FIXED INCOME PRODUCTS AND MARKETS

III – Fixed Income Derivatives and Models

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III – Fixed Income Derivatives and Models

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5. Futures Options, Caps, Floors and Swaptions

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5.1 Futures Options

- **Definition**

An option on a future contract gives the buyer the right to buy (call option) from or sell (put option) to the seller one unit of a designated futures contract, at a determined price, called the strike price (expressed in the same basis as the futures price) on the maturity date of the option (European option) or at any time during the option life (American option)

- **Position in the underlying futures contracts after the option exercise**

	Call option	Put option
Buyer of the option	Long	Short
Seller of the option	Short	Long

The future's position is open at the strike price of the option

Example: Option on Euribor 3 month future contract

Style: American

Underlying instrument: one Euribor 3 month future contract

Quotation method: in percentage

Tick size: 0,005% (0,5 basis point); tick value: 12,5€

Strike prices intervals:

Eurex: 0,1 (e.g. 95,5; 95,6; 95,7 etc.)

NYSE Euronext: 0,125 (e.g. 95,5; 95,625; 95,75 etc.)

Premium: no payment made up-front, *Future-Style Margining*

Example: Option on Euribor 3 month future contract

Delivery:

NYSE Euronext: 10 months, 3 consecutive and 7 of quarterly cycle

Option expiry month	Delivery month of the underlying future
January, February and March	March
April, May and June	June
July, August and September	September
October, November and December	December

Eurex: 4 months of quarterly cycle

Option expiry month	Delivery month of the underlying future
March	March
June	June
September	September
December	December

Example: **Option on Euribor 3 month future contract**

Quotation

- 95,50 Call Junho 08: 0,185

Option value: $$\frac{0,185}{0,005} \times 12,5€ = 462,5€$$

- 95,50 Put Junho 08: 0,070

Option value: $$\frac{0,070}{0,005} \times 12,5€ = 175€$$

Valuing **options** on futures

Distinguish: - Premium paid *upfront*

- *Future-Style Margining*

Put-call parity

$$C - P = B(0,T)(F - E) \quad \text{(\textit{upfront})}$$

$$C - P = F - E \quad \text{(\textit{future-style})}$$

C – call option
P – put option
F – future price
E – strike price
T – time to expiry

Valuing european options on futures – Black 76 model

- Options on short term interest rate futures

Future-style: $P = R_F N(d_1) - R_E N(d_2)$ $C = R_E N(-d_2) - R_F N(-d_1)$

Upfront: $P = B(0,T)\left[R_F N(d_1) - R_E N(d_2)\right]$ $C = B(0,T)\left[R_E N(-d_2) - R_F N(-d_1)\right]$

$$d_1 = \frac{\ln\left(\frac{R_F}{R_E}\right) + 0{,}5\sigma^2 T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T} \qquad (N(-a) = 1 - N(a))$$

R_F – implied interest rate in futures price
R_E – implied interest rate in strike price
T –time to expiry (in years)
σ – volatility (annual) of the implied rate in futures price
N() – standard normal cumulative distribution

Valuing european options on futures – Black 76 model

- Options on Bond Futures

Future-style: $C = FN(d_1) - EN(d_2)$ $P = EN(-d_2) - FN(-d_1)$

Upfront: $C = B(0,T)\left[FN(d_1) - EN(d_2)\right]$ $P = B(0,T)\left[EN(-d_2) - FN(-d_1)\right]$

$$d_1 = \frac{\ln\left(\frac{F}{E}\right) + 0{,}5\sigma^2 T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

F – futures price
E – strike price
T – time to expiry (in years)
σ – volatility (anual)of futures price
N() – standard normal cumulative distribution

Hedging with options on short term interest rate futures

- Fixing a maximum financing rate

(expiration date = future delivery date = financing date)

Buy put option

- Maximum financing rate = Implied interest rate on strike price + Option premium

- Effective financing rate ≈ Euribor 3M (LTD) - Option result

Example:

On March 20th, a firm expect to borrow EUR 3 000 000 on June 18th for 3 months (financing rate: Euribor 3M).

