FIXED INCOME PRODUCTS AND MARKETS

III – Fixed Income Derivatives and Models

José Azevedo Pereira and Sergio F. Silva

III – Fixed Income Derivatives and Models

1. Swaps, Fra's and Short Term Interest Rate Futures
2. Bond Futures
3. Credit Spread Dynamics
4. Bonds with embedded options and Bond Options
5. Futures Options, Caps, Floors and Swaptions
6. Exotic Options and Credit Derivatives

3. Credit Spread Dynamics

Credit Spread Dynamics

- Analyzing Credit Spreads
  - Ratings
  - Probability of Default
  - Severity of Default
- Modeling Credit Spreads
  - Structural Models
  - Reduced-Form Models

Corporate Bonds

- Bonds issued by a corporation
- 3 Sources of Risk
  - Interest Rate Risk
  - Default Risk
  - Liquidity Risk
- Bond indenture contracts stipulate collateral and specify terms
- Different “seniority” classes

Default Risk

EL = Expected loss = EADxDPxLGD

DP = *Default Probability*

EAD = *Exposure At Default*

LGD = *Loss Given Default*

LGD = 1 – *recovery rate*

Rating Agencies

Investment Grade (High Creditworthiness)		
Moody's	S&P	Definition
Aaa	AAA	Gilt-edged, best quality, extremely strong creditworthiness
Aa1 Aa2 Aa3	AA+ AA AA-	Very high grade, high quality, very strong creditworthiness
A1 A2 A3	A+ A A-	Upper medium grade, strong creditworthiness
Baa1 Baa2 Baa3	BBB+ BBB BBB-	Lower medium grade, adequate creditworthiness

Speculative Grade (Low Creditworthiness)		
Moody's	S&P	Definition
Ba1 Ba2 Ba3	BB+ BB BB-	Low grade, speculative, vulnerable to non-payment
B1 B2 B3	B+ B B-	Highly speculative, more vulnerable to non-payment
Caa	CCC+ CCC CCC-	Substantial risk, in poor standing, currently vulnerable to non-payment
Ca C	CC C D	May be in default, extremely speculative, currently highly vulnerable to non-payment Even more speculative Default

The modifiers 1, 2, 3 or +, - account for relative standing within the major rating categories.

Rating Agencies

- Default frequency / Cumulative default frequency
- Transition matrix
- Recovery rates

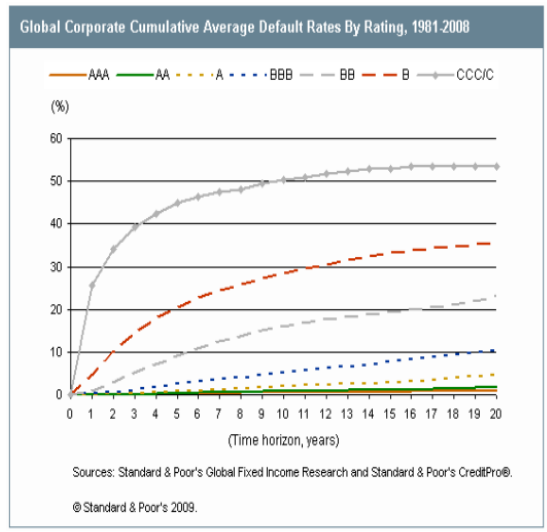
Descriptive Statistics On One-Year Global Default Rates

(%)	AAA	AA	A	BBB	BB	B	CCC/C
Minimum	0.00	0.00	0.00	0.00	0.00	0.24	0.00
Maximum	0.00	0.38	0.38	1.00	4.22	13.84	44.55
Weighted long-term average	0.00	0.03	0.08	0.24	0.99	4.51	25.67
Median	0.00	0.00	0.00	0.21	0.85	3.72	22.25
Standard deviation	0.00	0.08	0.11	0.27	1.08	3.17	12.15
2002 default rates	0.00	0.00	0.00	1.00	2.78	8.10	44.12
Latest four quarters (Q1 2008Q1 - Q4 2008)	0.00	0.38	0.38	0.47	0.76	3.82	26.53
Difference between last four quarters and average	0.00	0.35	0.30	0.23	(0.23)	(0.68)	0.86
number of standard deviations	0.00	4.57	2.63	0.86	(0.21)	(0.21)	0.07

Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

Rating Agencies

Cumulative default frequency:



Rating Agencies

Cumulative default frequency:

