FIXED INCOME PRODUCTS AND MARKETS

III – Fixed Income Derivatives and Models

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III – Fixed Income Derivatives and Models

1. Swaps, Fras and Short Term Interest Rate Futures

1.0 Introduction

- **Spot Market**
  - Equity, bonds,…
  - Settlement of transactions occurs few days after

- **Forward Market**
  - Futures, options, *forwards*, *swaps*, *caps*, *floors*, *collars*, etc.
  - Settlement of transactions (delivery) occurs in the future, but the price is agreed today
  - Main purpose: **Hedging**

Forward Markets

- **OTC markets**
  - Taylor made
  - Counterparty risk
- **Organized markets (Exchanges)**
  - Standardized contracts
  - Liquidity
  - Transparency
  - No counterparty risk

Types of operations

- **OTC Market**
  - *Forwards, Fra's, repos*
  - *Swaps*
  - Options, *caps*, *floors*, *collars*
- **Exchanges**
  - Futures
  - Options

1.1 Swaps

- Definition
  - Agreement between two parties
    - They exchange interest payments
    - Computed on a “notional” principal
    - Principal is not exchanged
- Classic swap (plain vanilla)
  - One side pays a fixed rate
  - Counterpart pays floating rate
- Floating rate is usually Libor / Euribor
  - Rate is reset at every payment date

Example (plain vanilla swap)

- Date: 01/01/2008
- 6- months Euribor Swap with 2 years maturity, fixed rate r^f
- Notional amount: N

Schedule of payments (pay floating receive fixed)

Datas	01/07/2008	01/01/2009	01/07/2009	01/01/2010
Fixed leg		$r^f \times N$		$r^f \times N$
Floating leg	$-\frac{N}{2} \times E_{6M(01/01/2008)}$	$-\frac{N}{2} \times E_{6M(01/07/2008)}$	$-\frac{N}{2} \times E_{6M(01/01/2009)}$	$-\frac{N}{2} \times E_{6M(01/07/2009)}$

Where $E_{6M(i)}$ is the 6-months Euribor rate at date i

Swap specifications (fixed/floating):

- Principal amount

  A swap is composed of two legs a fixed leg whose payments depends on a fixed rate and a floating leg whose payments depends on a floating rate

  The notional, or principal, amount allows one to calculate the exact amount of the different payments on the two legs of the swap
- Maturity date: date of termination of the swap contract
- Frequency

  Payments on the fixed-leg take place either annually or semi-annually (e.g., in the US); Payments on the floating leg match the maturity of the reference rate (e.g., 4 times a year if the reference rate is a 3 months rate)
- Rate (day count convention) and payments

  The floating rate for each period is fixed at the start of the period

  The first interest payment of the swap is known in advance by both parties

  Note that even if both parties pay and receive interest payments, at a payment date only the net difference between the two interest payments change hands

Pricing Swaps

- Exchange of a fixed-rate (F) for a floating-rate (V) security
- Initially, both have the same value
  - Otherwise it would not be a fair deal

$$F_0 = V_0$$

- Later on, prices can differ depending on the evolution of the term structure
  - Fixed-rate notes have longer duration => a rise (decline) in interest rates tends to lower (increase) the value of the fixed-leg more than that of the variable leg
  - This raises (lowers) the value of the swap to the buyer and lowers (raises) the value of the swap to the seller
- Value of the swap (party that receives fixed): $F_t - V_t$

Swap value

- Party that receives fixed

$$SWAP_t = N \times \left(\sum_{i=1}^{n} F \left(\frac{T_{ki} - T_{k(i-1)}}{360} \right) B(t, T_{ki}) - \sum_{j=1}^{m} V_{j-1} \left(\frac{T_j - T_{j-1}}{360} \right) B(t, T_j) \right)$$

(considering Actual/360 convention)

N – Notional principal

V_{j-1} - Floating rate at date T_{j-1}, paid at date T_j

m – number of cash-flows of the floating leg

F – Fixed rate

n – number of cash-flows of the fixed leg (k is a coeficient equal to the payment frequency on the variable leg divided by the payment frequency on the fixed leg: kn = m)

T_j-T_{j-1} – number of days between the j*th* and the (j-1)*th* payments

Swap value – zero coupon

- Assume that the notional is also exchanged (the fixed leg corresponds to a fixed rate bond and the variable leg to a FRN)

- Then, the floating will pay notional plus market rate

  - The present value of notional plus market rate is:

    => Notional

The price of a floating rate note on each and every coupon date (T_{j-1}, j =2,3,..m, incluiding T_0) is equal to par, thus:

$$SWAP_{T_{j-1}} = \underbrace{N \times \left(\sum_{i=a}^{n} F \left(\frac{T_{ki} - T_{k(i-1)}}{360} \right) B(T_{j-1}, T_{ki}) + B(T_{j-1}, T_m) \right)}_{\text{fixed part value}} - N$$

Swap value – zero coupon method

Swap value at date t e $T_{j-1} < t < T_j$

$$SWAP_t = \underbrace{N \times \left(\sum_{i=a}^{n} F \left(\frac{T_{ki} - T_{k(i-1)}}{360} \right) B(t, T_{ki}) + B(t, T_m) \right)}_{\text{fixed part value}} - \underbrace{N \times \left(V_{j-1} \left(\frac{T_j - T_{j-1}}{360} \right) + 1 \right) B(t, T_j)}_{\text{variable part value}}$$

fixed part value

variable part value

(considers only the next coupon)

