

# Problems and Solutions

## 1 CHAPTER 1—Problems

### 1.1 Problems on Bonds

*Exercise 1.1* On 12/04/01, consider a fixed-coupon bond whose features are the following:

- face value: \$1,000
- coupon rate: 8%
- coupon frequency: semiannual
- maturity: 05/06/04

What are the future cash flows delivered by this bond?

*Solution 1.1* 1. The coupon cash flow is equal to \$40

$$\text{Coupon} = \frac{8\% \times \$1,000}{2} = \$40$$

It is delivered on the following future dates: 05/06/02, 11/06/02, 05/06/03, 11/06/03 and 05/06/04.

The redemption value is equal to the face value \$1,000 and is delivered on maturity date 05/06/04.

*Exercise 1.3* An investor has a cash of \$10,000,000 at disposal. He wants to invest in a bond with \$1,000 nominal value and whose dirty price is equal to 107.457%.

1. What is the number of bonds he will buy?
2. Same question if the nominal value and the dirty price of the bond are respectively \$100 and 98.453%.

*Solution 1.3* 1. The number of bonds he will buy is given by the following formula

$$\text{Number of bonds bought} = \frac{\text{Cash}}{\text{Nominal Value of the bond} \times \text{dirty price}}$$

Here, the number of bonds is equal to 9,306

$$n = \frac{10,000,000}{1,000 \times 107.457\%} = 9,306.048$$

2.  $n$  is equal to 101,562

$$n = \frac{10,000,000}{100 \times 98.453\%} = 101,571.31$$

*Exercise 1.4* On 10/25/99, consider a fixed-coupon bond whose features are the following:

- face value: Eur 100

- coupon rate: 10%
- coupon frequency: annual
- maturity: 04/15/08

Compute the accrued interest taking into account the four different day-count bases: Actual/Actual, Actual/365, Actual/360 and 30/360.

*Solution 1.4* The last coupon has been delivered on 04/15/99. There are 193 days between 04/15/99 and 10/25/99, and 366 days between the last coupon date (04/15/99) and the next coupon date (04/15/00).

- The accrued interest with the Actual/Actual day-count basis is equal to Eur 5.273

$$\frac{193}{366} \times 10\% \times \text{Eur } 100 = \text{Eur } 5.273$$

- The accrued interest with the Actual/365 day-count basis is equal to Eur 5.288

$$\frac{193}{365} \times 10\% \times \text{Eur } 100 = \text{Eur } 5.288$$

- The accrued interest with the Actual/360 day-count basis is equal to Eur 5.361

$$\frac{193}{360} \times 10\% \times \text{Eur } 100 = \text{Eur } 5.361$$

There are 15 days between 04/15/99 and 04/30/99, 5 months between May and September, and 25 days between 09/30/99 and 10/25/99, so that there are 190 days between 04/15/99 and 10/25/99 on the 30/360 day-count basis

$$15 + (5 \times 30) + 25 = 190$$

- Finally, the accrued interest with the 30/360 day-count basis is equal to Eur 5.278

$$\frac{190}{360} \times 10\% \times \text{Eur } 100 = \text{Eur } 5.278$$

*Exercise 1.8* An investor wants to buy a bullet bond of the automotive sector. He has two choices: either invest in a US corporate bond denominated in euros or in a French corporate bond with same maturity and coupon. Are the two bonds comparable?

*Solution 1.8* The answer is no. First, the coupon and yield frequency of the US corporate bond is semiannual, while it is annual for the French corporate bond. To compare the yields on the two instruments, you have to convert either the semiannual yield of the US bond into an equivalently annual yield or the annual yield of the French bond into an equivalently semiannual yield. Second, the two bonds do not necessarily have the same rating, that is, the same credit risk. Third, they do not necessarily have the same liquidity.

*Exercise 1.15* What is the price  $P$  of the certificate of deposit issued by bank X on 06/06/00, with maturity 08/25/00, face value \$10,000,000, an interest rate at issuance of 5% falling at maturity and a yield of 4.5% as of 07/31/00?

*Solution 1.15* Recall that the price  $P$  of such a product is given by

$$P = F \times \frac{\left(1 + c \times \frac{n_c}{B}\right)}{\left(1 + y_m \times \frac{n_m}{B}\right)}$$

where  $F$  is the face value,  $c$  the interest rate at issuance,  $n_c$  is the number of days between issue and maturity,  $B$  is the year-basis (360 or 365),  $y_m$  is the yield on a money-market basis and  $n_m$  is the number of days between settlement and maturity.

Then, the price  $P$  of the certificate of deposit issued by bank X is equal to

$$P = \$10,000,000 \times \frac{\left(1 + 5\% \times \frac{80}{360}\right)}{\left(1 + 4.5\% \times \frac{25}{360}\right)} = \$10,079,612.3$$

Indeed, there are 80 calendar days between 06/06/00 and 08/25/00, and 25 calendar days between 07/31/00 and 08/25/00.

## 2 CHAPTER 2—Problems

*Exercise 2.1* Suppose the 1-year continuously compounded interest rate is 12%. What is the effective annual interest rate?

*Solution 2.1* The effective annual interest rate is

$$\begin{aligned} R &= e^{0.12} - 1 = 0.1275 \\ &= 12.75\%. \end{aligned}$$

*Exercise 2.2* If you deposit \$2,500 in a bank account that earns 8% annually on a continuously compounded basis, what will be the account balance in 7.14 years?

*Solution 2.2* The account balance in 7.14 years will be

$$\$2,500 \cdot e^{8\% \times 7.14} = \$4,425.98$$

*Exercise 2.3* If an investment has a cumulative 63.45% rate of return over 3.78 years, what is the annual continuously compounded rate of return?

*Solution 2.3* The annual continuously compounded rate of return  $R$  is such that

$$1.6345 = e^{3.78R}$$

We find  $R^c = \ln(1.6345)/3.78 = 13\%$ .

- Exercise 2.7*
1. What is the price of a 5-year bond with a nominal value of \$100, a yield to maturity of 7% (with annual compounding frequency), a 10% coupon rate and an annual coupon frequency?
  2. Same question for a yield to maturity of 8%, 9% and 10%. Conclude.

*Solution 2.7* 1. The price  $P$  of a bond is given by the formula

$$P = \sum_{i=1}^n \frac{N \times c}{(1+y)^i} + \frac{N}{(1+y)^n}$$

which simplifies into

$$P = \frac{N \times c}{y} \left[ 1 - \frac{1}{(1+y)^n} \right] + \frac{N}{(1+y)^n}$$

where  $N$ ,  $c$ ,  $y$  and  $n$  are respectively the nominal value, the coupon rate, the yield to maturity and the number of years to maturity of the bond.

Here, we obtain for  $P$

$$P = \frac{10}{7\%} \left[ 1 - \frac{1}{(1+7\%)^5} \right] + \frac{100}{(1+7\%)^5}$$

$P$  is then equal to 112.301% of the nominal value or \$112.301. Note that we can also use the Excel function “Price” to obtain  $P$ .

2. Prices of the bond for different yields to maturity (YTM) are given in the following table

YTM (%)	Price (\$)
8	107.985
9	103.890
10	100

Bond prices decrease as rates increase.

*Exercise 2.14* We consider the following zero-coupon curve:

Maturity (years)	Zero-Coupon Rate (%)
1	4.00
2	4.50
3	4.75
4	4.90
5	5.00

1. What is the price of a 5-year bond with a \$100 face value, which delivers a 5% annual coupon rate?
2. What is the yield to maturity of this bond?
3. We suppose that the zero-coupon curve increases instantaneously and uniformly by 0.5%. What is the new price and the new yield to maturity of the bond? What is the impact of this rate increase for the bondholder?

4. We suppose now that the zero-coupon curve remains stable over time. You hold the bond until maturity. What is the annual return rate of your investment? Why is this rate different from the yield to maturity?

*Solution 2.14* 1. The price  $P$  of the bond is equal to the sum of its discounted cash flows and given by the following formula

$$P = \frac{5}{1 + 4\%} + \frac{5}{(1 + 4.5\%)^2} + \frac{5}{(1 + 4.75\%)^3} + \frac{5}{(1 + 4.9\%)^4} + \frac{105}{(1 + 5\%)^5} \\ = \$100.136$$

2. The yield to maturity  $R$  of this bond verifies the following equation

$$100.136 = \sum_{i=1}^4 \frac{5}{(1 + R)^i} + \frac{105}{(1 + R)^5}$$

Using the Excel function “Yield”, we obtain 4.9686% for  $R$ .

3. The new price  $P$  of the bond is given by the following formula:

$$P = \frac{5}{1 + 4.5\%} + \frac{5}{(1 + 5\%)^2} + \frac{5}{(1 + 5.25\%)^3} + \frac{5}{(1 + 5.4\%)^4} + \frac{105}{(1 + 5.5\%)^5} \\ = \$97.999$$

The new yield to maturity  $R$  of this bond verifies the following equation

$$97.999 = \sum_{i=1}^4 \frac{5}{(1 + R)^i} + \frac{105}{(1 + R)^5}$$

Using the Excel function “yield”, we obtain 5.4682% for  $R$ .