Average Cumulative Issuer-Weighted Global Default Rates, 1970-2008[1]

Rating	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Aaa	0.000	0.013	0.013	0.037	0.107	0.176	0.250	0.330	0.416	0.508
Aa	0.017	0.054	0.087	0.157	0.234	0.312	0.388	0.455	0.498	0.551
A	0.025	0.118	0.272	0.432	0.612	0.814	1.025	1.266	1.516	1.752
Baa	0.164	0.472	0.877	1.356	1.824	2.299	2.770	3.241	3.776	4.397
Ba	1.113	2.971	5.194	7.523	9.639	11.580	13.263	14.921	16.570	18.276
B	4.333	9.752	15.106	19.864	24.175	28.261	32.164	35.432	38.437	41.088
Caa-C	16.015	25.981	34.154	40.515	45.800	49.687	52.702	56.097	59.736	63.275
Inv-Grade	0.068	0.215	0.416	0.651	0.894	1.146	1.399	1.661	1.939	2.237
Spec-Grade	4.113	8.372	12.467	16.093	19.245	22.047	24.520	26.696	28.726	30.637
All Rated	1.401	2.844	4.193	5.360	6.344	7.197	7.938	8.594	9.211	9.802

Rating	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20
Aaa	0.608	0.714	0.824	0.887	0.955	1.029	1.110	1.139	1.139	1.139
Aa	0.612	0.712	0.850	0.979	1.074	1.208	1.381	1.565	1.866	2.194
A	2.012	2.266	2.529	2.786	3.111	3.462	3.873	4.300	4.702	5.102
Baa	5.047	5.749	6.502	7.281	8.009	8.778	9.492	10.166	10.740	11.303
Ba	20.042	21.900	23.676	25.443	27.220	28.951	30.585	32.024	33.512	34.845
B	43.531	45.968	48.093	50.226	52.190	53.770	54.640	55.336	55.690	56.101
Caa-C	65.727	65.840	66.809	67.953	68.873	70.486	70.922	70.922	70.922	70.922
Inv-Grade	2.552	2.886	3.245	3.602	3.966	4.359	4.771	5.175	5.563	5.952
Spec-Grade	32.477	34.314	36.017	37.704	39.343	40.884	42.205	43.348	44.473	45.498
All Rated	10.377	10.952	11.514	12.065	12.608	13.156	13.684	14.179	14.658	15.125

Corporate Default and Recovery Rates, 1920-2008, Moody's

Rating Agencies

Transition Matrix:

2008 One-Year Letter Rating Migration Rates[1]

From/To	Aaa	Aa	A	Baa	Ba	B	Caa	Ca-C	Default	WR
Aaa	84.066	10.440	0.000	0.549	0.000	0.000	0.000	0.000	0.000	4.945
Aa	0.126	80.252	13.459	0.377	0.252	0.000	0.126	0.126	0.503	4.780
A	0.000	1.371	87.258	4.758	0.161	0.081	0.081	0.000	0.323	5.968
Baa	0.000	0.176	2.109	85.940	4.306	0.264	0.176	0.000	0.439	6.591
Ba	0.000	0.000	0.169	4.237	75.254	9.153	1.017	1.356	1.017	7.797
B	0.000	0.000	0.166	0.166	2.566	71.440	14.238	0.993	1.904	8.526
Caa	0.000	0.000	0.000	0.000	0.000	3.210	67.407	8.642	12.099	8.642
Ca-C	0.000	0.000	0.000	0.000	0.000	0.000	5.263	26.316	52.632	15.789

Average One-Year Letter Rating Migration Rates, 1920-2008[1]

From/To	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	Default	WR
Aaa	87.365	7.487	0.845	0.176	0.032	0.001	0.001	0.000	0.000	4.092
Aa	1.243	84.949	6.623	0.697	0.165	0.036	0.007	0.004	0.068	6.208
A	0.082	3.000	84.972	5.304	0.671	0.105	0.025	0.008	0.078	5.755
Baa	0.042	0.291	4.576	81.239	5.000	0.780	0.146	0.016	0.285	7.624
Ba	0.007	0.082	0.483	5.897	73.790	6.683	0.570	0.058	1.320	11.109
B	0.007	0.052	0.159	0.605	5.952	71.791	5.106	0.498	3.769	12.062
Caa	0.000	0.026	0.039	0.231	0.785	8.133	64.080	3.552	11.438	11.717
Ca-C	0.000	0.000	0.114	0.000	0.447	3.232	7.822	55.109	20.445	12.831