Example:

Today is 1/1: remaining life of 9 months

Notional principal of $1,000

Receives 10% a year (Semiannually paid coupons, on 3/31 and 9/30)

Pays 6-month LIBOR (Next payment based on LIBOR at 6%)

Term structure: r(0;0,25) = 5% and r(0;0,75) = 7%

Swap value – zero coupon method

Example (Cont.):

Fixed: $F = \frac{50}{(1+0,05)^{0,25}} + \frac{1050}{(1+0,07)^{0,75}} = 1\ 047,44$

Floating: $V = \frac{1030}{(1+0,05)^{0,25}} = 1\ 017,51$

Swap value (receive fixed rate):

F – V = 1 047,44 – 1 017,51 = 29,93

Swap value – *Forward Projection Method*

Replace V_{j-1} by its forward value $F(t, V_{j-1})$

(Receives fixed)

$$SWAP_t = N \times \left(\sum_{i=1}^{n} F\left(\frac{T_{ki} - T_{k(i-1)}}{360}\right) B(t, T_{ki}) - \sum_{j=1}^{m} F(t, V_{j-1}) \left(\frac{T_j - T_{j-1}}{360}\right) B(t, T_j) \right)$$

For the plain vanilla *swaps*, where the payments frequency and the maturity of the variable rate is the same, we have:

$$F(t, V_{j-1}) = \left(\frac{B(t, T_{j-1})}{B(t, T_j)} - 1 \right) \frac{360}{T_j - T_{j-1}}$$

And both methods are equivalent

Quotation

Market convention:

- set the floating leg at flat rate (Libor or Euribor)

- quote the fixed rate, called the **swap rate**, that makes the value of the swap equal to zero

The ***swap rate*** is then the value of the fixed rate that makes the swap's fixed leg equal to its floating leg

When the swap is initiated, the value of the floating leg equals to par, thus the value of the fixed leg also equals to par

The Swap rate is a *par yield*

Quotation – example

A market maker is quoting a 5 years 3-months Libor swap:

Bid	Ask
4,25%	4,30%

The bank is willing to pay a fixed rate of 4,25% and to receive 3-months Libor

The bank is willing to pay 3-months Libor and to receive a fixed rate of 4,30%

A swap is also quoted as a *swap spread*. The swap spread of a swap with a given maturity is equal to the difference between the fixed rate of the swap and the benchmark treasury bond yield of the same maturity. It is expressed as a number of basis points

Example: A market maker quotes a 5 years 3-month Libor swap: 45-50

- Means that he is willing to enter a swap paying fixed 45 points above the 5-years benchmark bond yield and receiving the Libor
- And receiving fixed 50 basis points above the 5-years bond yield and paying the Libor

Euro

EURO

Ticker	TIME	Bid	Ask	Change	Open	High	Low	Prev Cls
Euro Swap Annu								
3) 1YR	18:25	4.6160	4.6340	+.0430	4.5830	4.6365	4.5410	4.5820
4) 1.5YR	18:40	4.4350	4.4440	+.0600	4.3910	4.4510	4.3755	4.3790
5) 2YR	18:40	4.3340	4.3530	+.0545	4.2900	4.3570	4.2825	4.2880
6) 3YR	18:42	4.2540	4.2740	+.0440	4.2190	4.2805	4.2040	4.2195
7) 4YR	18:00	4.2145	4.2410	+.0270	4.2040	4.2575	4.1795	4.2005
8) 5YR	18:42	4.2200	4.2390	+.0190	4.2100	4.2605	4.1850	4.2105
9) 6YR	18:42	4.2450	4.2630	+.0130	4.2410	4.2990	4.2190	4.2405
10) 7YR	18:42	4.2830	4.3020	+.0095	4.2850	4.3220	4.2635	4.2830
11) 8YR	18:42	4.3290	4.3480	+.0030	4.3360	4.3720	4.3210	4.3355
12) 9YR	18:42	4.3820	4.4000	+.0005	4.3910	4.4220	4.3730	4.3900
13) 10YR	18:42	4.4350	4.4540	+.0005	4.4360	4.4755	4.4255	4.4440
14) 12YR	18:42	4.5230	4.5620	-.0005	4.5430	4.5740	4.5240	4.5430
15) 15YR	18:42	4.6400	4.6610	-.0296	4.6800	4.6801	4.6210	4.6801
16) 20YR	18:42	4.6970	4.7360	-.0015	4.7180	4.7470	4.6955	4.7180
17) 25YR	18:41	4.6940	4.7320	-.0015	4.7150	4.7410	4.6925	4.7145
18) 30YR	18:41	4.6660	4.7040	-.0030	4.6890	4.7135	4.6665	4.6880
19) 40YR	18:41	4.6090	4.6460	-.0005	4.6290	4.6530	4.6055	4.6280
20) 50YR	18:41	4.5620	4.6000	-.0020	4.5840	4.6060	4.5570	4.5830

