LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

February 3, 2017
2h30m

Exam

Answer any 4 questions

Question 1

The utility function of a consumer is given by u(x_1, x_2)= $10\sqrt{x_1} + x_2$.

a) (2,5 marks) Formulate the consumer problem and determine this consumer's demand functions.

A: $x_1(p_1, p_2, y) = (\frac{5p_2}{p_1})^2$ and $x_2(p_1, p_2, y) = \frac{y}{p_2} - 25\frac{p_2}{p_1}$.

b) (1 mark) Determine the consumer's indirect utility function.

A: $v(p_1, p_2, y) = \frac{25p_2}{p_1} + \frac{y}{p_2}$.

c) (0,5 marks) Find the consumer's optimal bundle when y = 50 and both goods' prices are 1.

A: $x_1(1,1,50) = 25$ and $x_2(p_1, p_2, y) = 25$.

d) (1 mark) Now assume that the price of good 1 falls to 0.5 (everything else constant). What is the income effect in the demand of good 1?

A: Since $x_1(0.5,1,50) = 100$, the total change in the consumption of good 1 is 75, whereas the consumption of good 2 drops to 0. Given that preferences are quasilinear, there is no increase in income (keeping initial prices constant) that would make the consumer want to consume 100 units. Therefore, the substiutution effect is 0 and the income effect is 75.

Question 2

1. (2,5 marks) Assume that a consumer's income is €200, the price of good 2 is €2 and the price of good 1, initially equal to €1, increases to €4. Compute the equivalent variation and the compensating variation associated to the price change when the consumer's utility function is u(x_1, x_2) = min{2x_1, x_2}.

A: Solving the consumer problema, we obtain x_1= y/(p_1+2p_2) and x_2= 2y/(p_1+2p_2). Therefore, v(p_1, p_2, y) = 2y/(p_1+2p_2). We have v(1, 2, 200) = 80 and v(4, 2, 200) = 50. The CV is such that v(4, 2, 200+CV)=80, so that CV = 120. The EV is such that v(1, 2, 200-EV) = 50, so that EV = 75.

2. (2,5 marks) In the model of consumer choice under uncertainty, does a strictly increasing transformation of a Von Neumann Morgenstern utility representation yield another Von Neumann Morgenstern utility representation of the same preferences?

A: See Jehle and Reny, p 107 and 108.

Question 3

A firm's production function is f(l, k) = $\sqrt{2l + k}$, where l and k represent the amounts of labour and capital, whose prices are w and r, respectively.

a) (1,5 marks) If w = 2 and r = 8, what is the minimum cost of producing y? What if w = 6 and r = 2?

R: Inputs are perfect substitutes, so that for most prices, only one input is used.

When only labour is used (and k = 0), we have y = f(l,0) = $\sqrt{2l + 0}$⇔ln = y^2/2 and cost is c(y) = w. y^2/2.

When only capital is used (and l = 0), we have y = f(0,k) = $\sqrt{0 + k}$⇔k = y^2 and cost is c(y) = r. y^2.

Therefore, only labou ris used if and only if w.y^2/2 ≤ r.y^2 ⇔w ≤ 2r.

For w = 2 and r = 8, we have w < 2r. Therefore, c(y) = w. y^2/2 = y^2 and c(8) = 64.

F orw = 6 and r = 2, we have w > 2r. Therefore, c(y) = r. y^2 = $2y^2$ and c(8) = 128.

b) (1,5 marks) Determine the conditional input demand and the long run cost function.

R: Conditional input demands are:

$$(l(w,r,y), k(w,r,y)) = \begin{cases} \left(\frac{y^2}{2}, 0\right), \text{ if } w < 2r \\ (x, y^2 - 2x) \text{ with } x \in [0, \frac{y^2}{2}], \text{ if } w = 2r \\ (0, y^2), \text{ if } w > 2r \end{cases}$$

And the long run cost function:

$$c(y) = \begin{cases} w\frac{y^2}{2}, \text{ if } w \le 2r \\ ry^2, \text{ if } w > 2r \end{cases}.$$

c) If, in the short run, k = 4, determine the short run cost function.