Corporate Default and Recovery Rates, 1920-2008, Moody's

Rating Agencies

Transition Matrix:

Table 12

2008 Global Corporate Transition Rates (%)

From/To	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
AAA	81.82	6.06	3.03	0.00	0.00	1.01	2.02	0.00	6.06
AA	0.00	77.65	17.23	0.57	0.00	0.00	0.19	0.38	3.98
A	0.00	1.59	87.59	4.92	0.45	0.00	0.00	0.38	5.07
BBB	0.00	0.00	2.57	86.81	3.59	0.27	0.20	0.47	6.09
BB	0.00	0.09	0.00	4.94	77.21	8.26	1.04	0.76	7.69
B	0.00	0.00	0.00	0.14	3.68	73.16	8.08	3.82	11.11
CCC/C	0.00	0.00	0.00	0.00	0.00	11.22	41.84	26.53	20.41

Source: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

Rating Agencies

Transition Matrix:

Global Corporate Average Transition Rates, 1981-2008 (%)

From/To	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
Three yeara									
AAA	68.97	18.56	2.53	0.32	0.12	0.06	0.09	0.09	9.26
	(7.99)	(7.53)	(1.53)	(0.86)	(0.35)	(0.30)	(0.46)	(0.29)	(4.92)
AA	1.41	67.03	17.65	2.28	0.37	0.23	0.03	0.11	10.89
	(0.74)	(7.05)	(4.93)	(1.48)	(0.69)	(0.54)	(0.08)	(0.14)	(4.17)
A	0.09	4.83	67.24	11.85	1.47	0.59	0.11	0.31	13.51
	(0.12)	(2.31)	(6.24)	(2.98)	(1.17)	(0.86)	(0.17)	(0.27)	(3.79)
BBB	0.03	0.46	9.30	60.83	7.57	2.18	0.40	1.16	18.07
	(0.11)	(0.54)	(4.01)	(7.75)	(2.77)	(1.82)	(0.52)	(0.88)	(3.54)
BB	0.02	0.07	0.74	11.43	43.89	11.75	1.45	5.19	25.46
	(0.10)	(0.24)	(1.12)	(4.47)	(5.81)	(4.05)	(1.14)	(3.54)	(4.12)
B	0.01	0.06	0.39	1.27	11.61	37.87	4.46	15.00	29.34
	(0.13)	(0.17)	(0.82)	(0.94)	(3.68)	(6.19)	(2.45)	(7.17)	(6.23)
CCC/C	0.00	0.00	0.32	0.97	2.26	15.99	13.09	39.85	27.53
	(0.00)	(0.00)	(0.90)	(2.46)	(3.52)	(7.82)	(11.62)	(13.80)	(12.19)

Rating Agencies

Recovery rate:

Average Corporate Debt Recovery Rates Measured by Post-Default Trading Prices[1]

	Issuer-Weighted			Value-Weighted		
Lien Position	2008	2007	1982-2008	2008	2007	1982-2008
Bank Loans						
Sr. Secured	63.4%	68.6%	69.9%	49.0%	78.3%	62.1%
Second Lien	40.4%	65.9%	50.4%	36.6%	65.8%	49.8%
Sr. Unsecured[2]	29.8%	--	52.5%	22.6%	--	41.0%
Bonds						
Sr. Secured	58.0%	80.5%	52.3%	45.9%	81.7%	53.0%
Sr. Unsecured	33.8%	53.3%	36.4%	26.2%	56.9%	32.4%
Sr. Subordinated	23.0%	54.5%	31.7%	10.4%	67.7%	26.4%
Subordinated	23.6%	--	31.0%	7.3%	--	23.5%
Jr. Subordinated	--	--	24.0%	--	--	16.8%
Pref. Stock[3]						
Trust Pref.	--	--	11.7%	--	--	13.0%
Non-trust Pref.	8.6%	--	21.6%	1.7%	--	13.1%

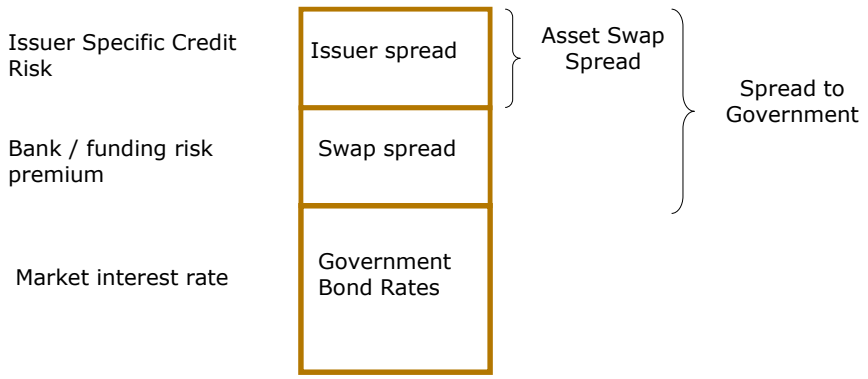
1. Based on 30-day post-default market prices.