Risk of interest rate increase

⇓

- Sell 3M Euribor futures (fix the financing rate)

or

- buy Put option on 3M Euribor futures
(fix a maximum financing rate, benefiting in case of a decrease in interest rates)

Dat2 20/03

Euribor 3 month: 4,674%

3 months Euribor futures (June): 95,615

95,50 Put (June): 0,07

Implied interest rate in strike price = 4,5%

$$\# \text{ of contracts} = \frac{3\ 000\ 000}{1\ 000\ 000} = 3$$

Buy 3 Put option contracts

Example:

Maximum financing rate = 4,5% + 0,07% = 4,57%

	Scenario 1	Scenario 2	Scenario 3
Euribor 3M	4,00%	4,50%	5,00%
Futures Euribor 3M	96,00	95,50	95,00
Option result	-0,07%	-0,07%	0,43%
Effective financing rate	4,07%	4,57%	4,57%

Scenario 3:

Euribor 3M = 5%

Futures Euribor 3M = 95%

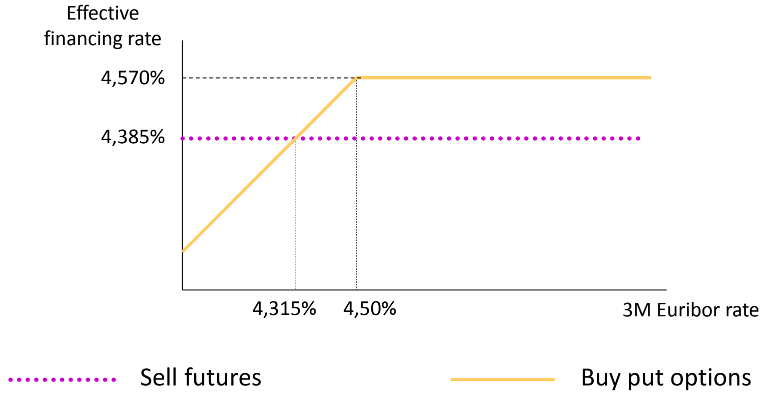
Option result = -0,07 + (95,50 – 95,00) =0,43

Effective financing rate= 5% - 0,43% = 4,57%

Using futures, the firm would had fixed the financing rate at 4,385% (100-95,615)

Example:

Hedging with Put options *versus* futures



Hedging with options on short term interest rate futures

- Fixing a maximum financing rate at a reduced cost
  (expiration date = future delivery date = financing date)

Buy Put - fix the maximum financing rate

Sell Call - limit the potential gain, reducing the hedging cost

(Call strike price > Put strike price)

Maximum financing rate = interest rate implied in Put strike price +
+ Put premium – Call premium

Minimum financing rate = interest rate implied in Call strike price +
+ Put premium – Call premium

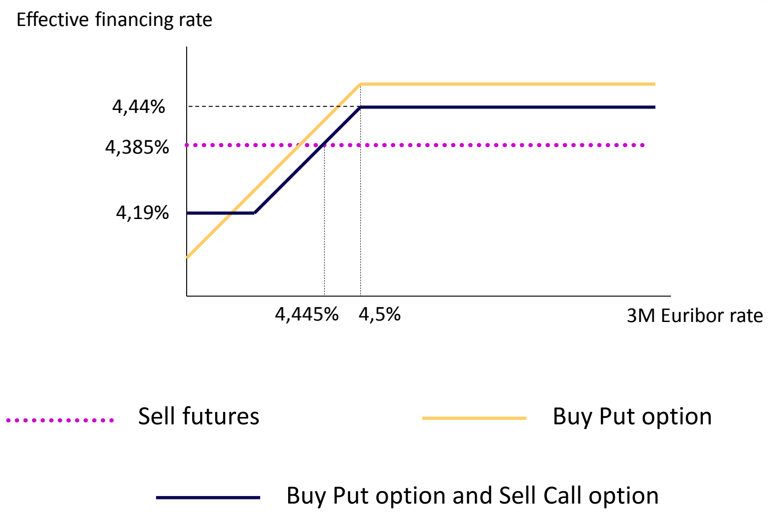
Sell call 95,75 Call Junho 08: 0,130

Maximum financing rate= 4,5% + 0,07% - 0,13% = 4,44%

Minimum financing rate= 4,25% + 0,07% - 0,13% = 4,19%

Effective financing rate = Euribor Rate (LTD) – options result

Euribor	Future	Options result Put	Call	Total	Effective financing rate
4	96	-0,070	-0,12	-0,190	4,190
4,1	95,9	-0,070	-0,02	-0,090	4,190
4,2	95,8	-0,070	0,08	0,010	4,190
4,3	95,7	-0,070	0,13	0,060	4,240
4,4	95,6	-0,070	0,13	0,060	4,340
4,5	95,5	-0,070	0,13	0,060	4,440
4,6	95,4	0,030	0,13	0,160	4,440
4,7	95,3	0,130	0,13	0,260	4,440
4,8	95,2	0,230	0,13	0,360	4,440
4,9	95,1	0,330	0,13	0,460	4,440
5	95	0,430	0,13	0,560	4,440



