FIXED INCOME PRODUCTS AND MARKETS

III – Fixed Income Derivatives and Models

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III – Fixed Income Derivatives and Models

2. Bond Futures

2.0 Bond Futures

Underlying assets: notional bond issued by the treasury with a given maturity

Contracts:

- Euro Bund (10 Y notional)
- Euro Bobl (5 Y notional)
- Euro Schatz (2 Y notional)

German Treasury

- Long Gilt (10 Y notional)

UK Treasury

- 30-Years US Treasury Bonds
- 10-Years US Treasury Notes
- 5 Years US Treasury Notes
- 2 Years US Treasury Notes
- …

US Treasury

16:24 **WORLD BOND FUTURES**

P Price/Yield

1) North/Latin America		Exch	Symbol	Last	Change	Trade Time	High	Low
4) US LONG BOND(CBT)	Jun08	CBT	USM8	118-27	- 18+	16:14	119-24+	118-23+
5) US 10YR NOTE FUT	Jun08	CBT	TYM8	117-24+	- 13+	16:14	118-14	117-22+
6) US 5YR NOTE (CBT)	Jun08	CBT	FVM8	113-21¾	- 05+	16:14	114-01¾	113-20¼
7) US 2YR NOTE (CBT)	Jun08	CBT	TUM8	107-05+	- 01¾	16:14	107-10¾	107-05
8) CAN 10YR BOND FUT	Jun08	MSE	CNM8	118.38	-.37	16:04	118.89	118.31
9) GLOBAL 2040 BOND	Jul08	BMF	BGN8	133.781y	n.a.	4/09		
2) Europe/Africa								
10) EURO-BUND FUTURE	Jun08	EUX	RXM8	115.40	+.49	16:09	115.72	115.14
11) EURO BUXL 30Y BND	Jun08	EUX	UBM8	91.42d	+.70	16:08	91.84	91.18
12) LONG GILT FUTURE	Jun08	LIF	G M8	110.24	+.40	16:09	110.72	109.91
13) EURO-BOBL FUTURE	Jun08	EUX	OEM8	110.025	+.295	16:09	110.175	109.865
14) EURO-SCHATZ FUT	Jun08	EUX	DUM8	104.400	+.115	16:09	104.470	104.320
15) SWISS FED BND FUT	Jun08	EUZ	FBM8	124.60s	+.36	Close	124.85	124.52
16) SPANISH 10YR FUTR	Jun08	MFM	NTM8	9806.00y	n.a.	4/09		
17) SWEDISH 5YR FUTR	Jun08	PMI	SKM8	3.835d	-.050	15:07	3.895	3.822
3) Asia/Pacific								
18) JPN 10Y BOND(TSE)	Jun08	TSE	JBM8	140.04d	-.04	10:00	140.07	139.94
19) KOREA 3YR BND FUT	Jun08	KFE	KEM8	108.47s	+.44	Close	108.47	107.95
20) AUST 10Y BOND FUT	Jun08	SFE	XMM8	93.905d	-.010	16:13	93.965	93.895
21) AUST 3YR BOND FUT	Jun08	SFE	YMM8	93.835d	-.005	15:51	93.885	93.825

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Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000
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2.1 Euro Bund Futures

Contract Specifications (Eurex):

Trading unit (contract size):

Notional bond issued by the Federal Republic of Germany

Maturity: 10 years

Coupon rate: 6%

Principal: 100 000 EUR

(Eligible bonds to delivery: Bunds with a remaining maturity, on the delivery date, of 8,5 years to 10,5 years)

Quotation:

as a percentage of the nominal value

Tick size

0,01% (tick value = 10 EUR)

Delivery months

The three nearest quarterly months of the March, June, September and December cycle.

Contract Specifications (cont.):

Delivery date

The tenth calendar day of the respective quarterly month, if this day is an exchange day; otherwise, the exchange day immediately succeeding that day.