2. 2008's average senior unsecured loan recoveries are based on three observations.

3. Only includes defaults on preferred stock that are associated or followed by a broader debt default. Average recovery rates for preferred stock only covers the period of 1983-2008.

Corporate - spreads

spread over a given ***benchmark***



Structural Models - Merton's Model

- Merton (1974)

Merton's model regards the equity as an option on the assets of the firm

Consider a firm with the following capital structure:

- Equity (E)
- Debt (B)
  - Zero coupon bond
  - Principal: F
  - Maturity: T

Value of the firm's assets: V

$$V_t = E_t + B_t$$

Structural Models - Merton's Model

- Merton (1974)

At maturity (t=T) two possible scenarios:

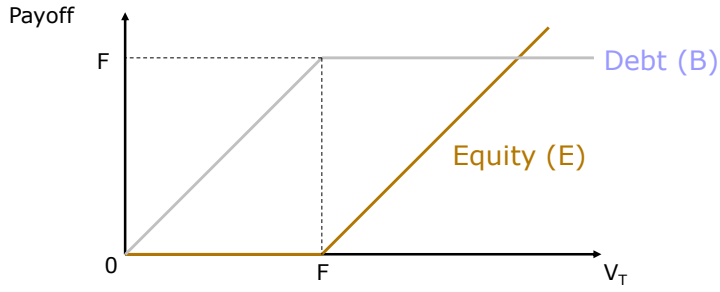
- $V_T < F$ –the value of the assets is not enough to pay the debt - default
- $V_T \geq F$ – no default

Maturity Payoff:

	Assets	Debt	Equity
No Default	$V_T \geq F$	F	$V_T - F$
Default	$V_T < F$	V_T	0

Structural Models - Merton's Model

- Merton (1974)



$E_T = Máx[V_T - F; 0]$ (call option on the assets of the firm)

$B_T = Min[F; V_T] = F - Máx[F - V_T; 0]$

(buy risk free zero coupon bond with face value of F and sell a put option on the assets of the firm)

Structural Models - Merton's Model

- Merton (1974)

Option terminology:

Call option (European style)	Equity value (E)
Underlying	Value of assets (V)
Strike price	Debt principal(F)
Expiration date	Debt maturity date (T)
Volatility	Asset value volatility

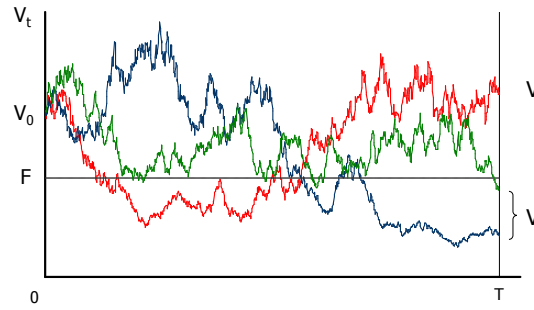
- Asset value stochastic process: GBM

$$dV_t/V_t = \mu dt + \sigma_V dW_t \qquad V_t = V_0 e^{\left(\left(\mu - \frac{\sigma_V^2}{2}\right)t + \sigma_V W_t\right)}$$

- Constant interest rate: r

Structural Models - Merton's Model

- Merton (1974)



$V_T \geq F$ (No Default)

$B_T = F$ e $E_T = V_T - F$

$V_T < F$ (Default)

$B_T = V_T$ e $E_T = 0$

Structural Models - Merton's Model

- Merton (1974)

$$E(V, t, T) = V_t N(d_1) - Fe^{-r(T-t)}N(d_2)$$

$$B(V, t, T) = Fe^{-r(T-t)}N(d_2) + V_t N(-d_1)$$

$$d_1 = \frac{\ln(V_t/F) + \left(r + \sigma_V^2/2\right)(T - t)}{\sigma_V \sqrt{T-t}} \qquad d_2 = d_1 - \sigma_V \sqrt{T-t}$$