Ticker	TIME	Bid	Ask	Change	Open	High	Low	Prev Cls
Euro Annual Swa								
23) 1YR	18:25	75.1363	76.9363	-.7507	76.8870	78.3035	72.0031	76.7870
24) 1.5YR	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	N.A.
25) 2YR	18:29	77.6622	79.5622	-.9763	79.3885	81.5300	78.0122	79.2885
26) 3YR	18:42	62.5360	64.5360	+.2414	63.2446	65.2963	61.6946	63.2946
27) 4YR	18:00	48.9156	51.5656	-1.5576	51.7982	52.9724	49.6482	51.7982
28) 5YR	18:40	50.2892	52.1892	-1.7655	53.0047	53.8515	50.4481	53.0047
29) 6YR	18:42	45.0787	46.8787	-1.5805	47.5592	48.4108	45.4287	47.5592
30) 7Y	18:42	41.9356	43.8356	-1.8504	44.7360	45.8572	42.3356	44.7360
31) 8YR	18:40	37.7191	39.6191	-1.6155	40.2846	41.4008	38.1191	40.2846
32) 9YR	18:42	38.6507	40.4507	-1.3194	40.8701	41.6213	38.9507	40.8701
33) 10YR	18:40	44.1000	46.5000	-1.8804	46.4278	48.1080	44.7974	47.2278
34) 15YR	18:42	14.9000	17.0000	-3.5600	19.5000	19.5100	14.6500	19.5100
35) 20YR	18:40	15.5556	19.4556	+.4699	17.0357	19.6310	16.2886	17.0357
36) 30YR	18:42	11.8310	15.6310	+1.1413	12.5897	15.5598	12.5332	12.5897

For Euro Benchmark Yield Curve, Type: {IYC1 I13 <Go>}
For Euro Swap Curve, Type: {IYC1 I53 <Go>}
Page Forward for All Swaps vs Euribor

Page 1 Page 2

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000
G771-205-2 02-Apr-08 18:43:20

Uses of Swaps

Swaps may be used to:

→ Optimize the financial conditions of a debt

→ Convert the financial conditions of a debt

→ Create new synthetic assets

→ Hedge a bond or another fixed-income security against any change of the yield curve

Optimize the financial conditions of a debt

Financial conditions for two firms:

	Fixed rate	Floating rate
A	8%	**Libor 6M + 0,50%**
B	**10%**	Libor 6M + 1,00%

- A would prefer floating and B fixed
- A has "comparative advantage" at fixed:
  - 200 basis points better than B for fixed
  - 50 basis points better than B for floating
- A borrows at fixed and B at floating
- They enter a swap

Optimize the financial conditions of a debt

- Terms of the swap
  - A will pay 6-month Libor
  - B will pay 8,5%

Libor 6M →

A ⇄ B

← 8,5%

A ↓ Fixed rate: 8%

B ↓ Floating rate: Libor 6M + 1%

- After the swap:
  - A pays: 8% + Libor 6M – 8,5% = Libor – 0,50%
  - B pays: Libor 6M + 1% + 8,5% - Libor 6M = 9,5%
- Advantage:
  - A saves 100 basis points (pays Libor-.5% instead of Libor +.5%)
  - B saves 50 basis points (pays 9,5% instead of 10%)

Convert the financial conditions of a debt

A firm issued a 5 years fixed rate (7%) bond.

At 2 years from maturity, the firm decides to transform its debt at a fixed rate into a floating-rate debt through a swap

Suppose the 2 years 6-months libor swap rate is 5,5% (or 7% against Libor 6M + 1,5%)

Libor 6M + 1,5% →

Firm A ⇄ Bank

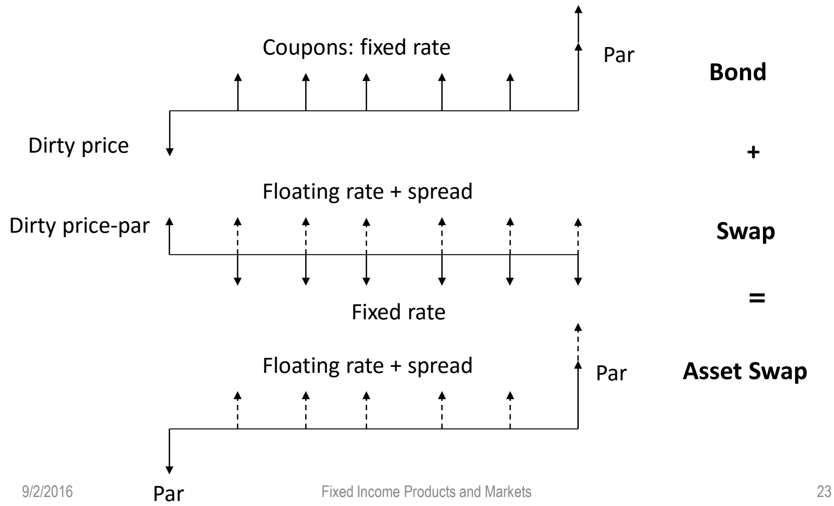
← 7%

Firm A ↓ 7%

After the swap, the firm pays Libor 6M + 1,5%

Create synthetic securities – Asset Swap

Combination of a long position in a fixed rate bond and a swap creates a synthetic floating rate bonds, whose value depends on the credit spread:



Hedging interest rate risk

- *Duration hedge:*

Hedge ratio: $$\phi_S = -\frac{\$Dur_P}{\$Dur_S}$$

Where $\$Dur_P$ e $\$Dur_S$ are, respectively, the \$duration of portfolio P and of the fixed-coupon bond contained in the swap

- *Duration/Convexity hedge:*

$$\begin{cases} \phi_1 \$Dur_{S_1} + \phi_2 \$Dur_{S_2} = -\$Dur_P \\ \phi_1 \$Conv_{S_1} + \phi_2 \$Conv_{S_2} = -\$Conv_P \end{cases}$$

Where $\$Dur_P$ ($\$Conv_P$), $\$Dur_{S1}$ ($\$Conv_{S1}$) e $\$Dur_{S2}$ ($\$Conv_{S2}$) are, respectively, the \$duration (\$Convexity) of portfolio P and of the fixed-coupon bond contained in swaps S_1 e S_2

Hedging interest rate risk

- *Hedge in a Three-Factor* model (Nelson e Siegel):

$$\begin{cases} \phi_1 \frac{\partial S_1}{\partial \beta_0} + \phi_2 \frac{\partial S_2}{\partial \beta_0} + \phi_3 \frac{\partial S_3}{\partial \beta_0} = -\frac{\partial P}{\partial \beta_0} \\ \phi_1 \frac{\partial S_1}{\partial \beta_1} + \phi_2 \frac{\partial S_2}{\partial \beta_1} + \phi_3 \frac{\partial S_3}{\partial \beta_1} = -\frac{\partial P}{\partial \beta_1} \\ \phi_1 \frac{\partial S_1}{\partial \beta_2} + \phi_2 \frac{\partial S_2}{\partial \beta_2} + \phi_3 \frac{\partial S_3}{\partial \beta_2} = -\frac{\partial P}{\partial \beta_2} \end{cases}$$

Where $\partial P/\partial \beta_i$, $\partial S_1/\partial \beta_i$, $\partial S_2/ \partial \beta_i$ e $\partial S_3/\partial \beta_i$ are, respectively, the sensitivity of portfolio P and of swaps S_1, S_2 e S_3 with respect to factors β_i (i = 0,1, 2)

Non Plain Vanilla Swaps

Accrediting swap: the notional principal increases in amount over time

Amortizing swap: the notional principal decreases in a predetermined way over the life of the swap

Roller Coaster swap: the notional principal can rise or fall from period to period

Basis swap: a floating-for-floating interest rate swap

- That exchanges floating rate of two different markets …
- …or that exchanges the same floating rate but with different maturities …
- …or that exchanges floating rate of two different markets and with different maturities

Other types of Swaps (*Nonplain Vanilla*)

Constant maturity swap (CMS): is a floating-for-floating interest rate swap exchanging a Libor/Euribor rate for a particular swap rate

Example: 6-months Euribor against 5-years Swap rate

Forward-start swap: allows the counterparties to initiate it at a specified deferred date

Back-set swap: the floating rate is fixed at the end of each period

Zero-coupon swap: is a swap which allows a counterparty to exchange:

- a fixed or floating index that delivers regular coupon
- for an index that delivers only one coupon at the beginning or at the end of the swap

1.2 Short Term Interest Rate Futures

Future contract:

- **negotiable**
- **in an exchange**
- where the parties agree to buy/sell
- an underlying asset
  - **with normalized quantity and quality**
  - **to be delivered in a future date**
- at an agreed-upon price today

Future contract – underlying asset

- **Commodities**
- **Bonds**
- **Index**
- **Equity**
- **Interest rate**
- **…**

Futures contracts - standardized characteristics

- Characteristics
  - Quantity (contract size)
  - Quality
  - Last trading day
  - Delivery day (delivery month)
  - Delivery place
  - Type of delivery (Settlement)
  - Quotation method / *tick size*

Short term interest rate futures contracts

- **Main Contracts**:
  - 3-month Euribor Futures
  - 3-month Eurodollar Futures
  - 3-month Euroswiss futures
  - 3-months EuroYen futures
  - 3-month Short Sterling futures
- **Main Exchanges**
  - NYSE Euronext (https://globalderivatives.nyx.com/nyse-liffe)
  - Eurex (www.eurexchange.com)
  - Chicago Mercantile Exchange (www.cme.com)

3-months Euribor future contract

Unit of trading (contract size)	European Interbank Offered Rate (EURIBOR) for three-month euro term deposits – 1 000 000€
Quotation	100,000 – implied interest rate
Tick size	0,005%, ie, 1/2 basis point (*tick value* = 12,50 euros)
Contract months (delivery months)	Nyse Euronext: 6 consecutive months and the next 22 months from the quarterly cycle: March, June, September and December Eurex: 12 months of the quarterly cycle
Settlement	Cash settlement
EDSP (Exchange Delivery Settlement Price)	100,000 – Euribor 3M
Last Trading Day	two exchange days prior to the third Wednesday of the respective delivery month
Final Settlement Day	Eurex: last trading day; Nyse Euronext: First business day after the Last Trading Day