The impact of this rate increase is an absolute capital loss of \$2.137 for the bondholder.

$$\text{Absolute Loss} = 97.999 - 100.136 = -\$2.137$$

and a relative capital loss of 2.134%

$$\text{Relative Loss} = \frac{-2.137}{100.136} = -2.134\%$$

4. Before maturity, the bondholder receives intermediate coupons that he reinvests in the market:

- after one year, he receives \$5 that he reinvests for 4 years at the 4-year zero-coupon rate to obtain on the maturity date of the bond

$$5 \times (1 + 4.9\%)^4 = \$6.0544$$

- after two years, he receives \$5 that he reinvests for 3 years at the 3-year zero-coupon rate to obtain on the maturity date of the bond

$$5 \times (1 + 4.75\%)^3 = \$5.7469$$

- after three years, he receives \$5 that he reinvests for 2 years at the 2-year zero-coupon rate to obtain on the maturity date of the bond

$$5 \times (1 + 4.5\%)^2 = \$5.4601$$

- after four years, he receives \$5 that he reinvests for 1 year at the 1-year zero-coupon rate to obtain on the maturity date of the bond

$$5 \times (1 + 4\%) = \$5.2$$

- after five years, he receives the final cash flow equal to \$105.  
The bondholder finally obtains \$127.4614 five years later

$$6.0544 + 5.7469 + 5.4601 + 5.2 + 105 = \$127.4614$$

which corresponds to a 4.944% annual return rate

$$\left( \frac{127.4614}{100.136} \right)^{1/5} - 1 = 4.944\%$$

This return rate is different from the yield to maturity of this bond (4.9686%) because the curve is not flat at a 4.9686% level. With a flat curve at a 4.9686% level, we obtain \$127.6108 five years later

$$6.0703 + 5.7829 + 5.5092 + 5.2484 + 105 = \$127.6108$$

which corresponds exactly to a 4.9686% annual return rate.

$$\left( \frac{127.6108}{100.136} \right)^{1/5} - 1 = 4.9686\%$$

*Exercise 2.20* We consider two bonds with the following features

Bond	Maturity (years)	Coupon Rate (%)	Price	YTM (%)
Bond 1	10	10	1,352.2	5.359
Bond 2	10	5	964.3	5.473

YTM stands for yield to maturity. These two bonds have a \$1,000 face value, and an annual coupon frequency.

1. An investor buys these two bonds and holds them until maturity. Compute the annual return rate over the period, supposing that the yield curve becomes instantaneously flat at a 5.4% level and remains stable at this level during 10 years.
2. What is the rate level such that these two bonds provide the same annual return rate? In this case, what is the annual return rate of the two bonds?

*Solution 2.20* 1. We consider that the investor reinvests its intermediate cash flows at a unique 5.4% rate.

For Bond 1, the investor obtains the following sum at the maturity of the bond

$$100 \times \sum_{i=1}^9 (1 + 5.4\%)^i + 1,100 = 2,281.52$$

which corresponds exactly to a 5.3703% annual return rate.

$$\left( \frac{2,281.52}{1,352.2} \right)^{1/10} - 1 = 5.3703\%$$

For Bond 2, the investor obtains the following sum at the maturity of the bond

$$50 \times \sum_{i=1}^9 (1 + 5.4\%)^i + 1,050 = 1,640.76$$

which corresponds exactly to a 5.4589% annual return rate.

$$\left( \frac{1,640.76}{964.3} \right)^{1/10} - 1 = 5.4589\%$$

2. We have to find the value  $R$ , such that

$$\frac{100 \times \sum_{i=1}^9 (1 + R)^i + 1,100}{1,352.2} = \frac{50 \times \sum_{i=1}^9 (1 + R)^i + 1,050}{964.3}$$

Using the Excel solver, we finally obtain 6.4447% for  $R$ .

The annual return rate of the two bonds is equal to 5.6641%

$$\left( \frac{100 \times \sum_{i=1}^9 (1 + 6.4447\%)^i + 1,100}{1,352.2} \right)^{1/10} - 1 = 5.6641\%$$

*Exercise 2.24* Assume that the following bond yields, compounded semiannually:

6-month Treasury Strip: 5.00%;

1-year Treasury Strip: 5.25%;

18-month Treasury Strip: 5.75%.

1. What is the 6-month forward rate in six months?
2. What is the 1-year forward rate in six months?
3. What is the price of a semiannual 10% coupon Treasury bond that matures in exactly 18 months?

*Solution 2.24* 1.

$$\begin{aligned} \left( 1 + \frac{R_2(0, 1)}{2} \right)^2 &= \left( 1 + \frac{R_2(0, 0.5)}{2} \right) \left( 1 + \frac{F_2(0, 0.5, 0.5)}{2} \right) \\ 1.02625^2 &= 1.025 \left( 1 + \frac{F_2(0, 0.5, 0.5)}{2} \right) \\ \implies F_2(0, 0.5, 0.5) &= 5.5003\% \end{aligned}$$

2.

$$\begin{aligned} \left(1 + \frac{R_2(0, 1.5)}{2}\right)^3 &= \left(1 + \frac{R_2(0, 0.5)}{2}\right) \left(1 + \frac{F_2(0, 0.5, 1)}{2}\right)^2 \\ 1.02875^3 &= 1.025 \left(1 + \frac{F_2(0, 0.5, 1)}{2}\right)^2 \\ \implies F_2(0, 0.5, 1) &= 6.1260\% \end{aligned}$$

3. The cash flows are coupons of 5% in six months and a year, and coupon plus principal payment of 105% in 18 months. We can discount using the spot rates that we are given:

$$P = \frac{5}{\left(1 + \frac{0.05}{2}\right)} + \frac{5}{\left(1 + \frac{0.0525}{2}\right)^2} + \frac{105}{\left(1 + \frac{0.0575}{2}\right)^3} = 106.0661$$

### 3 CHAPTER 3—Problems

*Exercise 3.1* We consider three zero-coupon bonds (strips) with the following features:

Bond	Maturity (years)	Price
Bond 1	1	96.43
Bond 2	2	92.47
Bond 3	3	87.97

Each strip delivers \$100 at maturity.

1. Extract the zero-coupon yield curve from the bond prices.
2. We anticipate a rate increase in one year so the prices of strips with residual maturity 1 year, 2 years and 3 years are respectively 95.89, 90.97 and 84.23. What is the zero-coupon yield curve anticipated in one year?

*Solution 3.1* 1. The 1-year zero-coupon rate denoted by  $R(0, 1)$  is equal to 3.702%

$$R(0, 1) = \frac{100}{96.43} - 1 = 3.702\%$$

The 2-year zero-coupon rate denoted by  $R(0, 2)$  is equal to 3.992%

$$R(0, 2) = \left(\frac{100}{92.47}\right)^{1/2} - 1 = 3.992\%$$

The 3-year zero-coupon rate denoted by  $R(0, 3)$  is equal to 4.365%

$$R(0, 3) = \left(\frac{100}{87.97}\right)^{1/3} - 1 = 4.365\%$$

2. The 1-year, 2-year and 3-year zero-coupon rates become respectively 4.286%, 4.846% and 5.887%.



*Exercise 3.3* We consider the following decreasing zero-coupon yield curve:

Maturity (years)	$R(0, t)$ (%)	Maturity (years)	$R(0, t)$ (%)
1	7.000	6	6.250
2	6.800	7	6.200
3	6.620	8	6.160
4	6.460	9	6.125
5	6.330	10	6.100

where  $R(0, t)$  is the zero-coupon rate at date 0 with maturity  $t$ .

1. Compute the par yield curve.
2. Compute the forward yield curve in one year.
3. Draw the three curves on the same graph. What can you say about their relative position?

*Solution 3.3* 1. Recall that the par yield  $c(n)$  for maturity  $n$  is given by the formula

$$c(n) = \frac{1 - \frac{1}{(1+R(0,n))^n}}{\sum_{i=1}^n \frac{1}{(1+R(0,i))^i}}$$

Using this equation, we obtain the following par yields:

Maturity (years)	$c(n)$ (%)	Maturity (years)	$c(n)$ (%)
1	7.000	6	6.293
2	6.807	7	6.246
3	6.636	8	6.209
4	6.487	9	6.177
5	6.367	10	6.154

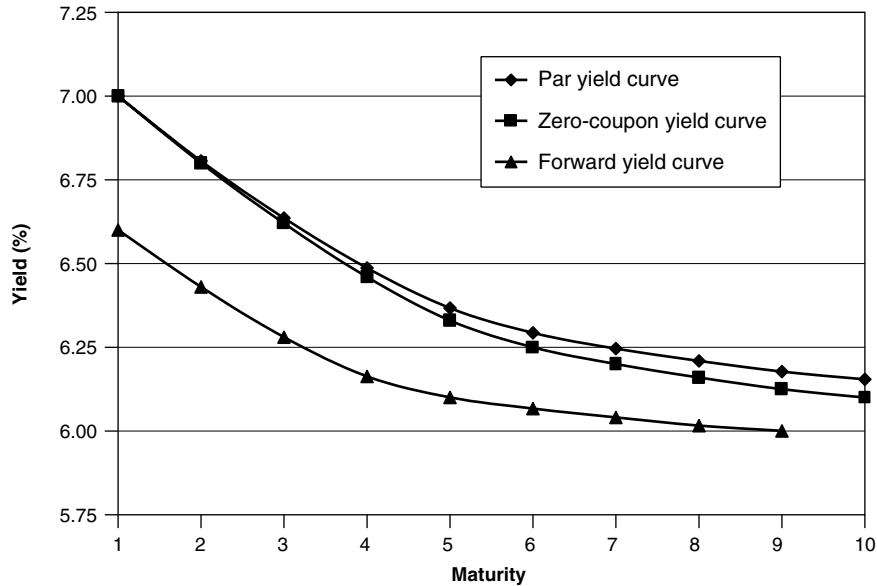
2. Recall that  $F(0, x, y - x)$ , the forward rate as seen from date  $t = 0$ , starting at date  $t = x$ , and with residual maturity  $y - x$  is defined as

$$F(0, x, y - x) \equiv \left[ \frac{(1 + R(0, y))^y}{(1 + R(0, x))^x} \right]^{\frac{1}{y-x}} - 1$$

Using the previous equation, we obtain the forward yield curve in one year

Maturity (years)	$F(0, 1, n)$ (%)	Maturity (years)	$F(0, 1, n)$ (%)
1	6.600	6	6.067
2	6.431	7	6.041
3	6.281	8	6.016
4	6.163	9	6.000
5	6.101	—	—

3. The graph of the three curves shows that the forward yield curve is below the zero-coupon yield curve, which is below the par yield curve. This is always the case when the par yield curve is decreasing.