Last trading day

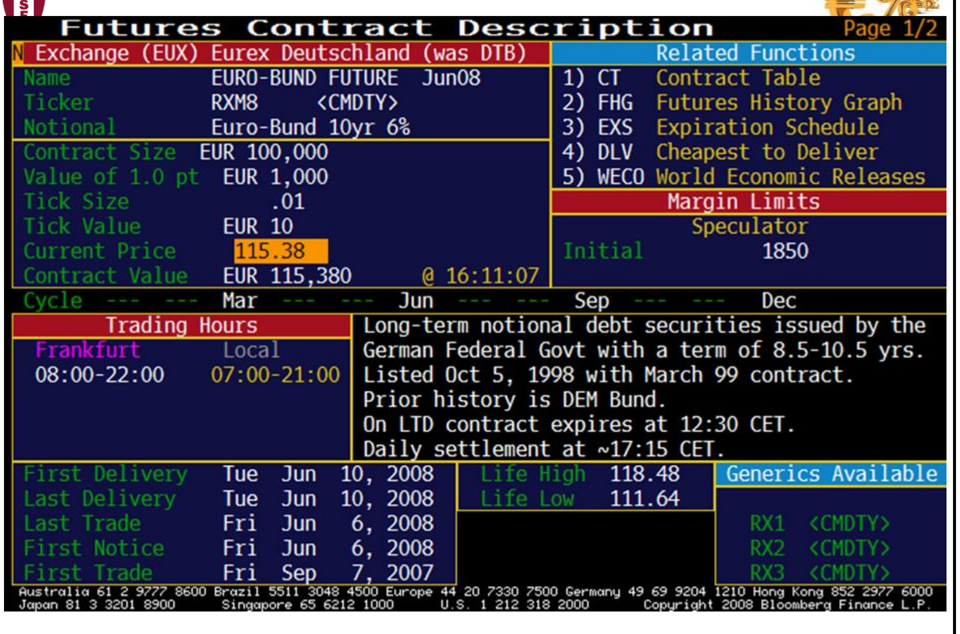
Two exchange days prior to the Delivery Day of the relevant maturity month.

Settlement

Physical settlement (deliverable bonds list: Bunds with a remaining maturity between 8,5 and 10,5 years and with a minimum issue amount of EUR 5 billion)

Deliverable bonds list on April 2008, for June 2008 delivery month future contract:

- Bund 3,75% Jan. 2017
- Bund 4,25% Jul. 2017
- Bund 4,00% Jan. 2018



Settlement

- The seller of the contract selects the bond to deliver from the deliverable bonds list
- Delivers EUR 100 000 of principal of the bond per contract
- Amount received by the seller (short position) and paid by the buyer (long position) per contract: the invoice price

$$[\ PF \times CF + AI\ ] \times 100\ 000$$

PF – Final settlement price of the future contract (in %)
CF - Conversion factor of the delivered bond
AI - Accrued interest (in %) (convention: Act/Act)

Settlement

Example: Settlement of March 08 Euro Bund contract (10/03/2008)

PF = 117,40

Deliverable bonds:

Bonds	Conversion Factors
Bund 3,75% Jan. 2017	0,849146
Bund 4,25% Jul. 2017	0,877457
Bund 4,00% Jan. 2018	0,854343

Suppose the seller delivers the Bund 3,75% Jan. 2017:

Last coupon date	04/01/2008	66 days (Act/Act)
Delivery date (futures):	10/03/2008	
Next coupon date:	04/01/2009	

$$AI = 3,75\% \frac{66}{366} = 0,67623\%$$

Invoice price (per contract):

$$\left[117,40\% \times 0,849146 + 0,67623\%\right] \times 100\ 000 = 100\ 365,97$$

2.2 Conversion Factors

Deliverable bond list: Bund with 8,5 to 10,5 years of remaining maturity, from the delivery date and with a minimum issue amount of EUR 5 billion

For June 2008 contract, the deliverable bond are:

Bund 3,75% Jan. 2017
Bund 4,25% Jul. 2017
Bund 4,00% Jan. 2018

EURO-BUND FUTURE Jun08 RXM8 115.36

Order DR re-sort? Y	(Mid) Price	Source	Conv. Yield	C.Factor
1) DBR 3 ¾ 01/04/17	98.240	BGN	3.991	.852348
2) DBR 4 ¼ 07/04/17	102.020	BGN	3.982	.880218
3) DBR 4 01/04/18	100.060	BGN	3.989	.857079