3-months Euribor future contract

Type # <GO> For Related Function

Futures Contract Description Page 1/2

Exchange (LIF) LIFFE		Related Functions
Name	3MO EURO EURIBOR Jun08	1) CT Contract Table
Ticker	ERM8 <CMDTY>	2) EXS Expiration Schedule
Price is 100 - Yield		3) SFR Synthetic FRA Matrix
Contract Size	EUR 1,000,000	4) WECO World Economic Releases
Value of 1.0 pt	EUR 2,500	
Tick Size	.005	Margin Limits
Tick Value	EUR 12.5	Speculator
Current Price	95.430 100 - yield	Initial 700
Pt. Val x Price	EUR 238,575 @ 15:51:41	

Cycle --- --- Mar --- --- Jun --- --- Sep --- --- Dec

Trading Hours	
London	Local
01:00-21:00	01:00-21:00

Three Month Euro Euribor Interest Rate Future.
As of Nov.22,1999, trading only on Liffe Connect.
On last trading day, trading ceases at 10am.
Delivery date is 1 business day after LTD.

For LIFFE Euribor Analysis please run EUS <go>

Cash Settled		Life High 97.330	Generics Available
Valuation Date	Mon Jun 16, 2008	Life Low 95.055	ER1 <CMDTY>
Last Trade	Mon Jun 16, 2008		Through
First Trade	Tue Jun 17, 2003		ER24 <CMDTY>

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3-months Euribor future contract

Contract Table

3MO EURO EURIBOR Delayed monitoring enabled
Exchange Web Page Pricing Date: 4/ 7/08 Price Display: 2
EURONEXT.LIFFE Delayed prices --LATEST AVAILABLE--- 2
Grey date = options trading

Scroll		Last	Pct Chg	Time	High	Low	OpenInt (3758339)	TotVol (406082)	Previous Close
1) ERJ8	Apr08	95.280	-.03%	15:41	95.330	95.275	164061	3783	95.305
2) ERK8	May08						1957	0	95.375
3) ERM8	Jun08	95.435	-.01%	15:42	95.450	95.425	793724	76839	95.445
4) ERN8	Jul08						0	0	95.525
5) ERQ8	Aug08						0	0	95.680
6) ERU8	Sep08	95.730	-.04%	15:42	95.760	95.715	658842	94444	95.765
7) ERZ8	Dec08	95.880	-.02%	15:42	95.900	95.850	498295	80799	95.900
8) ERH9	Mar09	96.065	-.03%	15:42	96.080	96.040	464716	61422	96.090
9) ERM9	Jun09	96.125	-.03%	15:42	96.140	96.100	338172	39104	96.155
10) ERU9	Sep09	96.170	-.04%	15:42	96.185	96.150	278412	21895	96.205
11) ERZ9	Dec09	96.125	-.04%	15:42	96.150	96.105	219168	15198	96.160
12) ERH0	Mar10	96.110	-.04%	15:40	96.140	96.090	122409	5411	96.145
13) ERM0	Jun10	96.070	-.03%	15:37	96.095	96.045	66091	3041	96.100
14) ERU0	Sep10	96.040	-.03%	15:35	96.050	96.020	62909	2526	96.065
15) ERZ0	Dec10	95.995	-.03%	15:21	95.995	95.975	54387	1081	96.020
16) ERH1	Mar11	95.995	-.03%	15:10	96.000	95.980	20807	539	96.020
17) ERM1	Jun11						6848	0	96.015

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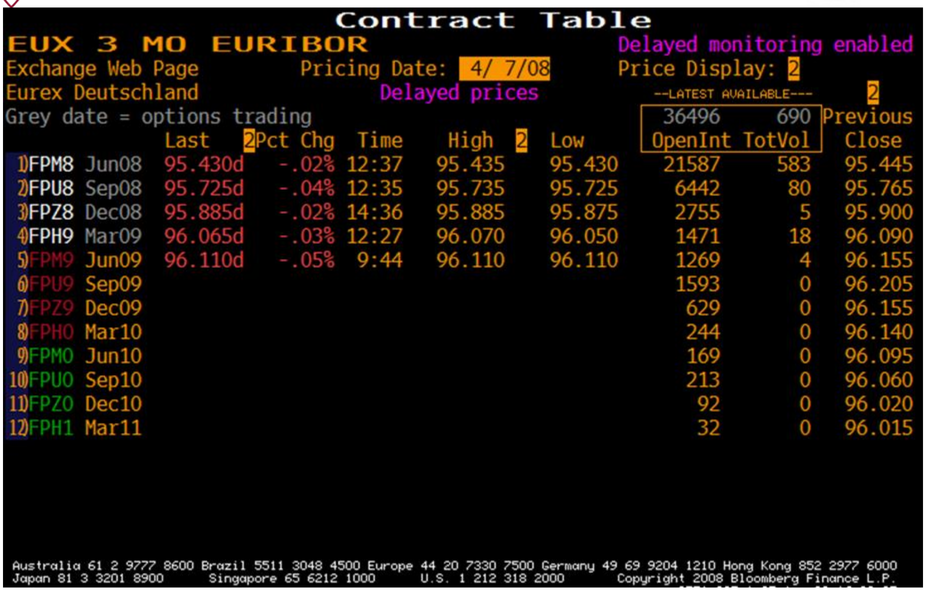
3-months Euribor future contract