*Exercise 3.13* Explain the basic difference that exists between the preferred habitat theory and the segmentation theory.

*Solution 3.13* In the segmentation theory, investors are supposed to be 100% risk-averse. So risk premia are infinite. It is as if their investment habitat were strictly constrained, exclusive.

In the preferred habitat theory, investors are not supposed to be 100% risk averse. So, there exists a certain level of risk premia from which they are ready to change their habitual investment maturity. Their investment habitat is, in this case, not exclusive.

#### 4 CHAPTER 4—Problems

*Exercise 4.1* At date  $t = 0$ , we consider five bonds with the following features:

	Annual Coupon	Maturity	Price
Bond 1	6	1 year	$P_0^1 = 103$
Bond 2	5	2 years	$P_0^2 = 102$
Bond 3	4	3 years	$P_0^3 = 100$
Bond 4	6	4 years	$P_0^4 = 104$
Bond 5	5	5 years	$P_0^5 = 99$

Derive the zero-coupon curve until the 5-year maturity.

*Solution 4.1* Using the no-arbitrage relationship, we obtain the following equations for the five bond prices:

$$\begin{cases} 103 = 106B(0, 1) \\ 102 = 5B(0, 1) + 105B(0, 2) \\ 100 = 4B(0, 1) + 4B(0, 2) + 104B(0, 3) \\ 104 = 6B(0, 1) + 6B(0, 2) + 6B(0, 3) + 106B(0, 4) \\ 99 = 5B(0, 1) + 5B(0, 2) + 5B(0, 3) + 5B(0, 4) + 105B(0, 5) \end{cases}$$

which can be expressed in a matrix form as

$$\begin{pmatrix} 103 \\ 102 \\ 100 \\ 104 \\ 99 \end{pmatrix} = \begin{pmatrix} 106 & & & & \\ 5 & 105 & & & \\ 4 & 4 & 104 & & \\ 6 & 6 & 6 & 106 & \\ 5 & 5 & 5 & 5 & 105 \end{pmatrix} \times \begin{pmatrix} B(0, 1) \\ B(0, 2) \\ B(0, 3) \\ B(0, 4) \\ B(0, 5) \end{pmatrix}$$

We get the following discount factors:

$$\begin{pmatrix} B(0, 1) \\ B(0, 2) \\ B(0, 3) \\ B(0, 4) \\ B(0, 5) \end{pmatrix} = \begin{pmatrix} 0.97170 \\ 0.92516 \\ 0.88858 \\ 0.82347 \\ 0.77100 \end{pmatrix}$$

and we find the zero-coupon rates

$$\begin{pmatrix} R(0, 1) = 2.912\% \\ R(0, 2) = 3.966\% \\ R(0, 3) = 4.016\% \\ R(0, 4) = 4.976\% \\ R(0, 5) = 5.339\% \end{pmatrix}$$

- Exercise 4.4*
1. The 10-year and 12-year zero-coupon rates are respectively equal to 4% and 4.5%. Compute the  $11^{1/4}$  and  $11^{3/4}$ -year zero-coupon rates using linear interpolation.
  2. Same question when you know the 10-year and 15-year zero-coupon rates that are respectively equal to 8.6% and 9%.

*Solution 4.4* Assume that we know  $R(0, x)$  and  $R(0, z)$  respectively as the  $x$ -year and the  $z$ -year zero-coupon rates. We need to get  $R(0, y)$ , the  $y$ -year zero-coupon rate with  $y \in [x; z]$ . Using linear interpolation,  $R(0, y)$  is given by the following formula:

$$R(0, y) = \frac{(z - y)R(0, x) + (y - x)R(0, z)}{z - x}$$

1. The  $11^{1/4}$  and  $11^{3/4}$ -year zero-coupon rates are obtained as follows:

$$R(0, 11^{1/4}) = \frac{0.75 \times 4\% + 1.25 \times 4.5\%}{2} = 4.3125\%$$

$$R(0, 11^{3/4}) = \frac{0.25 \times 4\% + 1.75 \times 4.5\%}{2} = 4.4375\%$$

2. The  $11^{1/4}$  and  $11^{3/4}$ -year zero-coupon rates are obtained as follows:

$$R(0, 11^{1/4}) = \frac{3.75 \times 8.6\% + 1.25 \times 9\%}{5} = 8.70\%$$

$$R(0, 11^{3/4}) = \frac{3.25 \times 8.6\% + 1.75 \times 9\%}{5} = 8.74\%$$

*Exercise 4.7* From the prices of zero-coupon bonds quoted in the market, we obtain the following zero-coupon curve:

Maturity (years)	Zero-coupon Rate $R(0, t)$ (%)	Discount Factor $B(0, t)$
1	5.000	0.95238
2	5.500	0.89845
3	5.900	0.84200
4	6.200	0.78614
5	?	?
6	6.550	0.68341
7	6.650	0.63720
8	?	?
9	6.830	0.55177
10	6.900	0.51312

where  $R(0, t)$  is the zero-coupon rate at date 0 for maturity  $t$ , and  $B(0, t)$  is the discount factor at date 0 for maturity  $t$ .

We need to know the value for the 5-year and the 8-year zero-coupon rates. We have to estimate them and test four different methods.

1. We use a linear interpolation with the zero-coupon rates. Find  $R(0, 5)$ ,  $R(0, 8)$  and the corresponding values for  $B(0, 5)$  and  $B(0, 8)$ .
2. We use a linear interpolation with the discount factors. Find  $B(0, 5)$ ,  $B(0, 8)$  and the corresponding values for  $R(0, 5)$  and  $R(0, 8)$ .
3. We postulate the following form for the zero-coupon rate function  $\bar{R}(0, t)$ :

$$\bar{R}(0, t) = a + bt + ct^2 + dt^3$$

Estimate the coefficients  $a$ ,  $b$ ,  $c$  and  $d$ , which best approximate the given zero-coupon rates using the following optimization program:

$$\text{Min}_{a,b,c,d} \sum_i (B(0, i) - \bar{B}(0, i))^2$$

where  $B(0, i)$  are the zero-coupon rates given by the market.

Find the value for  $R(0, 5) = \bar{R}(0, 5)$ ,  $R(0, 8) = \bar{R}(0, 8)$ , and the corresponding values for  $B(0, 5)$  and  $B(0, 8)$ .

4. We postulate the following form for the discount function  $\bar{B}(0, t)$ :

$$\bar{B}(0, t) = a + bt + ct^2 + dt^3$$

Estimate the coefficients  $a$ ,  $b$ ,  $c$  and  $d$ , which best approximate the given discount factors using the following optimization program:

$$\text{Min}_{a,b,c,d} \sum_i (B(0, i) - \bar{B}(0, i))^2$$

where  $B(0, i)$  are the discount factors given by the market.

Obtain the value for  $B(0, 5) = \bar{B}(0, 5)$ ,  $B(0, 8) = \bar{B}(0, 8)$ , and the corresponding values for  $R(0, 5)$  and  $R(0, 8)$ .

5. Conclude.

*Solution 4.7*

1. Consider that we know  $R(0, x)$  and  $R(0, z)$  respectively as the  $x$ -year and the  $z$ -year zero-coupon rates and that we need  $R(0, y)$ , the  $y$ -year zero-coupon rate with  $y \in [x; z]$ . Using linear interpolation,  $R(0, y)$  is given by the following formula:

$$R(0, y) = \frac{(z - y)R(0, x) + (y - x)R(0, z)}{z - x}$$

From this equation, we find the value for  $R(0, 5)$  and  $R(0, 8)$

$$\begin{aligned} R(0, 5) &= \frac{(6 - 5)R(0, 4) + (5 - 4)R(0, 6)}{6 - 4} \\ &= \frac{R(0, 4) + R(0, 6)}{2} = 6.375\% \end{aligned}$$

$$\begin{aligned} R(0, 8) &= \frac{(9 - 8)R(0, 7) + (8 - 7)R(0, 9)}{9 - 7} \\ &= \frac{R(0, 7) + R(0, 9)}{2} = 6.740\% \end{aligned}$$

Using the following standard equation in which lies the zero-coupon rate  $R(0, t)$  and the discount factor  $B(0, t)$

$$B(0, t) = \frac{1}{(1 + R(0, t))^t}$$

we obtain 0.73418 for  $B(0, 5)$  and 0.59345 for  $B(0, 8)$ .

2. Using the same formula as in question 1 but adapting to discount factors

$$B(0, y) = \frac{(z - y)B(0, x) + (y - x)B(0, z)}{z - x}$$

we obtain 0.73478 for  $B(0, 5)$  and 0.59449 for  $B(0, 8)$ .

Using the following standard equation

$$R(0, t) = \left( \frac{1}{B(0, t)} \right)^{1/t} - 1$$

we obtain 6.358% for  $R(0, 5)$  and 6.717% for  $R(0, 8)$ .