WIth k = 4, we have y = f(l,4) = $\sqrt{2l + 4}$⇔l = y^2/2 – 4. Therefore cs(y) = w(y^2/2 – 4) + 4r.

Question 4

In a perfectly competitive market, demand is given by $Q^D(p) = 205 - 20p$, where p represents the market price. Firms have the following cost function:

$$c(q) = \begin{cases} 8 + \frac{q^2}{2}, & q > 0 \\ 0, & q = 0 \end{cases}$$

where q is the quantity of output produced by a firm.

a) Determine the firm's long run supply curve and illustrate graphically. (1.5 marks)

A: Solving the profit maximization problema, we obtain q = p if p ≥ 4 (=minAC).

b) Determine the long run equilibrium (price, quantity and number of firms). Do firms have postive profit in the long run equilibrium? Explain. (3.5 marks)

A: Solving for long run equilibrium we get 31,25 firms. Therefore, there are 31 firms in the market and each firm has positive profit (even though the entry of na additional firm would lead to losses).

Question 5

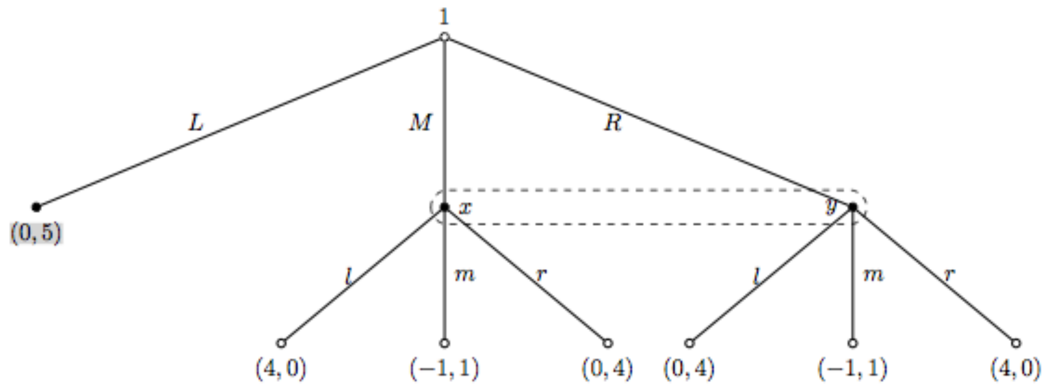
Two firms are engaged in Bertrand price competition (and therefore choose prices to maximize profits). Firm 1's marginal cost of production is 0 and this is common knowledge. However, firm 1 is uncertain about firm 2's constant marginal cost, which can either be 4 (high) or 1 (low), with each possibility being equally likely. There are no fixed costs. Assume demand is given by Q = 8 –p if the lowest price charged is p. Also, assume that if both firms charge a common price, then: (i) if both firms' costs are strictly less than the common price, the market is split evenly between them, (ii) otherwise, firm 1 captures the entire market at the common price. Finally, to keep things simple, assume that firms can only choose between 3 different prices: 1, 4, and 6 (so that, the strategy sets of each firm contain only these 3 numbers).

Compute one Bayesian-Nash equilibrium in pure strategies of this incomplete information game.

A: See Jehle and Reny, p. 323-325.

Question 6

Consider the following game tree:



a) Write the game in normal form (0.5 marks)

A: I = {1,2}, S1 = {L,M,R}, S2 = {l,m,r}.

b) Determine all Nash equilibria. (2.5 marks)

A: In pure strategies: S*={(L,l)}.

c) Determine a subgame perfect Nash equilibria in pure strategies. (1 mark)

A: There are no proper subgames, so (L,l) is subgame perfect.

d) Check whether the equilibrium found in c) is sequential. (1 mark)

A: No. Whatever beliefs 2 may have, m is never part of a sequential equilibrium. It is dominated by, e.g., the mixed strategy 0.5l+0.5r.