RXM6 Comdty | 97) Export to Excel | 98) Settings

EURO-BUND FUTURE Jun16 Price 163.48 Trade Settle

Sort By

Implied Repo | Decreasing

	Cash Security	Price	Source	Conven Yield	Conver Factor
	Adjust Value				
1)	DBR 0 ½ 02/15/25	104.1600	BGN	0.0278	0.635989
2)	DBR 1 08/15/25	108.4100	BGN	0.0932	0.654708
3)	DBR 0 ½ 02/15/26	103.3100	BGN	0.1601	0.604688

The deliverable bonds have:

- different maturities
- different coupon rates

It makes no sense to deliver different bonds and receive the same amount

Need for a method that transforms the price of the futures contract into an equivalent price for each Bond, and vice versa

For each deliverable bond, of a given delivery month, there exist a **conversion factor** that homogenize the value of the different bonds

The bond **Conversion factor**, for a given delivery month, is:

The clean price of the bond on the delivery date, assuming a *yield-to-maturity* equal to the contract coupon rate (6,0%)

Bond clean price:

Bund 4,00% Jan. 2018 $_{(\text{YTM} = 6{,}0\%)}$ = 85,7079

CF = 0,857079

Example:

Conversion factor calculation for the bond: Bund 4,00% Jan. 2018, for June 2008 delivery month: 10/06/2008

Coupon rate	4,00%
Maturity	4/01/2018
Last coupon date / Issue date	16/11/2007
Next coupon date	4/01/2009
Number of coupons	10

Long first coupon:

$$\left.\begin{array}{l} 16/11/2007 \\ 04/01/2008 \\ 04/01/2009 \end{array}\right\} \quad \left.\begin{array}{l} 4\% \dfrac{49}{365} = 0{,}5370\% \\ 4\% \end{array}\right\} \quad 4{,}5370\%$$

Date: 10/06/2008

Dates	t_i	Discount factor	Cash-flows	Discounted Cash-flows
04-01-2009	0,568306	0,967428	4,5370%	4,3892%
04-01-2010	1,568306	0,912668	4%	3,6507%
04-01-2011	2,568306	0,861007	4%	3,4440%
04-01-2012	3,568306	0,812271	4%	3,2491%
04-01-2013	4,568306	0,766293	4%	3,0652%
04-01-2014	5,568306	0,722918	4%	2,8917%
04-01-2015	6,568306	0,681998	4%	2,7280%
04-01-2016	7,568306	0,643395	4%	2,5736%
04-01-2017	8,568306	0,606976	4%	2,4279%
04-01-2018	9,568306	0,572619	104%	59,5524%
			Total	**87,9717%**

Accrued Interest:

04/01/2007 16/11/2007 04/01/2008 10/06/2008 04/01/2009

49 days 158 days

365 days 366 days

$$AI = \left(\frac{49}{365} + \frac{158}{366}\right) \times 4\% = 2,2638\%$$

Price (clean) = 87,9717% -2,2638% = 85,7079%

Conversion Factor	**0,857079**

Cheapest to Delivery (CTD) Bond

On the delivery date, the seller will choose the bond, within the deliverable bond list, to deliver for settlement. What bond?

Cheapest to delivery

The bond that maximizes the difference between the amount received from the contract settlement and the bond's acquisition cost

$$\underset{i}{\text{Máx}}\left[PF \times CF_i + AI_i - (P_i + AI_i)\right] \Leftrightarrow \underset{i}{\min}\left[\frac{P_i}{CF_i}\right]$$

Example: on 5/03/2008 we observe the march 2008 (10/03/2008) deliverable bond's prices.