3. Using the Excel function “Linest”, we obtain the following values for the parameters:

Parameters	Value
$a$	0.04351367
$b$	0.00720757
$c$	-0.000776521
$d$	3.11234E-05

which provide us with the following values for the zero-coupon rates and associated discount factors:

Maturity	$R(0, t)$ (%)	$\bar{R}(0, t)$ (%)	$B(0, t)$	$\bar{B}(0, t)$
1	5.000	4.998	0.95238	0.95240
2	5.500	5.507	0.89845	0.89833
3	5.900	5.899	0.84200	0.84203
4	6.200	6.191	0.78614	0.78641
5	?	6.403	?	0.73322
6	6.550	6.553	0.68341	0.68330
7	6.650	6.659	0.63720	0.63681
8	?	6.741	?	0.59339
9	6.830	6.817	0.55177	0.55237
10	6.900	6.906	0.51312	0.51283

4. We first note that there is a constraint in the minimization because we must have

$$B(0, 0) = 1$$

So, the value for  $a$  is necessarily equal to 1.

Using the Excel function “Linest”, we obtain the following values for the parameters:

Parameters	Value
$a$	1
$b$	-0.04945479
$c$	-0.001445358
$d$	0.000153698

which provide us with the following values for the discount factors and associated zero-coupon rates:

Maturity	$B(0, t)$	$\bar{B}(0, t)$	$R(0, t)$ (%)	$\bar{R}(0, t)$ (%)
1	0.95238	0.94925	5.000	5.346
2	0.89845	0.89654	5.500	5.613
3	0.84200	0.84278	5.900	5.867
4	0.78614	0.78889	6.200	6.107
<b>5</b>	<b>?</b>	<b>0.73580</b>	<b>?</b>	<b>6.328</b>
6	0.68341	0.68444	6.550	6.523
7	0.63720	0.63571	6.650	6.686
<b>8</b>	<b>?</b>	<b>0.59055</b>	<b>?</b>	<b>6.805</b>
9	0.55177	0.54988	6.830	6.871
10	0.51312	0.51461	6.900	6.869

5. The table below summarizes the results obtained using the four different methods of interpolation and minimization

—	Rates Interpol.	DF Interpol.	Rates Min.	DF Min.
$R(0, 5)$	6.375%	6.358%	6.403%	6.328%
$R(0, 8)$	6.740%	6.717%	6.741%	6.805%
$B(0, 5)$	0.73418	0.73478	0.73322	0.73580
$B(0, 8)$	0.59345	0.59449	0.59339	0.59055

“Rates Interpol.” stands for interpolation on rates (question 1). “DF Interpol.” stands for interpolation on discount factors (question 2). “Rates Min” stands for minimization with rates (question 3). “DF Min.” stands for minimization with discount factors (question 4).

The table shows that results are quite similar according to the two methods based on rates. Differences appear when we compare the four methods. In particular, we can obtain a spread of 7.5 bps for the estimation of  $R(0, 5)$  between “Rates Min.” and “DF Min.”, and a spread of 8.8 bps for the estimation of  $R(0, 8)$  between the two methods based on discount factors. We conclude that the zero-coupon rate and discount factor estimations are sensitive to the method that is used: interpolation or minimization.

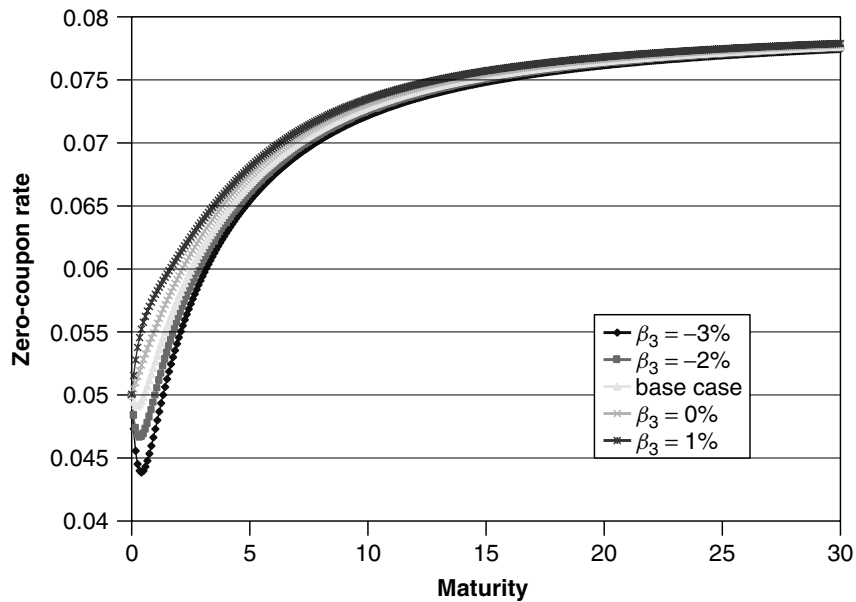
*Exercise 4.15* Consider the Nelson and Siegel Extended model

$$R^c(0, \theta) = \beta_0 + \beta_1 \left[ \frac{1 - \exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}} \right] + \beta_2 \left[ \frac{1 - \exp\left(-\frac{\theta}{\tau_1}\right)}{\frac{\theta}{\tau_1}} - \exp\left(-\frac{\theta}{\tau_1}\right) \right] \\ + \beta_3 \left[ \frac{1 - \exp\left(-\frac{\theta}{\tau_2}\right)}{\frac{\theta}{\tau_2}} - \exp\left(-\frac{\theta}{\tau_2}\right) \right]$$

with the following base-case parameter values:  $\beta_0 = 8\%$ ,  $\beta_1 = -3\%$ ,  $\beta_2 = 1\%$ ,  $\beta_3 = -1\%$ ,  $1/\tau_1 = 0.3$  and  $1/\tau_2 = 3$ .

We give successively five different values to the parameter  $\beta_3$ :  $\beta_3 = -3\%$ ,  $\beta_3 = -2\%$ ,  $\beta_3 = -1\%$ ,  $\beta_3 = 0\%$  and  $\beta_3 = 1\%$ . The other parameters are fixed. Draw the five different yield curves to estimate the effect of the curvature factor  $\beta_3$ .

*Solution 4.15* The following graph shows clearly the effect of the curvature factor  $\beta_3$  for the five different scenarios:



## 5 CHAPTER 5—Problems

*Exercise 5.1* Calculate the percentage price change for 4 bonds with different annual coupon rates (5% and 10%) and different maturities (3 years and 10 years), starting with a common 7.5% YTM (with annual compounding frequency), and assuming successively a new yield of 5%, 7%, 7.49%, 7.51%, 8% and 10%.

*Solution 5.1* Results are given in the following table:

New Yield (%)	Change (bps)	5%/3yr	10%/3yr	5%/10yr	10%/10yr
5.00	-250	6.95	6.68	20.71	18.31
7.00	-50	1.34	1.29	3.76	3.34
7.49	-1	0.03	0.03	0.07	0.07
7.51	+1	-0.03	-0.03	-0.07	-0.07
8.00	+50	-1.32	-1.26	-3.59	-3.19
10.00	+250	-6.35	-6.10	-16.37	-14.65



*Exercise 5.7* Compute the dirty price, the duration, the modified duration, the \$duration and the BPV (basis point value) of the following bonds with \$100 face value assuming that coupon frequency and compounding frequency are (1) annual; (2) semiannual and (3) quarterly.

Bond	Maturity (years)	Coupon Rate (%)	YTM (%)
Bond 1	1	5	5
Bond 2	1	10	6
Bond 3	5	5	5
Bond 4	5	10	6
Bond 5	5	5	7
Bond 6	5	10	8
Bond 7	20	5	5
Bond 8	20	10	6
Bond 9	20	5	7
Bond 10	20	10	8

*Solution 5.7* We use the following Excel functions “Price”, “Duration” and “MDuration” to obtain respectively the dirty price, the duration and the modified duration of each bond. The \$duration is simply given by the following formula:

$$\text{\$duration} = -\text{price} \times \text{modified duration}$$

The BPV is simply

$$\text{BPV} = \frac{-\text{\$duration}}{10,000}$$

1. When coupon frequency and compounding frequency are assumed to be annual, we obtain the following results:

	Price	Duration	Modified Duration	\$Duration	BPV
Bond 1	100	1	0.95	-95.24	0.00952
Bond 2	103.77	1	0.94	-97.90	0.00979
Bond 3	100	4.55	4.33	-432.95	0.04329
Bond 4	116.85	4.24	4	-467.07	0.04671
Bond 5	91.8	4.52	4.23	-388.06	0.03881
Bond 6	107.99	4.2	3.89	-420.32	0.04203
Bond 7	100	13.09	12.46	-1,246.22	0.12462
Bond 8	145.88	11.04	10.42	-1,519.45	0.15194
Bond 9	78.81	12.15	11.35	-894.72	0.08947
Bond 10	119.64	10.18	9.43	-1,127.94	0.11279

2. When coupon frequency and compounding frequency are assumed to be semi-annual, we obtain the following results:

—	Price	Duration	Modified Duration	\$Duration	BPV
Bond 1	100	0.99	0.96	−96.37	0.009637
Bond 2	103.83	0.98	0.95	−98.45	0.009845
Bond 3	100	4.49	4.38	−437.60	0.04376
Bond 4	117.06	4.14	4.02	−470.04	0.047004
Bond 5	91.68	4.46	4.31	−394.87	0.039487
Bond 6	108.11	4.1	3.94	−425.73	0.042573
Bond 7	100	12.87	12.55	−1,255.14	0.125514
Bond 8	146.23	10.77	10.46	−1,529.39	0.152939
Bond 9	78.64	11.87	11.47	−902.13	0.090213
Bond 10	119.79	9.87	9.49	−1,136.91	0.113691

3. When coupon frequency and compounding frequency are assumed to be quarterly, we obtain the following results:

—	Price	Duration	Modified Duration	\$Duration	BPV
Bond 1	100	0.98	0.97	−96.95	0.009695
Bond 2	103.85	0.96	0.95	−98.72	0.009872
Bond 3	100	4.45	4.40	−439.98	0.043998
Bond 4	117.17	4.08	4.02	−471.53	0.047153
Bond 5	91.62	4.42	4.35	−398.40	0.03984
Bond 6	108.18	4.04	3.96	−428.51	0.042851
Bond 7	100	12.75	12.60	−1,259.67	0.125967
Bond 8	146.41	10.64	10.48	−1,534.44	0.153444
Bond 9	78.56	11.73	11.53	−905.89	0.090589
Bond 10	119.87	9.71	9.52	−1,141.47	0.114147

### Exercise 5.11 Zero-coupon Bonds

1. What is the price of a zero-coupon bond with \$100 face value that matures in seven years and has a yield of 7%? We assume that the compounding frequency is semiannual.
2. What is the bond's modified duration?
3. Use the modified duration to find the approximate change in price if the bond yield rises by 15 basis points.

