LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomia Avançada

Doutoramento em Economia

11 de Janeiro de 2016
Duração: 2h30m

Exam

Answer any 4 questions

Question 1

A firm has the production function $f(x_1, x_2)=(x^a_1 +x^a_2)^{s/a}$, where $a = -1$ and where $0 < s < 1$.

a) (1 mark) Is this production function homogeneous? If so, of what degree is it homogeneous? Does it have positive marginal products? (Prove your answers.)

A: It is homogeneous of degree s. It has positive marginal products.

b) (1,5 marks) Find the conditional input demand functions for inputs 1 and 2 with prices w_1 and w_2, respectively, and output y.

A: $x_1(w_1, w_2,y) = y^{1/s}[1+(w_2/w_1)^{½}]$ and $x_2(w_1, w_2,y) = y^{1/s}[1+(w_1/w_2)^{½}]$.

c) (1 mark) Find the cost function $c(w_1,w_2,1)$ for producing 1 unit of output.

A: $c(w_1, w_2,1) = (w_1^{½} + w_2^{½})^2$ as $c(w_1, w_2,y) = y^{1/s}(w_1^{½} + w_2^{½})^2$.

d) (1,5 marks) Suppose that $s = 1/2$, $w_1 = 4$ and $w_2 =1$. If the firm can sell its output at a competitive price of €72 per unit, how many units should it produce to maximize its profits?

A: The firm solves Max $72y - c(4, 1,y)$ to obtain $y = 4$.

Question 2

a) (2 marks) Comment on the following sentence: "In a strategic form game G, all players have a strictly dominant strategy if and only if the Nash equilibrium is unique."

A: If each player has a strictly dominant strategy, this strategy gives a strictly higher payoff against any strategy profile of the others. Thus, it is his/her unique best reply. It follows that there is a unique Nash equilibrium. Nevertheless, it is easy to find a counterexample for the converse statement, so that we may have a unique Nash equilibrium even though there is at least one player that does not have a strictly dominant strategy.

b) (3 marks) Two firms are engaged in Bertrand price competition (and therefore choose prices to maximize profits). Firm 1's marginal cost of production is 0 and this is common knowledge. However, firm 1 is uncertain about firm 2's constant marginal cost, which can either be 4 (high) or 1 (low), with each possibility being equally likely. There are no fixed costs. Assume demand is given by $Q = 8 – p$ if the lowest price charged is p. Also, assume that if both firms charge a common price, then: (i) if both firms' costs are strictly less than the common price, the market is split evenly between them, (ii) otherwise, firm 1 captures the entire market at the common price. Finally, to keep things simple, assume that firms can only choose between 3 different prices: 1, 4, and 6 (so that, the strategy sets of each firm contain only these 3 numbers).

Compute one Bayesian-Nash equilibrium in pure strategies of this incomplete information game.

A: See pages 323-325 of Jehle and Reny.

Question 3

Consider the following signalling game:

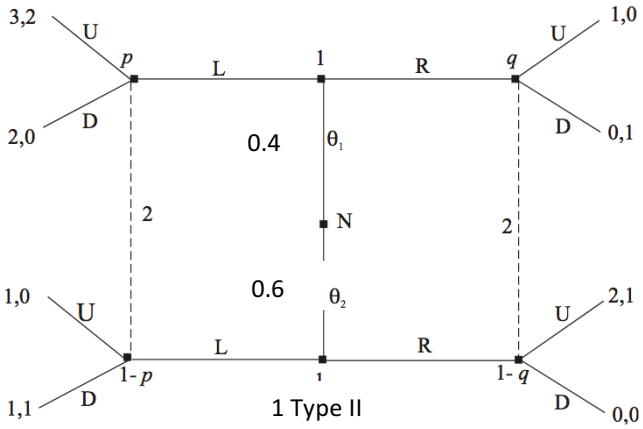
1 Type I

Microeconomia Avançada

Doutoramento em Economia

11 de Janeiro de 2016

Duração: 2h30m



1 Type II

a) (1 mark) Determine the sets of strategies of each player.

A: S_1={LL,LR,RL,RR}, S_2={UU,UD,DU,DD}.

b) (4 marks) Compute all perfect Bayesian equilibria.

A: PBE={[LR,UU,p=1,q=0], [LL,UD,p=0.4,q≥0.5]}.