PF = 117,40

Deliverable bonds	Price	CF	$PFxFC_i$ - P_i	P_i/FC_i	
Bund 3,75% Jan. 2017	99,715	0,849146	-0,025	117,43	← CTD
Bund 4,25% Jul. 2017	103,465	0,877457	-0,452	117,91	
Bund 4,00% Jan. 2018	101,326	0,854343	-1,026	118,60	

2.3 Contract valuation

Forward price vs Future Equivalent price

Buying forward the bond ⇔ Buy spot with borrowed money

$$\text{Forward price} = \text{Spot price} + \underbrace{\text{Cost of carry}}$$

(financial cost – income received from holding the bond) t = 0 (Spot settlement date) → t = T (future contract delivery date)

Forward (clean) price (considering up to 1 coupon payment until the future contract delivery date):

$$P_{forward} = P_{spot} + \underbrace{AI_0 + (P_{spot} + AI_0)\, r_0 \left(\frac{T}{360}\right) - AI_T - X_1 C\left(1 + r_1 \frac{T - T_1}{360}\right)}_{\text{Net financial cost}}$$

$P_{forward}$ – Forward (clean) price

P_{spot} – Spot (clean) price (on the transaction date)

AI_0 – Accrued interest, on the spot settlement date (t=0)

r_0 – borrowing rate for T days

AI_T –Accrued interest, on the delivery date (t=T)

C – coupon

X_1 – Binary variable which assumes the value of 1 if a coupon payment is made within T_1 days (T_1<T), and 0 otherwise

r_1 - coupon reinvestment rate for $T\text{-}T_{i\ days}$

Future Equivalent price (PF_e):

Convert the forward price of the bond into an equivalent future price using the respective conversion factor:

$$PF_i^e = \frac{P_{forward,i}}{CF_i}$$

Implied repo rate: the return that is achieved through the acquisition of the bond in the cash market and simultaneously selling it through the future contract. Corresponds to the (annual) rate **r** that solves the following equation (considering up to 1 coupon payment until the delivery date of the contract)

$$(P_{spot} + AI_0)\left(1 + r\frac{T}{360}\right) = PF(CF) + AI_T + X_1 C\left(1 + r_1 \frac{T - T_1}{360}\right)$$

$$r = \left[\frac{PF(CF) + AI_T + X_1 C\left(1 + r_1 \frac{T - T_1}{360}\right)}{(P_{spot} + AI_0)} - 1\right]\frac{360}{T}$$

Note: the *implied repo rate may also be calculated* (e.g. Bloomberg) as:

$$\left(P_{spot}+AI_0\right)\left(1+r\frac{T}{360}\right)=PF\left(CF\right)+AI_T+X_1C\left(1+r\frac{T-T_1}{360}\right)$$

$$r=\left[\frac{PF\left(CF\right)+AI_T-\left(P_{spot}+AI_0\right)+X_1C}{\left(P_{spot}+AI_0\right)\left(\frac{T}{360}\right)+X_1C\left(\frac{T-T_1}{360}\right)}\right]$$

(Note: when X_1=0, no coupon payments during the period, the results are the same)

Before the delivery date, the **Cheapest to Delivery** is the bond with:

- **the lower equivalent future price**
- **the highest *Implied repo rate***

Example: On 10 April (***settlement*: 15/04/2008**), the deliverable Bonds for the June 2008 Euro Bund future contract are:

	Bund 3,75% Jan. 2017	**Bund 4,25% Jul. 2017**	**Bund 4,00% Jul. 2018**
Coupon rate	3,75%	4,25%	4,00%
Last coupon date/issue date	04/01/2008	25/05/2007	16/11/2007
Next coupon date	04/01/2009	04/07/2008	04/01/2009
Accrued Interest* (year base)	102 (366)	286 (366) + 40 (365)	102 (366) + 49 (365)
Price	**98,24%**	**102,02%**	**100,06%**
Accrued Interest*	**1,045082%**	**3,786792%**	**1,651740%**
Conversion factor	**0,852348**	**0,880218**	**0,857079**

*at date 15/04/2008

Number of days until the future's delivery date (10/06/2008): 56 days

Financing rate: 4,52%

Future equivalent price for Bund 3,75% Jan. 2017:

Acquisition cost (gross price): $98,24\% + 1,045082\% = 99,285082\%$

Financial cost: $99,285082\%\,(4,52\%)\left(\frac{56}{360}\right) = 0,698084\%$

Accrued interest (delivery date): $3,75\%\left(\frac{102+56}{366}\right) = 1,618852\%$

Forward price: $99,285082\% + 0,698084\% - 1,618852\% = 98,364314\%$

Future equivalent price: $\frac{98,364314\%}{0,852348} = 115,404\%$

Note: Cost of carry = $1,045082\% + 0,698084\% - 1,618852\% = 0,124314\%$

Implied repo rate calculation for Bund 3,75% Jan. 2017, considering that on 10/04/2007 the future was quoted at 115,36%:

