = \frac{\widehat{\rho_1}}{se(\widehat{\rho_1})} \sim t_{(n-k-1)}$ (Under H_0) since the sample is large is it possible to write

$t = \frac{\widehat{\rho_1}}{se(\widehat{\rho_1})} \sim N(0,1)$ (Under H_0).

Observed value of the Test Statistic:

$t_{obs} = 2.247058$

Rejection Rule:

Reject H_0 if $|t_{obs}| > c$, where c is the critical value.

$$\alpha = 1\% \Rightarrow c = z_{\frac{0.01}{2}} = 2.576;$$

$$\alpha = 5\% \Rightarrow c = z_{0.05/2} = 1.96;$$

$$\alpha = 10\% \Rightarrow c = z_{0.10/2=} z_{0.05} = 1.645$$

Conclusion:

For $\alpha = 1\%, t_{obs} < c \Rightarrow Do\ not\ reject\ H_0$

Hence there is not enough evidence to assume that the errors are serially correlated, at a level of 1%.

For $\alpha = 5\%\ and\ 10\%$, $t_{obs} > c \Rightarrow Reject\ H_0$

Hence there is enough evidence to assume that the errors are serially correlated, at a level of 5% and 10%.

c) For a level of $\alpha = 5\%$ the null hypothesis was rejected which means that the errors are serially correlated.
As a consequence of this, the OLS estimators are no longer BLUE and the usual OLS standard errors and test statistics are not valid, even asymptotically.

d)

$$\log(C02_t) = \beta_0 + \beta_1 t + \beta_2 \log(GDP_t) + \beta_3 \log(CO2_{t-1}) + u_t$$

$$u_t = \alpha_0 + \alpha_1 t + \alpha_2 \log(GDP_t) + \alpha_3 \log(CO2_{t-1}) + \rho_1 u_{t-1} + \rho_2 u_{t-2}$$

$$\log(\widehat{C02}_t) = -21.50146 - 0.016883t + 1.289284 \log(GDP_t) + 0.386230 \log(CO2_{t-1})$$

$$\widehat{\widehat{u_t}} = 6.709184 + 0.002153t - 0.413802 \log(GDP_t) + 0.264243 - 0.389510\widehat{u_{t-1}} - 0.122787\widehat{u_{t-2}}$$

<u>Eview's Outputs:</u>

Dependent Variable: LCO2
Method: Least Squares
Date: 12/22/16 Time: 12:26
Sample (adjusted): 1981 2012
Included observations: 32 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-21.50146	6.495962	-3.309973	0.0026
T	-0.016883	0.005344	-3.159494	0.0038
LGDP	1.289284	0.384780	3.350702	0.0023
LCO2(-1)	0.386230	0.184978	2.087982	0.0460

R-squared	0.958658	Mean dependent var	3.834325
Adjusted R-squared	0.954229	S.D. dependent var	0.314242
S.E. of regression	0.067229	Akaike info criterion	-2.444941
Sum squared resid	0.126554	Schwarz criterion	-2.261724
Log likelihood	43.11906	Hannan-Quinn criter.	-2.384210
F-statistic	216.4280	Durbin-Watson stat	1.875661
Prob(F-statistic)	0.000000		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.941018	Prob. F(2,26)	0.4031
Obs*R-squared	2.159999	Prob. Chi-Square(2)	0.3396

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 12/22/16 Time: 12:40
Sample: 1981 2012
Included observations: 32
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.709184	8.510974	0.788298	0.4377
T	0.002153	0.005892	0.365351	0.7178
LGDP	-0.413802	0.510892	-0.809959	0.4253
LCO2(-1)	0.264243	0.273837	0.964964	0.3435
RESID(-1)	-0.389510	0.291123	-1.337960	0.1925
RESID(-2)	-0.122787	0.204160	-0.601424	0.5528

R-squared	0.067500	Mean dependent var	1.24E-15
Adjusted R-squared	-0.111827	S.D. dependent var	0.063894
S.E. of regression	0.067372	Akaike info criterion	-2.389827
Sum squared resid	0.118012	Schwarz criterion	-2.115002
Log likelihood	44.23724	Hannan-Quinn criter.	-2.298730
F-statistic	0.376407	Durbin-Watson stat	1.573473
Prob(F-statistic)	0.860231		

Test of hypothesis:

$H_0: \rho_1 = \rho_2 = 0 \; vs \; H_1: \exists \rho_1, \rho_2 \neq 0$

Test Statistic:

$LM = nR^2{}_{\hat{u}_t} \sim X^2_{(q)}$ (Under H_0) in this case $LM \sim X^2_{(2)}$

Observed value of the test statistic:

$LM_{obs} = 2.159999$

Rejection Rule

Reject H_0 if $LM_{obs} > c$, where c is the critical value.

$\alpha = 1\% \;\Rightarrow X^2_{(2);0.01} = 9.21034;$

$\alpha = 5\% \;\Rightarrow X^2_{(2);0.05} = 5.991465;$

$\alpha = 10\% \;\Rightarrow X^2_{(2);0.10} = 4.60517;$

Also, p-value=0.3396

Conclusion:

For $\alpha = 1\%\ 5\%\ and\ 10\%,\ LM_{obs} < c \Rightarrow Do\ not\ reject\ H_0$

Alternatively $pvalue = 0.3396 > \alpha, \forall\alpha \Rightarrow Do\ not\ reject\ H_0: \rho_1 = \rho_2 = 0$.

Therefore, there is not enough evidence to conclude that the errors are serially correlated.

Hence, at a level of 1%, 5% and 10% it is assumed that the errors are not serially correlated.

e) $E\left[LCO2_t \middle| LCO2_{t-1,} LCO2_{t-2,\ldots,} LGDP_{t,} LGDP_{t-1,\ldots}\right] = E[LCO2_t|LCO2_{t-1}, LGDP_t]$