*Solution 5.11* 1. The price  $P$  is given by

$$P = \frac{\$100}{\left(1 + \frac{7\%}{2}\right)^{2 \times 7}} = \$61.77818$$

2. The modified duration  $MD$  is given by

$$MD = \frac{1}{P \left(1 + \frac{7\%}{2}\right)} \times \sum_i t_i \text{PV}(\text{CF}_i) = 6.763$$

3. The approximate change in price is  $-\$0.627$

$$\Delta P \simeq -MD \times \Delta y \times P = -6.763 \times 0.0015 \times \$61.77818 = -\$0.627$$

*Exercise 5.19* An investor holds 100,000 units of a bond whose features are summarized in the following table. He wishes to be hedged against a rise in interest rates.

Maturity	Coupon Rate	YTM	Duration	Price
18 Years	9.5%	8%	9.5055	\$114,181

Characteristics of the hedging instrument, which is here a bond are as follows:

Maturity	Coupon Rate	YTM	Duration	Price
20 Years	10%	8%	9.8703	\$119.792

Coupon frequency and compounding frequency are assumed to be semiannual. YTM stands for yield to maturity. The YTM curve is flat at an 8% level.

1. What is the quantity  $\phi$  of the hedging instrument that the investor has to sell?
2. We suppose that the YTM curve increases instantaneously by 0.1%.
  - (a) What happens if the bond portfolio has not been hedged?
  - (b) And if it has been hedged?
3. Same question as the previous one when the YTM curve increases instantaneously by 2%.
4. Conclude.

*Solution 5.19* 1. The quantity  $\phi$  of the hedging instrument is obtained as follows:

$$\phi = -\frac{11,418,100 \times 9.5055}{119.792 \times 9.8703} = -91,793$$

The investor has to sell 91,793 units of the hedging instrument.

2. Prices of bonds with maturity 18 years and 20 years become respectively \$113.145 and \$118.664.
  - (a) If the bond portfolio has not been hedged, the investor loses money. The loss incurred is given by the following formula (exactly  $-\$103,657$  if we take all the decimals into account):

$$\text{Loss} = \$100,000 \times (113.145 - 114.181) = -\$103,600$$

- (b) If the bond portfolio has been hedged, the investor is quasi-neutral to an increase (and a decrease) of the YTM curve. The P&L of the position is given by the following formula:

$$\text{P\&L} = -\$103,600 + \$91,793 \times (119.792 - 118.664) = -\$57$$

3. Prices of bonds with maturity 18 years and 20 years become respectively \$95.863 and \$100.

- (a) If the bond portfolio has not been hedged, the loss incurred is given by the following formula:

$$\text{Loss} = \$100,000 \times (95.863 - 114.181) = -\$1,831,800$$

- (b) If the bond portfolio has been hedged, the P&L of the position is given by the following formula:

$$\text{P\&L} = -\$1,831,800 + \$91,793 \times (119.792 - 100) = -\$15,032$$

4. For a small move of the YTM curve, the quality of the hedge is good. For a large move of the YTM curve, we see that the hedge is not perfect because of the convexity term that is no more negligible (see Chapter 6).

## 6 CHAPTER 6—Problems

*Exercise 6.1* We consider a 20-year zero-coupon bond with a 6% YTM and \$100 face value. Compounding frequency is assumed to be annual.

1. Compute its price, modified duration, \$duration, convexity and \$convexity?
2. On the same graph, draw the price change of the bond when YTM goes from 1% to 11%
  - (a) by using the exact pricing formula;
  - (b) by using the one-order Taylor estimation;
  - (c) by using the second-order Taylor estimation.

*Solution 6.1* 1. The price  $P$  of the zero-coupon bond is simply

$$P = \frac{\$100}{(1 + 6\%)^{20}} = \$31.18$$

Its modified duration is equal to  $20/(1 + 6\%) = 18.87$

Its \$duration, denoted by \$Dur, is equal to

$$\text{\$Dur} = -18.87 \times 31.18 = -588.31$$

Its convexity, denoted by  $RC$ , is equal to

$$RC = 20 \times 21 \times \frac{100}{(1 + 6\%)^{22}} = 373.80$$

Its \$convexity, denoted by \$Conv, is equal to

$$\text{\$Conv} = 373.80 \times 31.18 = 11,655.20$$

2. Using the one-order Taylor expansion, the new price of the bond is given by the following formula:

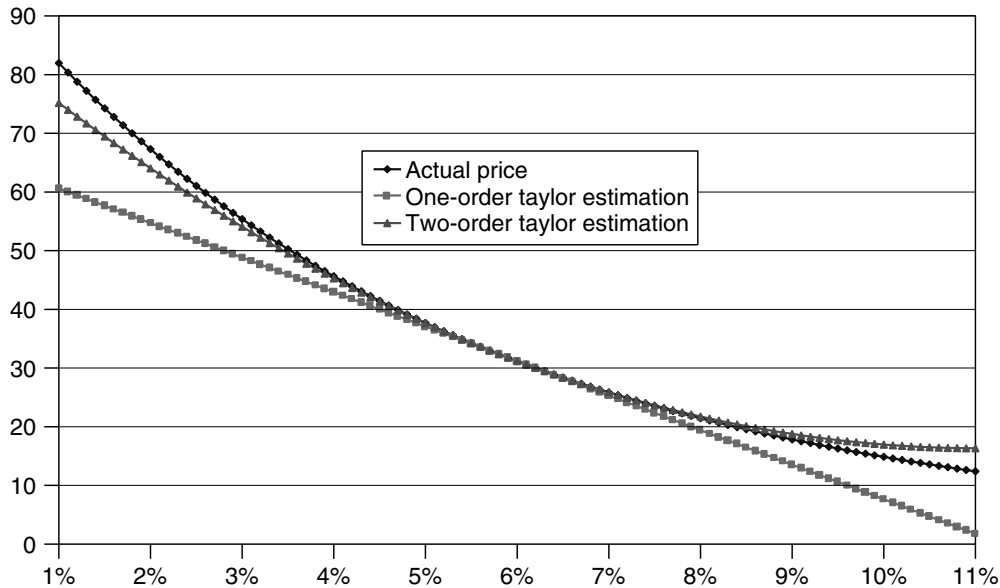
$$\text{New Price} = 31.18 + \text{\$Dur} \times (\text{New YTM} - 6\%)$$

Using the two-order Taylor expansion, the new price of the bond is given by the following formula:

$$\text{New Price} = 31.18 + \$\text{Dur} \times (\text{New YTM} - 6\%) + \frac{\$Conv}{2} \times (\text{New YTM} - 6\%)^2$$

We finally obtain the following graph.

The straight line is the one-order Taylor estimation. Using the two-order Taylor estimation, we underestimate the actual price for YTM inferior to 6%, and we overestimate it for YTM superior to 6%.



*Exercise 6.6* Assume a 2-year Euro-note, with a \$100,000 face value, a coupon rate of 10% and a convexity of 4.53. If today's YTM is 11.5% and term structure is flat. Coupon frequency and compounding frequency are assumed to be annual.

1. What is the Macaulay duration of this bond?
2. What does convexity measure? Why does convexity differ among bonds? What happens to convexity when interest rates rise? Why?
3. What is the exact price change in dollars if interest rates increase by 10 basis points (a uniform shift)?
4. Use the duration model to calculate the approximate price change in dollars if interest rates increase by 10 basis points.
5. Incorporate convexity to calculate the approximate price change in dollars if interest rates increase by 10 basis points.

## Solution 6.6 1. Duration

$$D = \frac{1 \times \frac{10,000}{1.115} + 2 \times \frac{10,000}{1.115^2} + 2 \times \frac{100,000}{1.115^2}}{\frac{10,000}{1.115} + \frac{10,000}{1.115^2} + \frac{100,000}{1.115^2}}$$

$$= \frac{1 \times \frac{10,000}{1.115} + 2 \times \frac{10,000}{1.115^2} + 2 \times \frac{100,000}{1.115^2}}{97,448.17} = 1.908$$

2. Convexity measures the change in modified duration or the change in the slope of the price-yield curve. Holding maturity constant, the higher the coupon, the smaller the duration. Hence, for low duration levels the change in slope (convexity) is small. Alternatively, holding coupon constant, the higher the maturity, the higher the duration, and hence, the higher the convexity. When interest rates rise, duration (sensitivity of prices to changes in interest rates) becomes smaller. Hence, we move toward the flatter region of the price-yield curve. Therefore, convexity will decrease parallel to duration.

