FIXED INCOME PRODUCTS AND MARKETS

III – Fixed Income Derivatives and Models

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4. Bonds with embedded options and Bond Options

- Callable Bonds and Putable Bonds

- Options on Bonds

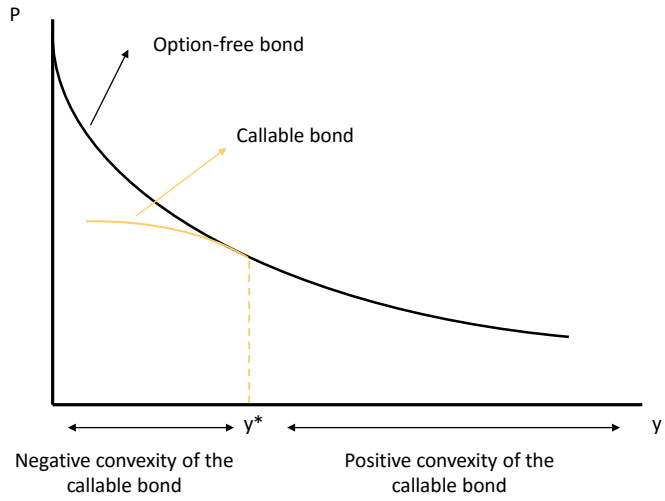
- Convertible Bonds

4.1 Callable Bonds and Putable Bonds

- **Callable bonds**
  - Issuer may repurchase at a pre-specified call price
  - Typically called if interest rates fall

- **Disadvantages for an investor**
  - If it is effectively called, the investor will have to invest in another bond yielding a lower rate
  - A callable bond has the unpleasant property for an investor to appreciate less than a normal similar bond when interest rates fall (negative convexity

Therefore, an investor will be willing to buy such a bond at a lower price than a comparable option-free bond



Callable bond = *option-free bond* - call option

- **Putable bonds**

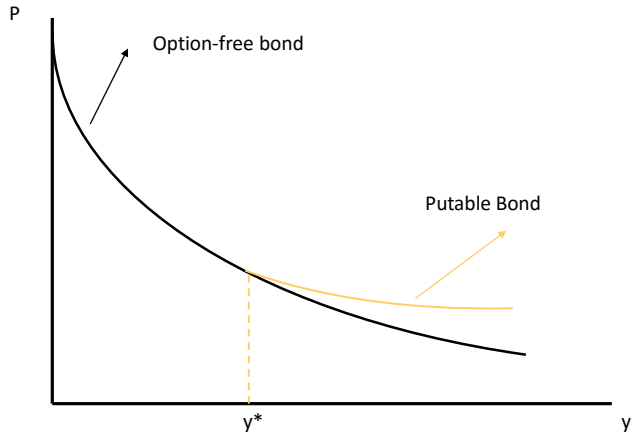
-The bond holder can sell the bond prior to maturity on specific dates at a pre-specified price

-Typically the option is exercised after an increase in interest rates, giving the opportunity to investors to buy new bonds at a higher coupon rates

- **Disadvantages for the issuer**

- The issuer of the bond will have to issue another bond at a higher coupon rate if the put option is exercised

A putable bond trades at a higher price than a comparable option-free bond



Putable bond = *option-free bond* + put option

Callable/Putable bonds:

- *Yield* to call/put

We can calculate the bond yields for every possible exercise dates of the options

- *Yield* to worst

The yield-to-worst is the lowest of the yield-to-maturity and all yields-to-call / yields-to-put

Contrary to fixed coupon option-free bond, callable/putable bonds are contingent securities, that is, the future cash-flows are not known with certainty because they depend on the future values of interest rate

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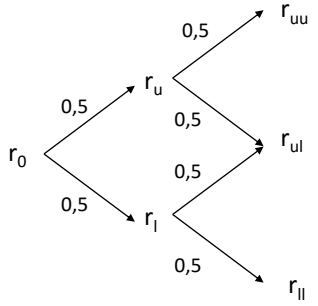
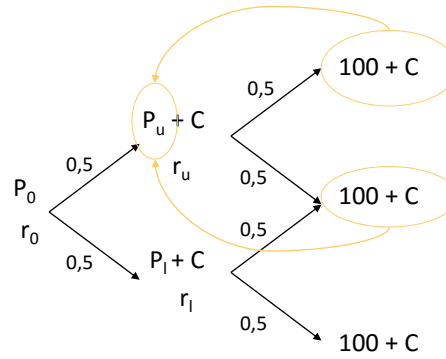
Interest rate model