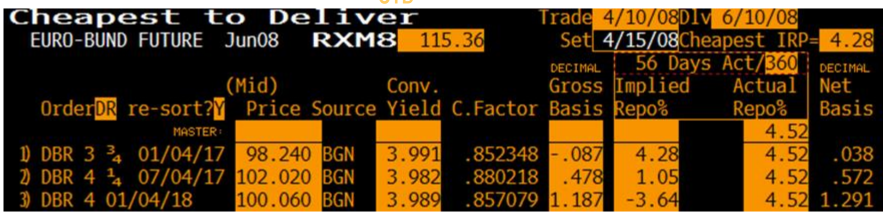
Since there isn't any coupon payment betwen 10/04 e 10/06, the *implied repo rate* will be given by:

$$r = \left[\frac{PF\,(FC) + JD_T}{(P_{vista} + JD_0)} - 1\right]\frac{360}{T}$$

$$r = \left[\frac{115,36\%\,(0,852348) + 1,618852\%}{(98,24\% + 1,045082\%)} - 1\right]\frac{360}{56} = 4,28\%$$

	Bund 3,75% Jan. 2017	Bund 4,25% Jul. 2017	Bund 4,00% Jul. 2018
Price	98,24%	102,02%	100,06%
Accrued interest (*settlement date*)	1,045082%	3,786792%	1,651740%
Financial cost	0,698084%	0,743939%	0,715147%
Accrued interest (delivery date)	1,618852%	4,437065%	2,263762%
Forward price	98,364314%	102,113666%	100,163125%
Conversion factor	0,852348	0,880218	0,857079
Future equivalent price	115,404%	116,010%	116,866%
Implied repo rate	4,28%	1,05%	-3,64%

CTD



Cheapest to Deliver Trade 4/10/08 Dlv 6/10/08
EURO-BUND FUTURE Jun08 RXM8 115.36 Set 4/15/08 Cheapest IRP= 4.28
56 Days Act/360

Order DR re-sort? Y	(Mid) Price	Source	Conv. Yield	C.Factor	DECIMAL Gross Basis	Implied Repo%	Actual Repo%	DECIMAL Net Basis
MASTER:							4.52	
1) DBR 3 ¾ 01/04/17	98.240	BGN	3.991	.852348	-.087	4.28	4.52	.038
2) DBR 4 ¼ 07/04/17	102.020	BGN	3.982	.880218	.478	1.05	4.52	.572
3) DBR 4 01/04/18	100.060	BGN	3.989	.857079	1.187	-3.64	4.52	1.291

Theoretical value of the Euro Bund future contract

Contract theoretical value = Min PF_i^e $\quad$ i = 1,2,...,m

m - # of Bonds of the deliverable list for a given delivery date

PF_i^e – future equivalent price of the i-th deliverable bond

Cheapest to delivery

(the future equivalent price of the bond that maximizes the cash & carry strategy result)

2.4 Arbitrage strategies

- Price distortions
- Does not require any expectations about future changes in interest rates
- Riskless strategies

⇨ Cash & Carry Arbitrage

⇨ Reverse Cash & Carry Arbitrage

Cash & Carry Arbitrage

⇨ Future contract overvalued

Conditions to implement:

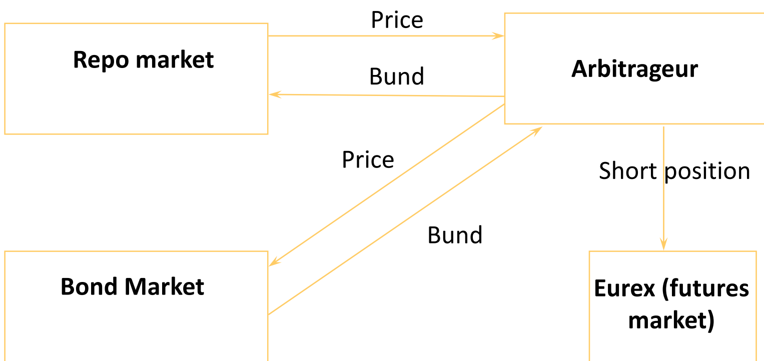
- Future price higher than the theoretical value
- *Implied repo rate* higher than the financing rate (repo rate)