Hedging with options on short term interest rate futures

- Fixing a minimum deposit rate

(expiration date = future delivery date = deposit date)

Buy call option

- Minimum deposit rate = Implied interest rate on strike price - Option premium - Spread

- Effective deposit rate = 3M Euribor (LTD) + Option result - Spread

Hedging with options on short term interest rate futures

- Fixing a minimum deposit rate at a reduced cost

(expiration date = future delivery date = deposit date)

Buy call – fix a minimum deposit rate

Sell Put – limit the potential gain, reducing the hedging cost

(Call strike price > Put strike price)

Minimum deposit rate = interest rate implied in call strike price –
- Call premium + Put premium - spread

Maximum deposit rate = interest rate implied in put strike price –
- Call premium + Put premium - spread

5.2 Caps, Floors e Collars

- **Cap**

Is an OTC contract by which the seller agrees to pay a positive amount to the buyer of the contract if the reference rate exceeds a pre-specified level called the exercise rate of the cap on given future dates

- **Floor**

Is an OTC contract by which the seller agrees to pay a positive amount to the buyer of the contract if the reference rate falls below the exercise rate on some future dates

- **Collar**

is a combination of a cap and a floor :

- buying a cap and selling a floor at the same time
- buying a floor and selling a cap at the same time

Terminology

- The notional or nominal amount (fixed in general)

- The reference rate is an interest rate index based for example on Libor, T-bill and T-bond yield to maturity, swap rates, ect

- The settlement frequency refers to the frequency with which the reference rate is compared to the exercise rate (most common: monthly, quarterly, semiannually and annually)

- the strike rate (fixed when the contract is initiated)

- The maturity

- The premium of caps, floors and collars is expressed as a percentage of the notional amount. (usually quoted in annual terms and paid *prorata* on the settlement dates)

Cap

Transaction date: t

Cap initial date: T_0

Cap maturity: $T_n - T_0$ (in years)

t	T_0	T_1	T_2	...	T_{n-1}	T_n
	$R^L(T_0,\delta)$	$R^L(T_1,\delta)$	$R^L(T_2,\delta)$		$R^L(T_{n-1},\delta)$	
		C_1	C_2	...	C_{n-1}	C_n

$R^L(T_{j-1},\delta)$ – reference rate observed on date T_{j-1} for a tenor of δ (years)

Settlement frequency: $1/\delta$

On each date T_j (j = 1, 2, …n), the cap buyer receives the cash-flow C_j:

$$C_j = N \times \delta \times \text{Máx}\left(R^L(T_{j-1}, \delta) - E; 0\right)$$

E – Cap exercise rate; N – cap notional value

C_j corresponds to the payoff of an european call option on the reference rate R^L, with strike rate E and expiration date T_{j-1}, whose settlement occurs on T_j

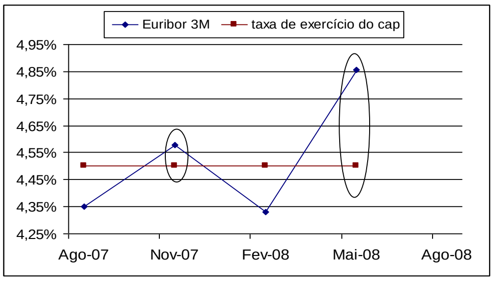
A Cap is a portfolio of such options named by ***caplets***

Example

On 1/08/2007, an investor bought a cap with the following characteristics:

Cap: 1 year maturity; quarterly frequency; reference rate: euribor 3M; day count convention: Act/360; exercise rate = 4,5%; initial date: 08-08-2007; reference value: 5 000 000 €; premium: 0,13%

Dates	Euribor 3m	# days
08-08-2007	4,352%	-
08-11-2007	4,579%	92,00
08-02-2008	4,331%	92,00
08-05-2008	4,855%	90,00
08-08-2008	-	92,00



Payoffs:

08-11-2007: $5000000 \times \frac{92}{360}\left[\text{máx}\left(4,352\% - 4,5\%;0\right) - 0,13\%\right] = -1661,1$

08-02-2008: $5000000 \times \frac{92}{360}\left[\text{máx}\left(4,579\% - 4,5\%;0\right) - 0,13\%\right] = -651,7$

08-05-2008: $5000000 \times \frac{90}{360}\left[\text{máx}\left(4,331\% - 4,5\%;0\right) - 0,13\%\right] = -1625,0$

08-08-2008: $5000000 \times \frac{92}{360}\left[\text{máx}\left(4,855\% - 4,5\%;0\right) - 0,13\%\right] = 2875,0$

Floor

Transaction date : t

Floor initial date: T_0

Floor maturity: $T_n - T_0$ (in years)