Binomial model: Valuing a fixed coupon bond

Example: 2 years bond with coupon C





$$P_u = \frac{0,5(100+C)+0,5(100+C)}{1+r_u} \qquad P_l = \frac{0,5(100+C)+0,5(100+C)}{1+r_l}$$

$$P_0 = \frac{0,5(P_u+C)+0,5(P_l+C)}{1+r_0}$$

Valuing a callable bond using the Binomial model

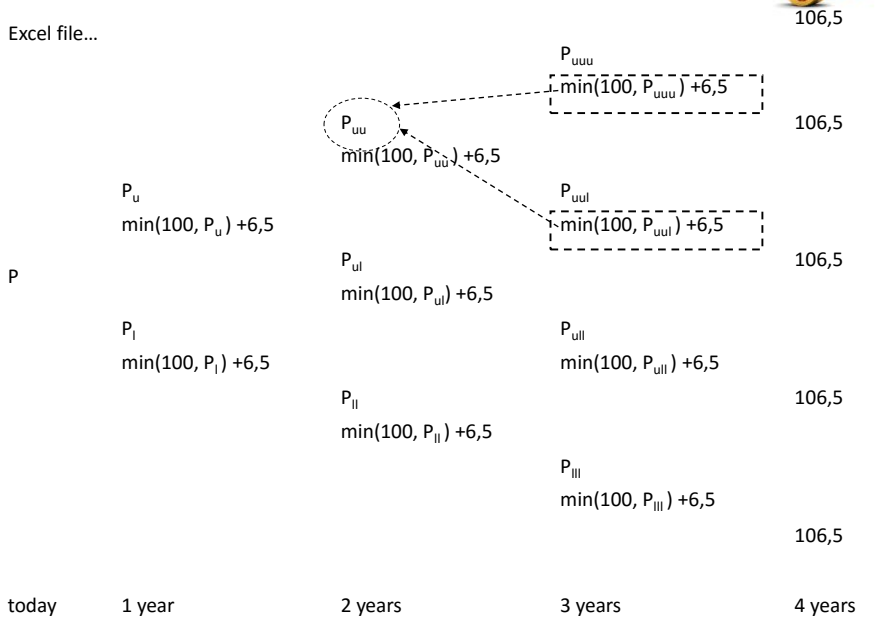
- Calculate, recursively, the bond value at each node

- noting that, at each node where the option exercise is possible, the bond value will correspond to the minimum of the call price and the value obtained recursively

Example:

4 years maturity callable bond, annual coupons, paying par at maturity
Coupon rate: 6,5%
The call option may be exercised at each coupon date, call price: 100%

Excel file…



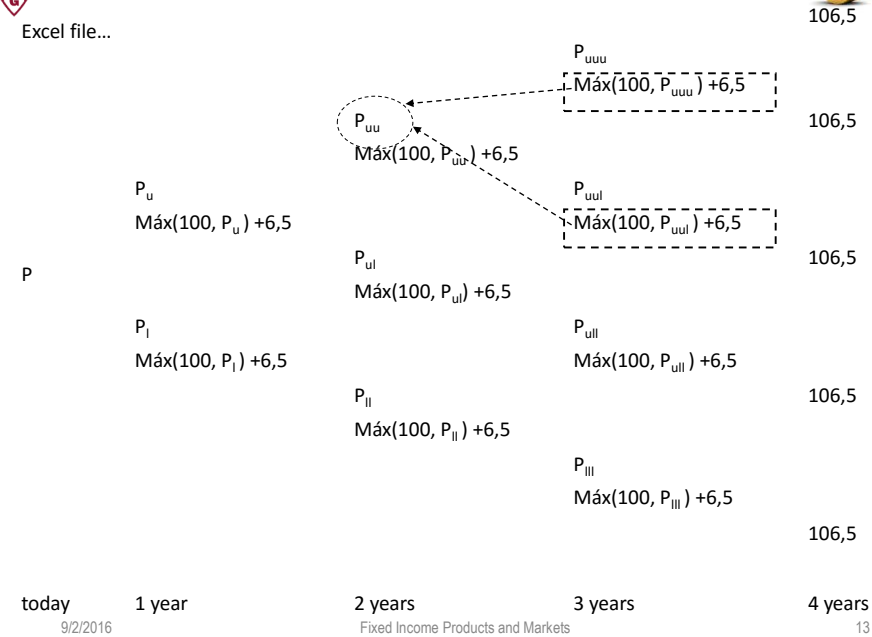
Valuing a putable bond using the Binomial model