3. Price for a 11.6% YTM is

$$P(11.6\%) = \frac{10,000}{1.116} + \frac{10,000}{1.116^2} + \frac{100,000}{1.116^2} = \$97,281.64$$

Price has decreased by \$166.53 from  $P(11.5\%) = \$97,448.17$  to \$97,281.64

4. We use

$$\Delta P \simeq -MD \times \Delta y \times P(11.5\%) = -\frac{D}{1+y} \times \Delta y \times P$$

$$= -\frac{1.908}{1.115} \times 97,448.17 \times 0.001 = -\$166.754$$

5. We use

$$\Delta P \simeq -MD \times \Delta y \times P + \frac{1}{2} \times RC \times (\Delta y)^2 \times P$$

$$= -\frac{1.908}{1.115} \times 97,448.17 \times 0.001 + \frac{1}{2}$$

$$\times 4.53 \times (0.001)^2 \times 97,448.17 = -\$166.531$$

Hedging error is smaller when we account for convexity.

## Exercise 6.8 Modified Duration/Convexity Bond Portfolio Hedge

At date  $t$ , the portfolio  $P$  to be hedged is a portfolio of Treasury bonds with various possible maturities. Its characteristics are as follows:

Price	YTM	MD	Convexity
\$28,296,919	7.511%	5.906	67.578

We consider Treasury bonds as hedging assets, with the following features:

Bond	Price (\$)	Coupon Rate (%)	Maturity date
Bond 1	108.039	7	3 years
Bond 2	118.786	8	7 years
Bond 3	97.962	5	12 years

Coupon frequency and compounding frequency are assumed to be annual. At date  $t$ , we force the hedging portfolio to have the opposite value of the portfolio to be hedged.

1. What is the number of hedging instruments necessary to implement a modified duration/convexity hedge?
2. Compute the YTM, modified duration and convexity of the three hedging assets.
3. Which quantities  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  of each of the hedging asset 1, 2, 3 do we have to consider to hedge the portfolio  $P$ ?

*Solution 6.8*

1. We need three hedging instruments.
2. We obtain the following results:

Bond	YTM (%)	MD	Convexity
Bond 1	6.831	2.629	9.622
Bond 2	7.286	5.267	36.329
Bond 3	7.610	8.307	90.212

3. We then are looking for the quantities  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  of each hedging instrument 1, 2, 3 as solutions to the following linear system:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 100.445 & 103.808 & 79.929 \\ -264.057 & -546.791 & -663.947 \\ 966.460 & 3,771.257 & 7,210.58 \end{pmatrix}^{-1} \begin{pmatrix} -28,296,919 \\ 167,143,615 \\ -1,912,260,201 \end{pmatrix} = \begin{pmatrix} -279,536 \\ 290,043 \\ -379,432 \end{pmatrix}$$

## 7 CHAPTER 7—Problems

*Exercise 7.1* Would you say it is easier to track a bond index or a stock index. Why or why not?

*Solution 7.1* As is often the case, the answer is yes and no.

On the one hand, it is harder to perform *perfect replication* of a bond index compared to a stock index. This is because bond indices typically include a huge number of bonds. Other difficulties include that many of the bonds in the indices are thinly traded and the fact that the composition of the index changes regularly, as they mature.

On the other hand, statistical replication on bond indices is easier to perform than statistical replication of stock indices, in the sense that a significantly lower tracking error can usually be achieved for a given number of instruments in the replicating portfolio. This is because bonds with different maturities tend to exhibit a fair amount of cross-sectional correlation so that a very limited number of factors account for a very large fraction of changes in bond returns. Typically, 2 or 3 factors (level, slope, curvature) account for more than 80% of these variations. Stocks typically exhibit much more idiosyncratic risk, so that one typically needs to use a large number of factors to account for not much more than 50% of the changes in stock prices.

## 8 CHAPTER 8—Problems

*Exercise 8.2* Choosing a Portfolio with the Maximum \$ Duration or Modified Duration Possible  
Consider at date  $t$ , five bonds delivering annual coupon rates with the following features:

Maturity (years)	CR (%)	YTM (%)	Price
5	7	4	113.355
7	6	4.5	108.839
15	8	5	131.139
20	5	5.25	96.949
22	7	5.35	121.042

CR stands for coupon rate and YTM for yield to maturity.

A portfolio manager believes that the YTM curve will very rapidly decrease by 0.3% in level. Which of these bonds provides the maximum absolute gain? Which of these bonds provides the maximum relative gain?

*Solution 8.2* We compute the modified duration and the \$duration of these five bonds. In the scenario anticipated by the portfolio manager, we then calculate the absolute gain and the relative gain that the portfolio manager will earn with each of these five bonds

Maturity (years)	$MD$	\$Dur	Absolute Gain (\$)	Relative Gain (%)
5	4.258	-482.7	1.448	1.28
7	5.711	-621.62	1.865	1.71
15	9.52	-1,248.4	3.745	2.86
20	12.322	-1,194.6	3.584	3.70
22	12.095	-1,464	4.392	3.63

$MD$  stands for modified duration and \$Dur for \$duration.

If he wants to optimize his absolute gain, the portfolio manager will choose the 22-year maturity bond. On the contrary, if he prefers to optimize his relative gain, he will invest in the 20-year maturity bond.

*Exercise 8.4* Rollover Strategy

An investor has funds to invest over one year. He anticipates a 1% increase in the curve in six months. The 6-month and 1-year zero-coupon rates are respectively 3% and 3.2%. He has two different opportunities:

- he can buy the 1-year zero-coupon T-bond and hold it until maturity,
- or he can choose a rollover strategy by buying the 6-month T-bill, holding it until maturity, and buying a new 6-month T-bill in six months, and holding it until maturity.



1. Calculate the annualized total return rate of these two strategies assuming that the investor's anticipation is correct.
2. Same question when interest rates decrease by 1% after six months.

*Solution 8.4*

1. The annualized total return rate of the first strategy is, of course, 3.2%

$$\frac{100 - \frac{100}{(1 + 3.2\%)}}{100} = \frac{100 - 96.899}{96.899} = 3.2\%$$

By doing a rollover, the investor will invest at date  $t = 0$ , \$98.533 to obtain \$100 six months later. Note that \$98.533 is obtained as follows:

$$\frac{\$100}{(1 + 3\%)^{\frac{6}{12}}} = \$98.533$$

Six months later, he will then buy a quantity  $\alpha = 100/98.058$  of a 6-month T-bill, whose price is \$98.058  $\left(\frac{\$100}{(1+4\%)^{\frac{6}{12}}} = \$98.058\right)$  and which pays \$100 at maturity so that the annualized total return rate of the second strategy is 3.5%

$$\frac{100 \times \alpha - 98.533}{98.533} = 3.5\%$$

2. The annualized total return rate of the first strategy is still 3.2% as it is only 2.5% for the rollover strategy

$$\frac{100 \times \beta - 98.533}{98.533} = 2.5\%$$

where  $\beta = 100/99.015$ .

In this case, the rollover strategy is worse than the simple buy-and-hold strategy.

*Exercise 8.7* Butterfly

We consider three bonds with short, medium and long maturities whose features are summarized in the following table:

Maturity (years)	Coupon Rate (%)	YTM (%)	Bond Price (\$)	\$Duration	Quantity
2	6	6	100	-183.34	$q_s$
10	6	6	100	-736.01	-10,000
30	6	6	100	-1,376.48	$q_l$

YTM stands for yield to maturity, bond prices are dirty prices, and we assume a flat yield-to-maturity curve in the exercise. We structure a butterfly in the following way:

- we sell 10,000 10-year bonds;
- we buy  $q_s$  2-year bonds and  $q_l$  30-year bonds.

1. Determine the quantities  $q_s$  and  $q_l$  so that the butterfly is cash-and-duration-neutral.
2. What is the P&L of the butterfly if the yield-to-maturity curve goes up to a 7% level? And down to a 5% level?
3. Draw the P&L of the butterfly depending on the value of the yield to maturity.