Contract Table

EUX 3 MO EURIBOR — Delayed monitoring enabled

Exchange Web Page — Pricing Date: 4/ 7/08 — Price Display: 2

Eurex Deutschland — Delayed prices — --LATEST AVAILABLE--- 2

Grey date = options trading — 36496 690

		Last	Pct Chg	Time	High	Low	OpenInt	TotVol	Previous Close
1) FPM8	Jun08	95.430d	-.02%	12:37	95.435	95.430	21587	583	95.445
2) FPU8	Sep08	95.725d	-.04%	12:35	95.735	95.725	6442	80	95.765
3) FPZ8	Dec08	95.885d	-.02%	14:36	95.885	95.875	2755	5	95.900
4) FPH9	Mar09	96.065d	-.03%	12:27	96.070	96.050	1471	18	96.090
5) FPM9	Jun09	96.110d	-.05%	9:44	96.110	96.110	1269	4	96.155
6) FPU9	Sep09						1593	0	96.205
7) FPZ9	Dec09						629	0	96.155
8) FPH0	Mar10						244	0	96.140
9) FPM0	Jun10						169	0	96.095
10) FPU0	Sep10						213	0	96.060
11) FPZ0	Dec10						92	0	96.020
12) FPH1	Mar11						32	0	96.015

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Hedging with 3-months Euribor futures contracts

Risk expousure:

	Risk of an increase in interest rate	Risk of a decrease in interest rate
Need to finance in the future	×	
Excess cash in the future		×
Bonds indexed to Euribor rate:		
- Issuer	×	
- Investors		×

Hedging with 3-months Euribor futures contracts

- Types of hedging:

  - Risk of an increase in interest rate ⇒ Sell Futures (*short hedge*)

  - Risk of a decrease in interest rate ⇒ Buy Futures (*long hedge*)

- Substitution of risks:

  Interest rate risk ⇒
  - Indivisibility risk
  - Basis Risk
  - Correlation risk
  - Spread risk

Hedging with 3-months Euribor futures contracts

- Fixing a loan rate

Example 1:

On 7/04, a firm knows that it will need 50 millions euros on 16/06 (last trading day of futures contracts) for 90 days. The loan rate is 3-months Euribor + 0,50%.

Cash-Euribor 3 Month: 4,742%

3-Months Euribor future (Junho): 95,43 (implied rate: 4,57%)

Risk of an increase in interest rates ⇒ **Sell futures**

Number of contracts to sell*: $\frac{50\ 000\ 000}{1\ 000\ 000} = 50$

* Without considering the *tailing*

Hedging with 3-months Euribor futures contracts

Scenario 1 On 16/06: 3-months Euribor = 4%

Futures = 96,00

Interest on the loan
(rate: 4,00%+0,50% = 4,50%): $50\ 000\ 000(0,045)\frac{90}{360} = 562\ 500$

Result on futures position: $\underbrace{\left(\frac{95,43 - 96,00}{0,005}\right)}_{\Delta \text{ ticks}} \underbrace{12,5}_{\text{Tick value}} \underbrace{(50)}_{\#\text{ of contracts}} = -71\ 250$

Result on futures positions
(capitalized at Euribor rate): $-71\ 250\left(1 + 0,04\frac{90}{360}\right) = -71\ 962,5$

Net financing costs: $562\ 500 + 71\ 962,5 = 634\ 462,5$

Net financing rate = $\frac{634\ 462,5}{50\ 000\ 000}\left(\frac{360}{90}\right) = 5,076\%$

Hedging with 3-months Euribor futures contracts

On 16/06:

	Scenario 1 Decrease in rate	Scenario 2	Scenario 3 Rise in rate
Euribor 3 m.	4,00%	4,74%	5,50%
Futures Euribor 3 m.	96,00	95,26	94,50
Financing rate	4,50%	5,242%	6,00%
Interest	562 500 €	655 250 €	750 000€
Results on futures*	-71 962,50 €	21 754,88 €	117 848,44 €
Net financing costs	634 462,50 €	633 495,12 €	632 151,56 €
Net financing rate	5,076%	5,068%	5,057%

* Capitalized for 90 days at Euribor 3 m rate

- The hedging with futures allowed to fix on 7/04 a value (the implied rate on the future price) for the 3 –months euribor rate on 16/06.

Hedging with 3-months Euribor futures contracts

- Fixing the Financing rate (cont.)