Which bond to use?
To maximize the arbitrage gain
⇓
CTD

Strategy implementation:

Buying bonds (bund's) in the cash market
and
Selling futures contracts

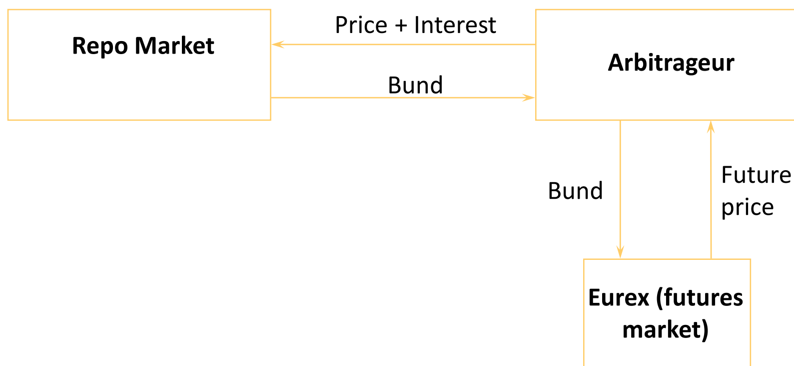
Start of operations (t=0)



Strategy implementation:

At the delivery date, deliver the bonds to settle the futures contracts, receiving the corresponding cash and liquidate the loan

Final operation (t = T)



Example: On April 10th (***settlement*: 15/04/2008**), the CTD bond is "Bund 3,75% Jan. 2017", and is quoted at 98,24%, while the price of the Euro Bund Future contract for June 2008 is 115,48%

	Bund 3,75% Jan. 2017
Price	98,24%
Accrued Interest (*settlement date*)	1,045082%
Accrued Interest (delivery date)	1,618852%
Repo rate	4,52%
Conversion factor	0,852348
Future equivalent price	115,404%
Implied repo rate	4,94%

Future price (115,48%) > Theoretical price (115,404%)

Implied repo rate (4,94%) > repo rate (4,52%)

Future contract is overvalued

Consider 1 000 000 € of nominal:

- Buy 1.000.000 € of nominal value of the CTD bond
- Sell 10 Euro Bund Futures contract (June): $\left(\frac{1\ 000\ 000}{100\ 000}=10\right)$
- Financing rate: 4,52%

Different scenarios for the Euro bund future contract price on the delivery date:

	Scenario 1	Scenario 2	Scenario 3
Future price	115,00	115,48	116,00

Scenario 1:

- Acquisition cost: $1\ 000\ 000(98,24\% + 1,045082\%) = 992\ 850,82$
- Financing cost: $992\ 850,82\left(4,52\%\ \frac{56}{360}\right) = 6\ 980,84$
- Margin adjustments: $\left(\frac{115,48 - 115,00}{0,01}\right) \times 10 \times 10 = 4\ 800$
- Future contract settlement:

$$\left[115,00\%(0,852348) + 1,618852\%\right] \times 10 \times 100\ 000 = 996\ 388,72$$

- Final Results:

$$996\ 388,72\ +\ 4\ 800\ -\ 6\ 980,84\ -\ 992\ 850,82 =\ 1\ 357,06$$

Different scenarios results :

	Scenario 1	Scenario 2	Scenario 3
Future price	115,00	115,48	116,00
Arbitrage result	1 357,06	648,33	-119,46

The results depend on the future price evolution – risk

Need to adjust the futures position by the conversion factor:

$$\frac{\text{NV cash position}}{\text{NV Euro Bund}} \times \text{Conversion Factor} = \frac{1\ 000\ 000}{100\ 000} \times 0,852348 = 8,52 \approx 9$$

At the initial date sell 9 contracts, and at the last trading day, sell 1 more contract:

	Scenario 1	Scenario 2	Scenario 3
Future price	115,00	115,48	116,00
Arbitrage Result	877,06	648,33	400,54