*Solution 8.7* 1. The quantities  $q_s$  and  $q_l$ , which are determined so that the butterfly is cash-and-duration-neutral, satisfy the following system:

$$\begin{cases} (q_s \times 183.34) + (q_l \times 1,376.48) = 10,000 \times 736.01 \\ (q_s \times 100) + (q_l \times 100) = 10,000 \times 100 \end{cases}$$

whose solution is

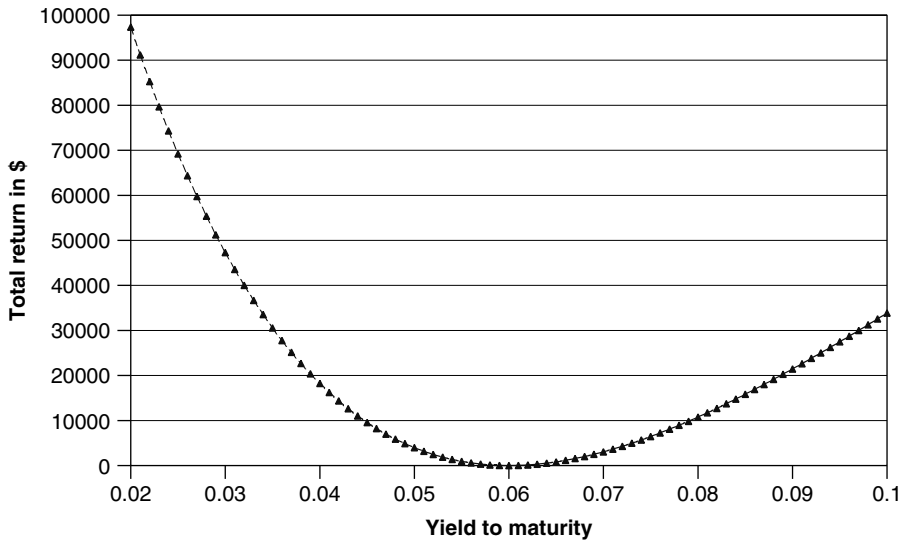
$$\begin{pmatrix} q_s \\ q_l \end{pmatrix} = \begin{pmatrix} 183.34 & 1,376.48 \\ 100 & 100 \end{pmatrix}^{-1} \times \begin{pmatrix} 7,360,100 \\ 1,000,000 \end{pmatrix} = \begin{pmatrix} 5,368 \\ 4,632 \end{pmatrix}$$

2. If the yield-to-maturity curve goes up to a 7% level or goes down to a 5% level, bond prices become

Maturity (years)	Price if $YTM = 7\%$	Price if $YTM = 5\%$
2	98.19	101.86
10	92.98	107.72
30	87.59	115.37

P&Ls are respectively \$3,051 and \$3,969 when the yield-to-maturity curve goes up to a 7% level or goes down to a 5% level.

3. We draw below the P&L profile of the butterfly depending on the value of the yield to maturity



The butterfly has a positive convexity. Whatever the value of the yield to maturity, the strategy always generates a gain. This gain is all the more substantial as the yield to maturity reaches a level further away from 6%.

## 9 CHAPTER 9—Problems

*Exercise 9.2* We have registered for each month the end-of-month value of a bond index, which is as follows:

Month	Index Value	Month	Index Value
January	98	July	112
February	101	August	110
March	104	September	111
April	107	October	112
May	111	November	110
June	110	December	113

At the beginning of January, the index value is equal to 100.

1. Compute the arithmetic average rate of return denoted by *AARR*, and calculate the final index value by using the following formula:

$$\text{Final Index Value} = 100 \times (1 + AARR)^{12}$$

2. Compute the time-weighted rate of return denoted by *TWRR*, and calculate the final index value by using the following formula:

$$\text{Final Index Value} = 100 \times (1 + TWRR)^{12}$$

3. Same question with the following index values:

Month	Index Value	Month	Index Value
January	85	July	115
February	110	August	130
March	120	September	145
April	135	October	160
May	115	November	145
June	100	December	160

4. Conclude.

*Solution 9.2* 1. We first compute the monthly rate of return

Month	Monthly Rate of Return (%)	Month	Monthly Rate of Return (%)
January	-2.00	July	1.82
February	3.06	August	-1.79
March	2.97	September	0.91
April	2.88	October	0.90
May	3.74	November	-1.79
June	-0.90	December	2.73

The arithmetic average rate of return is the average of the monthly rate of return, and is equal to

$$AARR = 1.045\%$$

The final index value is equal to

$$\text{Final Index Value} = 100 \times (1 + AARR)^{12} = 113.28$$

2. The time-weighted rate of return is given by the following formula:

$$TWRR = [(1 + R_J) \times (1 + R_F) \times \cdots \times (1 + R_D)]^{\frac{1}{12}} - 1$$

where  $R_J$ ,  $R_F$  and  $R_D$  are the monthly rate of return registered in January, February and December respectively.

We obtain

$$TWRR = 1.024\%$$

The final index value is equal to

$$\text{Final Index Value} = 100 \times (1 + TWRR)^{12} = 113$$

which, of course, corresponds to the actual final index value.

3. We first compute the monthly rate of return

Month	Monthly Rate of Return (%)	Month	Monthly Rate of Return (%)
January	-15.00	July	15.00
February	29.41	August	13.04
March	9.09	September	11.54
April	12.50	October	10.34
May	-14.81	November	-9.38
June	-13.04	December	10.34

The arithmetic average rate of return is the average of the monthly rate of return, and is equal to

$$AARR = 4.920\%$$

The final index value is equal to

$$\text{Final Index Value} = 100 \times (1 + AARR)^{12} = 177.95$$

The time-weighted rate of return is equal to

$$TWRR = 1.024\%$$

and the final index value is, of course, equal to 160.

4. It is incorrect to take the arithmetic average rate of return as a measure of the average return over an evaluation period. In fact, the arithmetic average rate of return is always superior or equal to the time-weighted rate of return. The difference between these two indicators of performance is all the more substantial than the variation in the subperiods returns are large over the valuation period.

## 10 CHAPTER 10—Problems

*Exercise 10.4* We consider two firms A and B that have the same financial needs in terms of maturity and principal. The two firms can borrow money in the market at the following conditions:

- Firm A: 11% at a fixed rate or Libor + 2% for a \$10 million loan and a 5-year maturity.
  - Firm B: 9% at a fixed rate or Libor + 0.25% for a \$10 million loan and a 5-year maturity.
1. We suppose that firm B prefers a floating-rate debt as firm A prefers a fixed-rate debt. What is the swap they will structure to optimize their financial conditions?
  2. If firm B prefers a fixed-rate debt as firm A prefers a floating-rate debt, is there a swap to structure so that the two firms optimize their financial conditions? Conclude.

*Solution 10.4* 1. Firm B has 2% better conditions at a fixed rate and 1.75% better conditions at a floating rate than firm A. The spread between the conditions obtained by firm A and firm B at a fixed rate and the spread between the conditions obtained by firm A and firm B at a floating rate is different from 0.25%.

To optimize their financial conditions:

Firm B borrows money at a 9% fixed rate, firm A contracts a loan at a Libor + 2% floating rate and they structure the following swap. Firm B pays Libor + 0.125% and receives the fixed 9% as firm A receives Libor + 0.125% and pays the fixed 9%. The financing operation is summarized in the following table:

	Firm A	Firm B
<b>Initial Financing</b>	(Libor + 2%)	(9%)
<b>Swap A to B</b>	(9%)	9%
<b>Swap B to A</b>	Libor + 0.125%	(Libor + 0.125%)
<b>Financing Cost</b>	(10.875%)	(Libor + 0.125%)
<b>Financing Cost without Swap</b>	(11%)	(Libor + 0.25%)
<b>Gain</b>	0.125%	0.125%

By structuring a swap, firm A and firm B have optimized their financial conditions and each firm has gained 0.125%.

2. There is no swap to structure between the two firms so that they can improve their financial conditions.

*Exercise 10.7* We consider at date  $T_0$ , a 1-year Libor swap contract with maturity 10 years and with the following cash-flow schedule:

	$F$	$F$	$F$	$F$	$F$	$F$	$F$	$F$	$F$	$F$
$T_0$	$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$	$T_7$	$T_8$	$T_9$	$T_{10}$
	$-V_0$	$-V_1$	$-V_2$	$-V_3$	$-V_4$	$-V_5$	$-V_6$	$-V_7$	$-V_8$	$-V_9$

Note that  $T_{i+1} - T_i = 1$  year,  $\forall i \in \{0, 1, 2, \dots, 9\}$ . We suppose that the swap nominal amount is \$10 million, and that the rate  $F$  of the fixed leg is 9.55%.

At date  $T_0$ , zero-coupon rates with maturities  $T_1, T_2, \dots, T_{10}$  are given in the following table:

Maturity	ZC rates (%)	Maturity	ZC rates (%)
$T_1$	8.005	$T_6$	9.235
$T_2$	7.856	$T_7$	9.478
$T_3$	8.235	$T_8$	9.656
$T_4$	8.669	$T_9$	9.789
$T_5$	8.963	$T_{10}$	9.883

1. Give the price of this swap.
2. What is the swap rate such that the price of this swap is zero?
3. An investor holds a bond portfolio whose price, yield to maturity and modified duration are respectively \$9,991,565,452, 9.2% and 5.92. He wants to be protected against an increase in rates. How many swaps must he sell to protect his bond portfolio?

*Solution 10.7*

1. The price of this swap is given by the following formula:

$$SWAP_{T_0} = \$10,000,000 \cdot \left( \sum_{i=1}^{10} 9.55\% \cdot B(T_0, T_i) - 1 + B(T_0, T_{10}) \right)$$

We first have to compute the discount factors  $B(T_0, T_i)$  for  $i = 1, 2, \dots, 10$  using the following formula:

$$B(T_0, T_i) = \frac{1}{[1 + R(T_0, T_i - T_0)]^{T_i - T_0}}$$

They are given below

Maturity	$B(T_0, T_i)$	Maturity	$B(T_0, T_i)$
$T_1$	0.92588	$T_6$	0.58861
$T_2$	0.85963	$T_7$	0.53053
$T_3$	0.78867	$T_8$	0.47834
$T_4$	0.71710	$T_9$	0.43149
$T_5$	0.65104	$T_{10}$	0.38967

Finally, the price of the swap is  $-\$28,598$ .

2. The swap rate is equal to 9.595%.
3. Assuming that the yield-to-maturity move of the bond portfolio to be hedged is equal to the yield to maturity of the bond contained in the swap, we find the number  $\phi_s$  of swaps that the investor has to sell according to the following

equation:

$$P_B \times MD_B = \phi_s \times N_S \times P_S \times MD_S$$

where  $P_B$  is the price in \$ of the bond portfolio held by the investor,  $MD_B$ , the modified duration of the bond portfolio,  $N_S$ , the nominal amount of the swap.  $P_S$  is the price in \$ of the bond contained in the swap.  $MD_S$  is the modified duration of the bond contained in the swap.