⇨ When the date of the future loan doesn't coincide with the future's last trading day

⇓

Basis Risk

(financing rate obtained ≈ initial financing rate (cash) + basis change)

(Basis = Future price – (100- Euribor 3M))

⇨ When the financing rate is related to other than 3-months Euribor reference rate

⇓

Correlation risk

(financing rate obtained ≈ initial financing rate (cash) + basis change + change in the difference between the financing rate and the 3-month euribor rate)

Hedging with 3-months Euribor futures contracts

- Fixing the lending rate

↳ Buy futures contracts

⇨ If the future lending date coincides with the future's last trading day :

Lending rate obtained ≈ futures implied rate – spread between Euribor 3M and the lending rate on the last trading day

⇨ If the future lending date doesn't coincide with the future's last trading day :

Lending rate obtained ≈ initial lending rate (cash) + basis change – change in spread (Euribor 3M - lending rate)

forward bid-ask rates

Remember that:

Borrowing for a short period (D_s days) + lending for a long period (D_L days)

Allows to fix today the rate for a future lending starting in D_s days from now, for a period of ($D_L - D_s$) days

(*forward bid rate*)

Lending for a short period (D_s days) + borrowing for a long period (D_L days)

Allows to fix today the rate for a future borrowing starting in Ds days from now, for a period of (DL – Ds) days

(*forward* ask rate)

Forward bid-ask rates

forward bid rate

$$= \left[\frac{1 + R_L^{bid} \dfrac{D_L}{360}}{1 + R_S^{ask} \dfrac{D_C}{360}} - 1 \right] \times \frac{360}{D_L - D_S}$$

forward ask rate

$$= \left[\frac{1 + R_L^{ask} \dfrac{D_L}{360}}{1 + R_S^{bid} \dfrac{D_C}{360}} - 1 \right] \times \frac{360}{D_L - D_S}$$

R_L^{bid} - bid rate for the long period

R_S^{bid} - bid rate for the short period

R_L^{ask} - ask rate for the long period

R_s^{ask} - ask rate for the short period

D_S - # of days in the short period

D_L - # of days in the long period

Forward bid-ask rates

Example:

Term	Euribid	Euribor
3 months (90d)	4,65%	4,75%
6 months (180d)	4,50%	4,60%

$$forward\ bid\ rate = \left[\frac{1+0,0450\frac{180}{360}}{1+0,0475\frac{90}{360}}-1\right]\times\frac{360}{180-90}=0,04200$$

$$forward\ ask\ rate = \left[\frac{1+0,0460\frac{180}{360}}{1+0,0465\frac{90}{360}}-1\right]\times\frac{360}{180-90}=0,044977$$

Arbitrage with 3-months Euribor futures

Compare the *forwards* rates obtained through the money market cash rates with the implied rates of the future's price

Relevant dates:

Moment 0 – Position opening in the futures market and the interbank market

Moment T – Futures contract delivery date

Moment T+ 90 – lending/borrowing settlement

Cash and Carry Arbitrage

Conditions to implement: overvalued future contract

Future price Implied rate < *forward bid rate*

Strategy:

- Borrow in the money market for T days (short period: (0;T))
- Sell ("expensive") futures contracts, with delivery in T days, ensuring the borrowing roll-over rate for 90 days
- Lend in the money market for T+90 days (long periord: (0;T+90))

Cash and Carry Arbitrage

Example:

Term	Euribid	Euribor
3 months (90d)	4,65%	4,75%
6 months (180d)	4,50%	4,60%

forward bid rate = 4,2%

Future (delivry in 90 days): 95,98

Future price implied rate (4,02%) < forward bid rate (4,20%)

Cash & carry arbitrage opportunity

Carry out the strategy considering 20 000 000 € of reference value.

Cash and Carry Arbitrage

Example (cont.):

Moment 0:

- Borrow for 90 days (rate: 4,75%) so as to liquidate 20.000.000€ at the end :

$$19\ 765\ 287{,}21 = \frac{20\ 000\ 000}{\left(1 + 0{,}0475\frac{90}{360}\right)}$$

- Lend for 180 days (4,5%): - 19 765 287,21

the final value will be: $19\ 765\ 287{,}21\left(1 + 0{,}045\frac{180}{360}\right) = 20\ 210\ 006{,}18$

- Sell futures contracts: $N = \frac{20\ 000\ 000}{1\ 000\ 000} = 20$

Cash and Carry Arbitrage

Example (cont.):

Moment T:

	Scenario 1 Euribor 3m = 4,00%	Scenario 2 Euribor 3m = 4,75%	Scenario 3 Euribor 3m = 5,50%
Second borrowing for 90 days*	20 009 907,11	19 972 828,83	19 935 887,72
maturity of first borrowing	-20 000 000,00	-20 000 000,00	-20 000 000,00
Result in futures	-1 000,00	36 500,00	74 000,00
Arbitrage gain	8 907,11	9 328,83	9 887,72

*The amount of the second borrowing is such that its maturity value matches the final value of the initial lending, (this way the arbitrage gain is available on T):

$$20\ 009\ 907{,}11 = \frac{20\ 210\ 006{,}18}{\left(1 + 0{,}04\frac{90}{360}\right)}$$

Reverse Cash and Carry Arbitrage

Conditions to implement: undervalued future contract

Future prices implied rate > **forward ask rate + bid-ask spread expected for the last trading day**

Strategy:

- Lend in the money market for T days (0;T)
- Buy ("cheap") futures, with delivery in T days, "ensuring" the lending roll-over rate for 90 days
- Borrow in the money market for T+90 days (0;T+90)

1.3 Fra – *Forward Rate Agreement*

Definition

FRA – is an agreement between two parties whereby:

- the seller of FRA agrees to pay floating interest rate and receive a fixed interest rate;
- the buyer of FRA agrees to pay the fixed interest rate and receive the floating interest;
- on an agreed notional amount
- over an agreed **forward period**