- Calculate, recursively, the bond value at each node
- noting that, at each node where the option exercise is possible, the bond value will correspond to the maximum of the put price and the value obtained recursively

Example:

4 years maturity putable bond, annual coupons, paying par at maturity
Coupon rate: 6,5%
The put option may be exercised at each coupon date, put price: 100%

Excel file…



Option Adjusted Spread (OAS)

Comparison of the theoretical price obtained by the model and the market price

Instead of calculating the price difference calculate the **yield spread**

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Option adjusted spread – corresponds to the constant spread added to all short term interest rates, that generates a theoretical price equal to the market price

In the previous example, if the callable bond was quoted at 102,218% we would have an OAS = 35 bp

Excel file…

4.2 Options on bonds

- **Call option**

Right to buy the underlying (bond) at a pre-specified price – strike price(K)

- **Put option**

Right to sell the underlying (bond) at a pre-specified price – strike price(K)

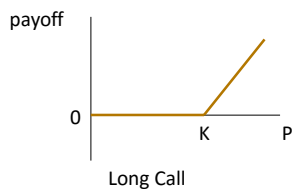
- **Exercise**

- At the expiration date – European options
- Until the expiration date – American options
- At fixed dates (e.g. coupon payments dates) – Bermudan options

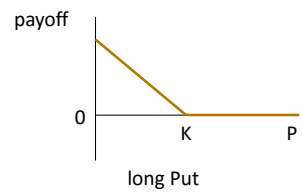
***Options Payoffs* at maturity/at the exercise date:**

Call option buyer: $\text{Máx}(P - K, 0)$

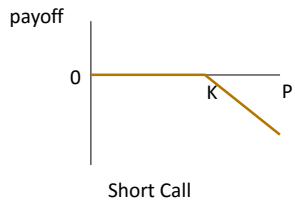
Put option buyer: $\text{Máx}(K - P, 0)$



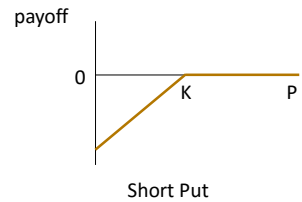
Long Call



long Put



Short Call



Short Put

Put-call parity for european options:

- Options on zero coupon bonds

$$c_0 = P_0 + p_0 - Ke^{-r(0,T)T}$$

- Options on fixed coupon bonds

$$c_0 = P_0 + p_0 - Ke^{-r(0,T)T} - \text{Discounted value of coupons paid until T}$$

We can replicate a call option through:

- buy a put option (same strike and same expiration date)
- buy the bond
- loan

c_0 – price of call option, with strike price K and expiration date T

p_0 – price of put option, with strike price K and expiration date T

P_0 – Bond Price

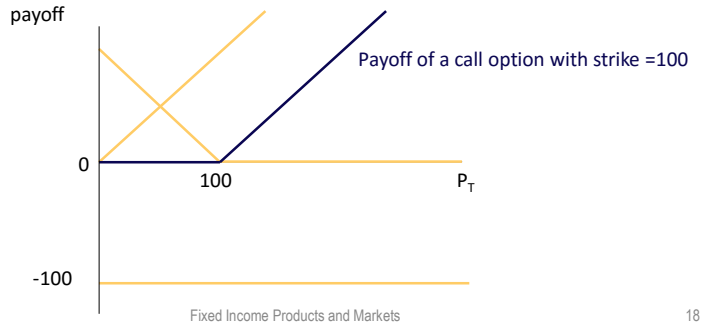
Put-call parity for european options:

Example: Option on a bond with nominal value of 100€, pays a semi-annual coupon of 3€; K = 100 and T= 1 year