Reverse Cash & Carry Arbitrage

⇨ Future contract undervalued

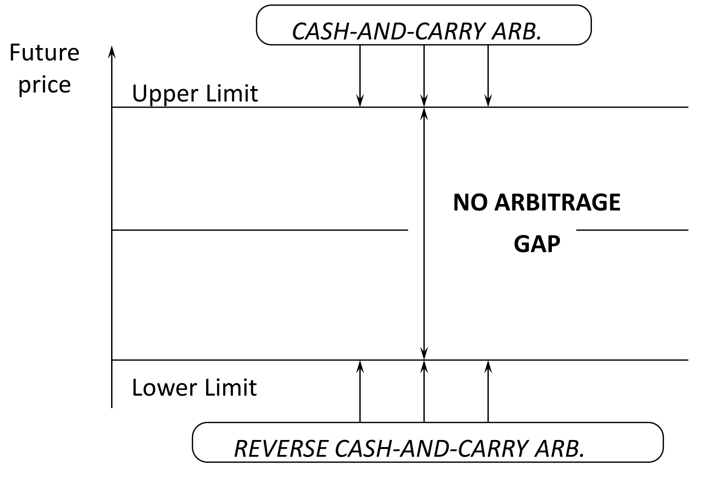
Conditions to implement:

- Future price lower than the theoretical value
- *Implied repo rate* lower than deposit rates

Strategy implementation:

Sell bonds (bund's) in the cash market
and
Buy Euro Bund futures contracts

Incorporating transaction costs ⇒ **No arbitrage gap**



2.5 Hedging

Futures contracts are low cost highly liquid instruments. They are frequently used for hedge purposes.

- *Duration hedge:*

Hedge ratio: $$\phi_f = -\frac{\$Dur_P}{\$Dur_B} \times CF$$

Where $\$Dur_P$ e $\$Dur_B$ correspond, respectively to the \$duration of the bond/portfolio to hedge and of the CTD bond of the future contract, being CF the respective conversion factor

Possibility to adjust the hedge ratio by the *yield* beta:

$$\Delta y_P = \alpha + \beta \Delta y_B + e$$

Example:

On 02/04/2008 an investor holds 2 000 000,00 € of NV of an OT 4,35% 16/10/2017 which has the following characteristics:

Price (%)	Coupon rate (%)	Yield (%)	\$Dur (%)
100,30	4,35	4,3080	-770,903

He wants to hedge his position using Euro bund futures contracts (June delivery)

The CTD bond is the Bund 3,75% 04/01/2017 which has the following characteristics:

Price (%)	Coupon Rate (%)	Yield (%)	\$Dur (%)	Conversion factor
98,20	3,75	3,9958	-719,210	0,852348

Future price: 115,18

Example:

How many contracts?

Hedge ratio: $$\phi = -\frac{2\ 000\ 000}{100\ 000} \times \frac{770,903}{719,210} \times 0,852348 = -18,27$$

Sell 18 futures contracts

Suppose that on 28/04/2008 the positions are closed:

	Price (%)	Yield (%)	Yield change
OT 4,35% 16/10/2017	98,53	4,5413	23,33 bp
Bund 3,75% 04/01/2017	96,75	4,2020	20,62 bp
Futures (June)	113,53		

Example:

Results on the futures: $$\underbrace{\left(\frac{115,18 - 113,53}{0,01}\right)}_{\Delta \text{ ticks}} \times \underbrace{10}_{\text{Tick value}} \times \underbrace{18}_{\text{\# of contracts}} = 29\ 700,00$$

Results analysis:

	02/04/2008		28/04/2008	
	Price (%)	Market value	Price (%)	Market value
OT 4,35% 16/10/2017	100,30	2 006 000,00	98,53	1 970 600,00
Result (without hedging)				**- 35 400,00**
Results on the futures				29 700,00
Result (with hedging)				**- 5 700,00**

Example:

The imperfect hedge is explained by:

- basis evolution (basis risk)

- Correlation risk

(equal changes in the YTM where assumed when the number of contracts was calculated)

- Indivisibility risk

Hedging:

- hedge a bond acquisition in the future
- Hedge a bond issue in the future
- Hedge the balance sheet structure in terms of *duration*
- …