Example: $V = 150$, $F = 100$, $r = 0{,}05$, $\sigma_V = 0{,}2$, $(T-t) = 5$

$d_1 = 1{,}689271$

$d_2 = 1{,}242058$

⇨ B=76,376

Structural Models - Merton's Model

- Merton (1974)

From the below equations and estimates for *E* and σ_E (from market prices of the stock and option on the stock), one may obtain *V* and σ_V

$$\sigma_E = (V/E)N(d_1)\sigma_V$$

$$E = V_t N(d_1) - Fe^{-r(T-t)}N(d_2)$$

Credit spreads:

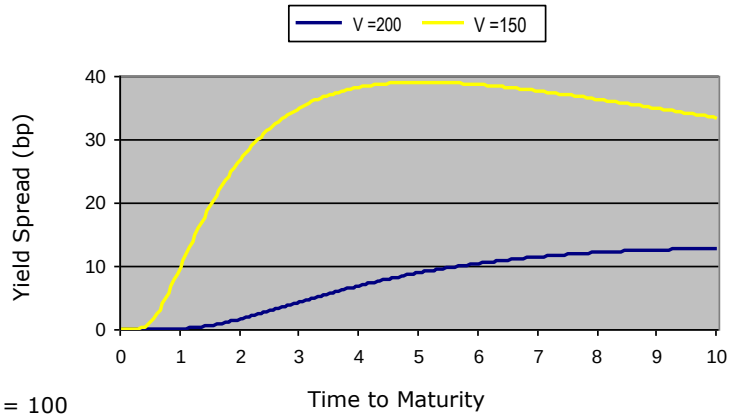
$$B(V, t, T) = e^{-y(t,T)(T-t)}F$$

$$\rightarrow y(t, T) = \frac{1}{T-t}(\ln F - \ln B(V, t, T))$$

$$s(t,T) = y(t,T) - r$$

Structural Models - Merton's Model

- Merton (1974)



F = 100

σ= 0,2

r = 0,05

Structural Models - Merton's Model

- Merton's interpretation of default is very narrow
  - Corporate bond is a zero-coupon
  - Default cannot occur before the debt matures
  - Focus on credit risk: interest rate is assumed to be constant
- Extensions
  - Geske (1977, 1979) extends the analysis to coupon bonds
  - Black and Cox (1976) extended Merton's model to cases where creditors can force the firm into bankruptcy at any time (when asset value falls below an exogenous threshold defined in the indenture)
  - Ramaswamy and Sundaresan (1993) and Briys and De Varenne (1997) introduce interest rate risk
  - See also Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001) for more recent models
- Empirical shortcoming of (most) structural models
  - Predicted spreads are too low
  - In particular, short-term spread is zero

Reduced –form models

- They don't try to explain why and how default occurs
- Just assume default occurs at a random time with instantaneous probability

$$\lambda = \lim_{dt \to 0} \frac{Pr\left(t < \tau \leq t + dt \middle| \tau > t\right)}{dt}$$

λ - intensity rate or hazard rate

τ - default time

Default probability until t (risk neutral) considering a constant intensity rate:

$$Pr(\tau \leq t) = Q(0,t) = 1 - e^{-\lambda t}$$

Reduced –form models

- Jarrow and Turnbull (1995): constant intensity; constant recovery rate (φ); revovey of treasury

Value of a defaultable zero coupon bond:

$$B^d\left(t,T\right) = B\left(t,T\right) - B\left(t,T\right)\left(1-\varphi\right)Q\left(t,T\right)$$

Yield spread: $$s\left(t,T\right) = \frac{1}{T-t}\ln\left(\frac{1}{\varphi + \left(1-\varphi\right)e^{-\lambda\left(T-t\right)}}\right)$$

With $\varphi = 0$: $$s\left(t,T\right) = \lambda$$

Reduced –form models

- Duffie and Singleton (1999): recovey of market (φ =1-L)

Value of a defaultable zero coupon bond:

$$B^d(t,T) = E\left[e^{-\int_t^T (r_s + L\lambda_s)ds}\right]$$

If the riskless rate process is independent of the default process:

$$B^d(t,T) = B(t,T)E\left[e^{-\int_t^T L\lambda_s ds}\right]$$

Yield spread (with constant intensity):

$$s(t,T) = L\lambda = (1-\varphi)\lambda$$