Stochastic Calculus – Master Program in Mathematical Finance

Program 2016/2017

1-Introduction
1.1. What is stochastic calculus?
1.2. A very brief history of stochastic calculus

2 – A review of basic concepts in probability and stochastic processes
2.1. Stochastic processes
2.2. Conditional expectation
2.3. Martingales in discrete time
2.4. Martingale transform (or discrete stochastic integral) in discrete time
2.5. Martingales in continuous time

3 – Brownian motion
3.1. Brownian motion: Definition, continuous and non-differentiable sample paths
3.2. Main properties of Brownian motion.
3.3. Brownian motion with drift, Geometric Brownian motion and the "Brownian bridge".
3.4. Quadratic variation of Brownian motion

4 – The stochastic integral
4.1. Motivation
4.2. Stochastic integral for simple processes
4.3. The stochastic integral for adapted processes.
4.5. Main properties of the stochastic integral as a process: martingale property, continuity of sample paths and the quadratic variation.
4.6. Extensions of the stochastic integral

5 – The Itô formula
5.1. One dimensional Itô formula
5.2. Multidimensional Itô formula
5.3. Itô integral representation Theorem
5.4. Martingale representation Theorem

6 – Stochastic Differential Equations
6.1. Motivation and examples
6.2. Some examples: stochastic differential equation for the geometric Brownian motion and the Langevin equation for the Ornstein-Uhlenbeck process.
6.3. Existence and uniqueness of solutions Theorem for SDE's
6.4. Examples: Ornstein-Uhlenbeck process with mean reversion and a financial application: the Vasicek model for interest rates
6.5. Linear stochastic differential equations
6.6. Strong and weak solutions for stochastic differential equations
6.7. Numerical approximations: the Euler and the Milstein schemes

6.8. Markov property of the solutions of SDE's
6.9. Stratonovich stochastic integral and Stratonovich SDE's

7 – Stochastic differential equations and partial differential equations
7.1. Infinitesimal generator for a diffusion
7.2. Stochastic representation for solutions of parabolic PDE's: Feynman-Kac formulas
7.3. The heat equation and the Brownian motion
7.4. The Kolmogorov backward equation

8 – Girsanov Theorem
8.1. Changes of probability measures
8.2. Girsanov Theorem: elementary version
8.3. Girsanov Theorem: a more general version

9 – Application to financial markets and derivatives pricing
9.1. The Black-Scholes model
9.2. No arbitrage and the Black-Scholes equation
9.3. The equivalent martingale measure and risk neutral valuation
9.4. The Black-Scholes formula

Recommended Bibliography

_J. Guerra, Stochastic Calculus, Lecture Notes, ISEG, 2016.

_B. Oksendal, Stochastic Differential Equations, 6th. Edition, Springer, 2003

_D. Nualart, Stochastic Calculus (Lecture notes, Kansas University):
http://www.math.ku.edu/~nualart/StochasticCalculus.pdf

Optional Bibliography

_Tomas Björk, Arbitrage Theory in Continuous Time, Oxford University Press, 3rd Editions, 2009.

_I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calculus, 2nd edition, Springer, 1991.

_F. Klebaner, Introduction to Stochastic Calculus with Applications, 3rd edition, Imperial College Press, 2012.

_T. Mikosch, Elementary Stochastic Calculus with Finance in view, World Scientific, 1998.

_Steven Shreve, Stochastic Calculus for Finance II: Continuous-Time Models, Springer, 2004.

Assessment

Two components:

1) A list of problems to solve at home (1 week for solving the problems at the end of April).
2) Final Exam

Final mark = Max {0.35 * (mark of problem list)+0.65*(Exam mark), Exam mark}, if the Exam mark is at least 8.

Final mark = Exam mark, if the Exam mark is less than 8.