E – floor exercise rate

t	T_0	T_1	T_2	…	T_{n-1}	T_n
	$R^L(T_0,\delta)$	$R^L(T_1,\delta)$	$R^L(T_2,\delta)$		$R^L(T_{n-1},\delta)$	
		F_1	F_2	…	F_{n-1}	F_n

$R^L(T_{j-1},\delta)$ – reference rate observed on date T_{j-1} for a tenor of δ (years)

Settlement frequency: $1/\delta$

On each date T_j (j = 1, 2, …n), the floor buyer receives the cash-flow C_j:

$$F_j = N \times \delta \times \text{Máx}\left(E - R^L\left(T_{j-1},\delta\right);0\right)$$

C_j corresponds to the payoff of an european put on the reference rate R^L, with strike rate E and expiration date T_{j-1}, whose settlement occurs on T_j

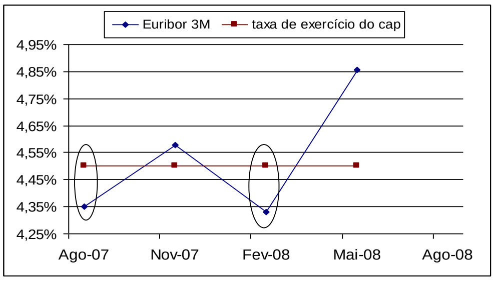
A Floor is a portfolio of such options named by ***floorlets***

Example

On 1/08/2007, an investor bought a fllor with the following characteristics:

Floor: 1 year maturity; quarterly frequency; reference rate: euribor 3M; day count convention: Act/360; exercise rate = 4,5%; initial date: 08-08-2007; reference value: 5 000 000 €; premium: 0,16%

Dates	Euribor 3m	# dayss
08-08-2007	4,352%	-
08-11-2007	4,579%	92,00
08-02-2008	4,331%	92,00
08-05-2008	4,855%	90,00
08-08-2008	-	92,00



Payoffs:

08-11-2007: $5000000 \times \frac{92}{360}\left[\text{máx}(4,5\% - 4,352\%;0) - 0,16\%\right] = -153,3$

08-02-2008: $5000000 \times \frac{92}{360}\left[\text{máx}(4,5\% - 4,579\%;0) - 0,16\%\right] = -2044,4$

08-05-2008: $5000000 \times \frac{90}{360}\left[\text{máx}(4,5\% - 4,331\%;0) - 0,16\%\right] = 112,5$

08-08-2008: $5000000 \times \frac{92}{360}\left[\text{máx}(4,5\% - 4,855\%;0) - 0,16\%\right] = -2044,4$

Valuing

Value of a *caplet,* for the period (T_{j-1}, T_j):

(t = 0)

$$\text{caplet}_j = N\delta B(0,T_j)\left[F_j N(d_1) - E N(d_2)\right]$$

$$d_1 = \frac{\ln\left(\frac{F_j}{E}\right) + 0,5\sigma_j^2 T_{j-1}}{\sigma_j\sqrt{T_{j-1}}} \qquad d_2 = d_1 - \sigma_j\sqrt{T_{j-1}}$$

Cap value:

$$\text{cap} = \sum_{j=1}^{n} \text{caplet}_j$$

F_j – forward rate for period (T_{j-1}, T_j)

E – strike rate

σ_j – volatility of forward rate F_j

Valuing

Value of a *floorlet,* for the period (T_{j-1}, T_j):

(t = 0)

$$floorlet_j = N\delta B(0,T_j)\left[EN(-d_2) - F_jN(-d_1)\right]$$

$$d_1 = \frac{\ln\left(\frac{F_j}{E}\right) + 0,5\sigma_j^2 T_{j-1}}{\sigma_j\sqrt{T_{j-1}}} \qquad d_2 = d_1 - \sigma_j\sqrt{T_{j-1}}$$

Floor value:

$$floor = \sum_{j=1}^{n} floorlet_j$$

F_j – forward rate for period (T_{j-1}, T_j)

E – strike rate

σ_j – volatility of forward rate F_j

Hedging

A cap enables the buyer to cap the reference rate associated to a liability

→ hedge against an increase in interest rates

Example A: A firm borrowed 2 000 000€; financing rate: euribor 6M + 50bp, maturity in 2 years. To hedge against an increase in interest rates, the firm bought a cap with reference value of 2 000 000€, reference rate: euribor 6M, semiannual frequency, 2 years maturity, with strike rate of 4,55% and premium (annual) of 0,24% (paid prorata)

The cap allowed the firm to fix a maximum financing rate:

