Probability Theory and Stochastic Processes

Solutions

Jan 15, 2016

1. b) 0

2. 2

3. e^{-1}

4.

$$E(X|Y)(\omega) = \begin{cases} \omega + \frac{1}{4}, & \omega < \frac{1}{2} \\ \omega - \frac{1}{4}, & \omega \geq \frac{1}{2} \end{cases}$$

5.

a) Recurrent non-null, period 1

b) Unique stationary distribution $\pi_i = \frac{1}{a}$, $i = 1, \ldots, a$, mean recurrence time $\mu_i = a$

6. Yes

Feb 1, 2016

1.

a) 0

b) $\{\emptyset, \Omega, X^{-1}(\{a\}), X^{-1}(\{b\})\}$

c) We don't know

2. 0

3.

a) states 1, 2: transient period=2; states 3,4: recurrent non-null period=2

b) $\pi_1 = \pi_2 = 0, \pi_3 = \pi_4 = 1/2$, $\mu_1 = \mu_2 = +\infty, \mu_3 = \mu_4 = 2$

4.

a) not a martingale

b) $-\infty$

Jan 18, 2017

1.

a)

$$F(x) = \begin{cases} 1, & x \geq \sqrt{2} \\ 0, & x < \sqrt{2} \end{cases}$$

$\phi(t) = e^{it\sqrt{2}}$. The distribution is the Dirac measure on $\mathbb{R}$ at $\sqrt{2}$.

b) Any that is equal to X a.e. Ex: $Y(x) = \sqrt{2}$.

2. Dirac distribution at 0.

3.

a) 1,2,3 non-recurrent (transient); 4 recurrent non-null

b) 1

c) $\pi = (0, 0, 0, 1)$, $\mu = (+\infty, +\infty, +\infty, 1)$

4.

a) not a martingale

b) $-\infty$