- buy the put,
- buy the bond
- loans: $100e^{-r(0,1)}$; $3e^{-r(0;0,5)0,5}$ e $3e^{-r(0;1)}$

these two loans will be settled with the received coupons

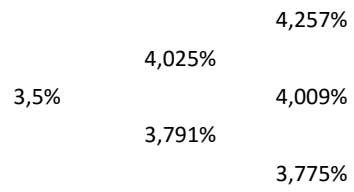
At the expiration date:



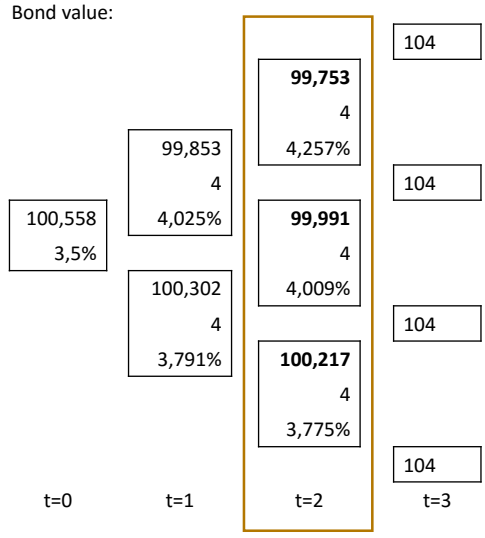
Valuing european options – binomial model

Example: 3 years to maturity bond, 4% annual coupon

Calculate the value of a call option on the bond, with a strike price of 99,80 and expiry date in 2 years, considering the following tree:



Bond value:



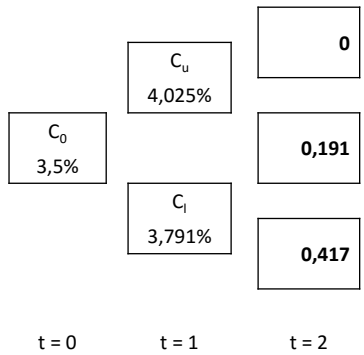
Option Payoff at the expiration date:

C_{uu}=Máx(99,753 - 99,80 ; 0) =0

C_{ul}=Máx(99,991 - 99,80 ; 0) =0,191

C_{ll}=Máx(100,217 - 99,80 ; 0) =0,417

t=2



$$C_u = \frac{0,5 \times 0 + 0,5 \times 0,191}{(1,04025)} = 0,092$$

$$C_l = \frac{0,5 \times 0,191 + 0,5 \times 0,417}{(1,03791)} = 0,293$$

$$C_0 = \frac{0,5 \times 0,092 + 0,5 \times 0,293}{(1,035)} = 0,186$$

Valuing american/bermudan options – binomial model

At each exercise date we must compare the value from the immediate exercise with the value of keeping the option alive until the next period

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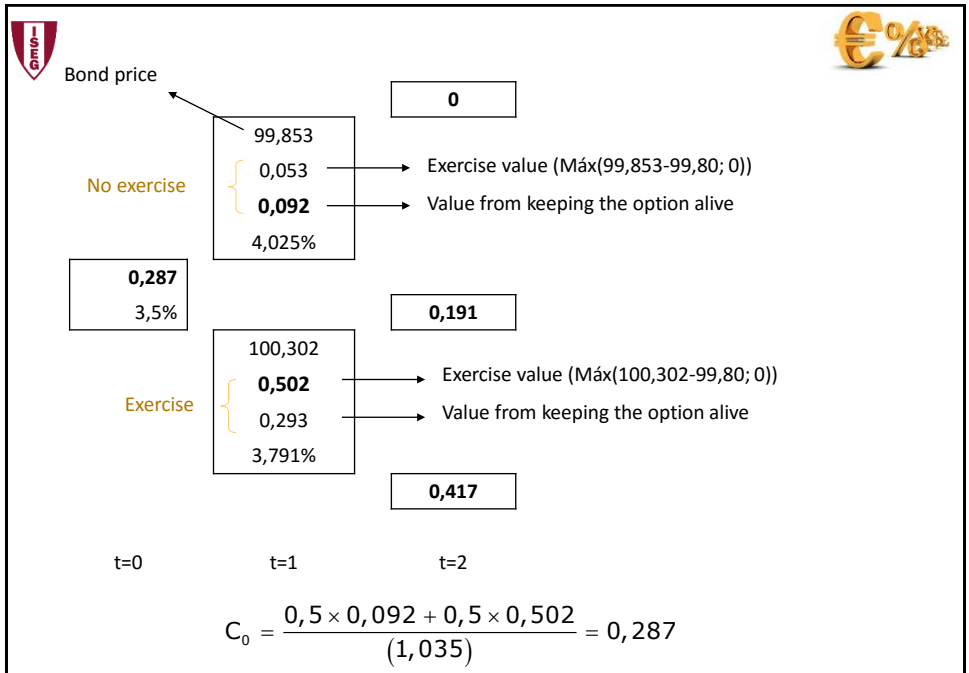
At each node, the option value will be the maximum of those values