Maximum rate = 4,55% + 0,50% + 0,24% = 5,29%

For each interest payment, the effective financing rate is:

Effective rate = min(5,29%; Euribor 6M + 0,50% + 0,24%)

Hedging

A floor enables the buyer to fix a minimum value for the reference rate associated with an asset return

→ hedge against a decrease in interest rates

Example B: An investor owns 1 000 000€ of nominal value of a FRN with coupon rate: Euribor 3M (quarterly payments), maturity 3 years from now. To hedge against a decrease in interest rates, he bought a floor with notional amount of 1 000 000€, reference rate: euribor 3M, quarterly frequency, 3 years maturity, strike rate: 4,30% and premium (annual) of 0,18% (paid prorata)

The floor allowed the investor to fix a minimum coupon rate :

Minimum rate = 4,30% - 0,18% = 4,12%

For each coupon received, the effective coupon rate is:

Effective rate = máx(4,12%; Euribor 3M - 0,18%)

Hedging

Reducing the cost of hedging using a collar

Example A: Besides buying a cap, the firm sold a floor with 2 000 000€ of notional value, reference rate: euribor 6M, semiannual frequency, maturity 2 years, exercise rate 4,00% (lower than the cap rate) and premium (annual): 0,10%.

Maximum rate = 4,55% + 0,50% + 0,24% - 0,10% = 5,19%

Minimum rate = 4,00% + 0,50% + 0,24% - 0,10% = 4,64%

Effective rate = máx(min(5,19%; Euribor 6M +0,50%+ 0,24% - 0,10%); 4,64%)

Hedging

Reducing the cost of hedging using a **collar**

Example B: Besides buying the floor, the investor sold a cap with 1 000 000€ of notional value, reference rate: euribor 3M,quarterly frequency, 3 years to maturity, exercise rate 4,80% (higher than the floor rate) and premium (annual) de 0,08%.

Minimum rate = 4,30% - 0,18% + 0,08% = 4,20%

Maximum rate = 4,80% - 0,18% + 0,08% = 4,70%

Effective rate = máx(min(4,70%; Euribor 3M - 0,18%% + 0,08%); 4,20%)

5.3 Swaptions

- ***European Swaption***

Is an OTC option contract allowing holder to enter a pre-specified swap contract on a pre-specified date – expiry date

Two kinds of *European swaptions*:

- Receiver Swaption

  Gives the buyer the right to receive the fixed leg of the swap

- Payer Swaption

  Gives the buyer the right to pay the fixed leg of the swap

- Exercise rate – the specified fixed rate at which buyer can enter into the swap
- Premium – expressed as a percentage of the principal amount of the swap

Example: buyer of a *payer swaption* with expiry date T_0

	t	T_0	T_1	T_2	...	T_n
fixed leg			$-F_1$	$-F_2$	...	$-F_n$
floating leg			V_1	V_2	...	V_n

Cash-fows after the option exercise at T_0

Considering cash-flows with payment frequency $1/\delta$ ($T_j - T_{j-1} = \delta$ years), for a reference value of N, exercise rate F and floating rate $R^L(T_{j-1},\delta)$ (rate on date T_{j-1} for a tenor of δ years):

Pay fixed: $-F_j = -F \times \delta \times N$

Receive floating: $V_j = R^L\left(T_{j-1}, 1\right) \times \delta \times N$ $\quad j = 1,2,\ldots, n$

Valuing

Payer swaption:

$$swaption_t = N\left(\sum_{j=1}^{n} \delta B\left(t, T_j\right)\left[F_S\left(t\right)N\left(d_1\right) - FN\left(d_2\right)\right]\right)$$

Receiver swaption:

$$swaption_t = N\left(\sum_{j=1}^{n} \delta B\left(t, T_j\right)\left[FN\left(-d_2\right) - F_S\left(t\right)N\left(-d_1\right)\right]\right)$$

$$d_1 = \frac{\ln\left(\frac{F_s(t)}{F}\right) + 0,5\sigma_S^2\left(T_0 - t\right)}{\sigma_S\sqrt{T_0 - t}} \qquad d_2 = d_1 - \sigma_S\sqrt{T_0 - t}$$

F – Exercise rate

$F_S(t)$ – *forward swap* rate calculated at t

σ_S – forward swap rate volatility

Hedging

Payer swaption:

- It enables a firm to fix a maximum limit to its floating rate debt

- It enables an investor to transform its fixed-rate assets into floating-rate assets to benefit from a rise in interest rates

Receiver swaption:

- It enables a firm to transform its fixed rate debt into a floating rate debt in a context of a decrase in interest rates

- It enables an investor to protect its floating rate investment