Buying Fras allows to fix a borrowing rate (to hedge against rising interest rates)

Selling Fra allows to fix a lending rate (to hedge against decreasing interest rates)

Fra - characteristics

- definition of two periods

  - "waiting period": period of time between today and the starting of the forward period – y months

  - "contract period": length of the forward period (term of the loan/deposit) - z months

  $Fra_{yx(y+z)}$: Fra_{1x7}: 6 months loan/deposit starting 1 month from now

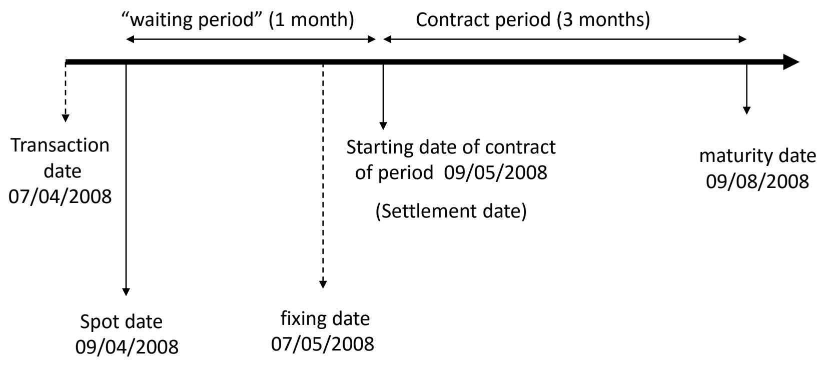
- Definition of two rates

  - the fixed rate (Fra rate)

  - the reference rate (exp: Euribor, Libor,…)

Fra - characteristics

Example: On 07/04/2008 an Fra_{1x4} was transacted :



FRA - Characteristics

- Interest difference:

$$N\left(r_{ref} - r_{Fra}\right)\frac{n}{360}$$

r_{ref} - reference rate

r_{Fra} - Fra rate

n - # of days of contract period

N - notional amount

- Settlement amount:

$$\frac{N\left(r_{ref} - r_{Fra}\right)\frac{n}{360}}{1 + r_{ref}\frac{n}{360}}$$

FRA - characteristics

If:

- $r_{ref} > r_{Fra}$ – the seller pays to the buyer the settlement amount
- $r_{ref} < r_{Fra}$ – The buyer pays to the seller the settlement amount

Example:

On 07/04/2008, a firm bought an Fra_{1x4} :

- Fra rate: 4,5%
- Reference rate: Euribor 3m
- Notional amount: 10 millions €

On 07/05/2008, the Euribor 3m rate is 4,15%, so the firm will pay to the bank:

$$\frac{10\ 000\ 000\left(0,045 - 0,0415\right)\frac{92}{360}}{1 + 0,0415\frac{92}{360}} = 8\ 850,58$$

FRA vs Futures

The 3-months Euribor future contract is equivalent to an FRA (with a contract period of 3 months), with daily mark to market.

Quotation:

		Bid	Ask
FRA : Interest rate	(exp.	4,25	4,28)
Future: 100 – implied rate	(exp.	95,54	95,555)

Hedging interest rate risk

Risk of...	Decrease	Increase
Futures	Buy	Sell
FRAs	Sell	buy

Futures vs FRAs Arbitrage

Implementation condition		Strategy
FRA bid rate > Bid future price implied rate	Future contract overvalued (relative to FRA)	Sell FRA Sell Futures
Fra ask rate < Ask future price implied rate	Future contract undervalued (relative to FRA)	Buy FRA Buy Futures

Futures vs FRAs Arbitrage

Example:

ON 16/04 the quotes of FRAs and 3-months euribor futures contracts for June were:

	Bid	*Ask*
FRA $_{2x5}$*	4,44%	4,47%
3-months Euribor futures	95,46	95,48

FRA *ask rate* (4,47%)

<

ask future price implied rate(4,52%)

Arbitrage opportunity:

buy FRA

buy Futures

*the reference rate is 3-months Euribor

Futures vs FRAs Arbitrage

Relevant dates:

Future last trading day:	16/06/2008	
FRA fixing date:	16/06/2008	
Futures settlement date:	17/06/2008	
FRA settlement date	18/06/2008	92 days
FRA maturity date	18/09/2008	

Considering an FRA notional amount of 20 millions, how many futures contracts should be transactd?

- Impact of 0,5 bp change on 3 months euribor rate:

- FRA value: $\dfrac{20\ 000\ 000(0{,}00005)\dfrac{92}{360}}{1+0{,}0453\dfrac{92}{360}} = 252{,}631$

- Value of 1 future contract: 12,5 (tick size)

\# of contracts to transact: $N = \dfrac{252{,}631}{12{,}5} = 20{,}21$ (20 contratos)

Futures vs FRAs Arbitrage

	Scenario 1 Euribor = 4,00% Futures = 96,00	Scenario 2 Euribor = 4,50% Futures = 95,50	Scenario 3 Euribor = 5,00% Futures = 95,00
FRA settlement	- 23 779,15	1 515,90	26 747,12
Result on futures	26 000,00	1 000,00	- 24 000,00
Total	2 220,85	2 515,90	2 747,12