Example:

Consider the last example but assume that the option can be exercised at each coupon payment date until the expiration date: at the end of the 1st and 2nd years



$$C_0 = \frac{0,5 \times 0,092 + 0,5 \times 0,502}{(1,035)} = 0,287$$

4.3 Convertible Bonds

- **Definition**

Bondholder has a right to convert the bond for pre-specified number of share of common stock

- **Terminology**

- *Convertible price:* is the price of the convertible bond

- *Bond floor* or *investment value:* is the price of the bond if there is no conversion option

- *Conversion ratio:* is the number of shares that is exchanged for a bond

- (market) conversion price*: convertible price* / conversion ratio

- *Conversion value* = current share price x conversion ratio

- *Conversion premium :* $\frac{\text{(convertible price – conversion value)}}{\text{conversion value}}$

- *Income pickup* – is the amount by which the yield to maturity of the convertible bond exceeds the dividend yield of the share

Example:

Current bond price : $930

Conversion ratio : 30

Current stock price : $25

(market) conversion price: $\frac{\$930}{30} = \$31$

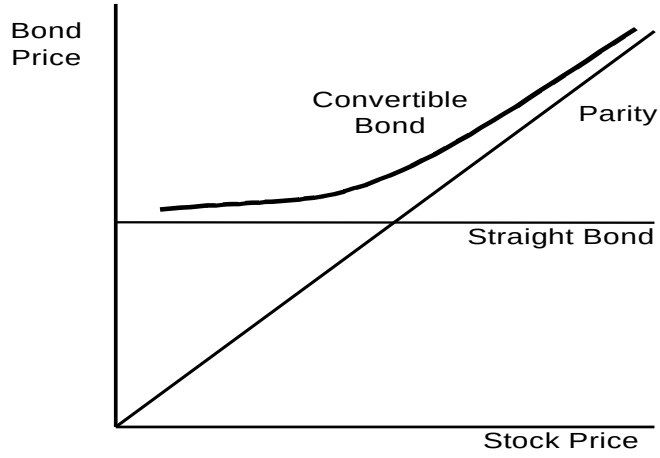
Market Conversion Value : $\$25 \times 30 = \$750$

Conversion Premium: $\frac{\$930 - \$750}{\$750} = 24\%$

Advantages

- **For the issuer**
  - Issuing convertible bonds enables a firm to obtain better financial conditions:
    Coupon rate of such a bond is always lower to that of a bullet bond with the same characteristics in terms of maturity and coupon frequency, this comes directly from the conversion advantage which is attached to this product
  - Besides the exchange of bonds for shares diminishes the liabilities of the firm issuer and increases in the same time its equity so that its debt capacity is improved
- **For the convertible bondholder**
  - The convertible bond is a defensive security very sensitive to a rise in the share price and protective when the share price decreases
    If the share price increases, the convertible price will also increase
    When share price decreases, price of convertible never gets below the bond floor, i.e., the price of an otherwise identical bullet bond with no conversion option

Valuing convertible bonds



Convertible bond is similar to a normal coupon bond plus a call option on the underlying stock with an important difference: the effective strike price of the call option will vary with the price of the bond

Valuing convertible bonds

Stock price evolution: binomial model (Cox, Ross e Rubinstein)

- up move: uS_0
- down move: dS_0

- $u = \frac{1}{d}$
- $u = e^{\sigma\sqrt{\Delta t}}$
- Up move risk neutral probability: $p = \frac{e^{r\Delta t} - d}{u - d}$

At each node, we analyze if the conversion is optimal