The price  $P_S$  is equal to 99.714%. Using the Excel functions “Yield” and MDuration”, we obtain the yield to maturity and the modified duration of the bond contained in the swap. They are respectively 9.596% and 6.25823.

We finally obtain

$$\phi_s = \frac{9,991,565,452 \times 5.92}{10,000,000 \times 99.714\% \times 6.25823} = 947.87$$

so that the investor has to sell 948 swaps.

## 11 CHAPTER 11—Problems

*Exercise 11.1* The Cheapest-to-Deliver on the Repartition Date

We are on the repartition date. Consider a futures contract with size Eur 100,000 whose price is 95% and three bonds denoted by A,B and C with the following features. What is the bond that the seller of the futures contract will choose to deliver?

	Clean Price (%)	Conversion Factor (%)
Bond A	112.67	119.96
Bond B	111.54	118.66
Bond C	111.47	119.78

*Solution 11.1* The seller of the futures contract will choose to deliver the bond that maximizes the difference between the invoice price  $IP$  and the cost of purchasing the bond  $CP$ , which is called the *cheapest-to-deliver*. The quantity  $IP - CP$  is given by the following formula:

$$IP-CP = Size \times [futures\ price \times CF - clean\ price]$$

where  $CF$  is the conversion factor. Here, we obtain the following results:

	IP-CP
Bond A	1,292
Bond B	1,187
Bond C	2,321

The seller of the futures contract will choose to deliver Bond C.

*Exercise 11.4* An investor holds a bond portfolio with principal value \$10,000,000 whose price and modified duration are respectively 112 and 9.21. He wishes to be hedged against a rise in interest rates by selling futures contracts written on a bond.

Suppose the futures price of the contract is 105.2. The contract size is \$100,000. The conversion factor for the cheapest-to-deliver is equal to 98.1%. The cheapest-to-deliver has a modified duration equal to 8.

1. Give a proof of the hedge ratio  $\phi_f$  as obtained in equation (11.4).
2. Determine the number of contracts  $\phi_f$  he has to sell.

*Solution 11.4* 1. To be immunized, the investor has to sell  $\phi_f$  contracts so that

$$(N_P \times dP) + (\phi_f \times N_F \times dF) = 0$$

where  $N_P$  and  $N_F$  are respectively the principal amount of the bond portfolio to be hedged and of the futures contract, whose prices (in % of the principal amount) are denoted by  $P$  and  $F$ .  $dP$  and  $dF$  represent price changes of  $P$  and  $F$ .

We now suppose that

$$dP_{CTD} = CF \times dF$$

where  $P_{CTD}$  is the price (in % of the size) of the cheapest-to-deliver,  $CF$  its conversion factor.

We then write

$$(N_P \times dP) + \left( \phi_f \times N_F \times \frac{dP_{CTD}}{CF} \right) = 0$$

We now denote by  $y$  and  $y_1$ , the yield to maturity of the bond portfolio to be hedged and of the cheapest-to-deliver, with the assumption that  $dy = dy_1$  (the yield-to-maturity curve is only affected by parallel shifts).

Using the one-dimensional version of Taylor expansion, we obtain

$$[N_P \times P'(y)] + \left[ \phi_f \times \frac{N_F}{CF} \times P'_{CTD}(y_1) \right] = 0$$

and

$$[N_P \times P \times MD_P] + \left[ \phi_f \times \frac{N_F}{CF} \times P_{CTD} \times MD_{CTD} \right] = 0$$

where  $MD_P$  and  $MD_{CTD}$  are respectively the modified duration of the bond portfolio  $P$  and of the cheapest-to-deliver bond.

Finally,  $\phi_f$  is given by

$$\begin{aligned} \phi_f &= -\frac{N_P \times P \times MD_P}{N_F \times P_{CTD} \times MD_{CTD}} \times CF \\ &= -\frac{\$Dur_P}{\$Dur_{CTD}} \times CF \end{aligned}$$

2. The investor has to sell 125 contracts as given in the following formula:

$$\phi_f = -\frac{10,000,000 \times 112\% \times 9.21}{1,000,000 \times 98.1\% \times 105.2\% \times 8} = -124.94$$

*Exercise 11.6* Hedging a Bond Position Using Eurodollar Futures



Suppose that you are a bond trader. You have just sold \$10,000,000 par value of a 10% coupon bond that matures in exactly three-fourth of a year. The yield is 6.5%. Compounding frequency is semiannual. You do not have the bond in inventory, but think you can purchase it in the market tomorrow. You want to hedge your interest-rate risk overnight with Eurodollar futures. Do you buy or sell? How many contracts do you need?

*Solution 11.6* To answer those questions, we first need to find the price  $P$  and duration  $D$  of the bond.

$$P = \frac{\frac{1}{2} \times 10\% \times 10,000,000}{\left(1 + \frac{6.5\%}{2}\right)^{2 \times 0.25}} + \frac{10,000,000 \times \left(1 + \frac{1}{2} \times 10\%\right)}{\left(1 + \frac{6.5\%}{2}\right)^{2 \times 0.75}}$$

$$= 492,067.8 + 10,008,159 = 10,500,227$$

$$\sum_{i=1}^2 \theta_i PV(F(\theta_i)) = 0.25 \times 492,067.8 + 0.75 \times 10,008,159$$

$$= 123,017 + 7,506,119 = 7,629,136$$

$$D = \frac{7,629,136}{10,500,227} = 0.726569$$

If interest rates rise by one basis point, the value of the bond loses approximately

$$-MD \times \Delta y = -D \times \frac{1}{\left(1 + \frac{6.5\%}{2}\right)} \times P \times \Delta y$$

$$= \underbrace{-0.726569 \times \frac{1}{\left(1 + \frac{6.5\%}{2}\right)}}_{=MD=0.703689} \times 10,500,227 \times 0.0001 = \$738.8984$$

Note that since we are short the bond, we will gain when rates go up. We also know that the Eurodollar futures contract loses \$25 when Libor goes up by one basis point. That means we want to purchase futures. We gain on our short position in the bond and lose on the futures when rates go up, and vice versa when rates go down. We need to get enough contracts to cover our position. The proper number of contracts is  $\frac{738.8984}{25} = 29.55598$ . Since we can only buy a whole number of contracts, we should buy 30 contracts.

*Exercise 11.10* Speculation with Futures

Today, the gross price of a 10-year bond with \$1,000 principal amount is 116.277. At the same moment, the price of the 10-year futures contract which expires in two months is 98.03. Its nominal amount is \$100,000, and the deposit margin is \$1,000. One month later, the price of the bond is 120.815 as the futures price is 102.24.

1. What is the leverage effect on this futures contract?
2. An investor anticipates that rates will decrease in a short-term period. His cash at disposal is \$100,000.
  - (a) What is the position he can take on the market using the bond? What is his absolute gain after one month? What is the return rate of his investment?
  - (b) Same question as the previous one using the futures contract.
  - (c) Conclude.

*Solution 11.10* 1. The leverage effect of the futures contract is 100

$$\text{Leverage Effect} = \frac{\text{No min al Amount}}{\text{Deposit Margin}} = \frac{100,000}{1,000} = 100$$

2. (a) The investor anticipates a decrease in rates so he will buy bonds. His cash at disposal is \$100,000. Then he buys 86 bonds

$$\text{Number of bonds bought} = \frac{\$100,000}{\$1,000 \times 116.277\%} = 86$$

His absolute gain over the period is \$3,902.68

$$\text{Absolute Gain} = 86 \times \$1,000 \times [120.815 - 116.277]\% = \$3,902.68$$

The return rate of his investment over the period is 3.903%

$$\text{Return Rate} = \frac{\$3,902.68}{\$100,000} = 3.903\%$$

- (b) Futures contracts move in the same way as bonds when interest rates change, so the investor will buy futures contracts. His cash at disposal is \$100,000, then he buys 102 futures contracts

$$\begin{aligned} \text{Number of futures contracts bought} &= \frac{\$100,000}{\text{deposit margin} \times \text{futures price}} \\ &= \frac{\$100,000}{\$1,000 \times 98.03\%} = 102 \end{aligned}$$

His absolute gain over the period is

$$\text{Absolute Gain} = 102 \times \$100,000 \times [102.24 - 98.03]\% = \$429,420$$

The return rate of his investment over the period is 429.42%

$$\text{Return Rate} = \frac{\$429,420}{\$100,000} = 429.42\%$$

- (c) The difference of performance between the two investments is explained by the leverage effect of the futures contract.

## 12 CHAPTER 12—Problems

*Exercise 12.1* Today's term structure of par Treasury yields is assumed to have the following values:

Maturity (in years)	Par yield (%)
1	3.50
2	3.70
3	3.80

Derive the corresponding recombining binomial interest-rate tree, assuming a 1-year interest-rate volatility of 3%.

*Solution 12.1* Let us determine  $r_u$  and  $r_l$ . We have

$$\frac{1}{2} \left( \frac{\frac{100+3.7}{(1+r_l \exp(6/100))} + 3.7}{1 + 3.5\%} + \frac{\frac{100+3.7}{(1+r_l)} + 3.7}{1 + 3.5\%} \right) = 100$$

which gives

$$r_l = 3.79\%$$

and

$$r_u = r_l \exp(6/100) = 4.03\%$$

Let us now move on to the computation of  $r_{uu}$ ,  $r_{ul}$  and  $r_{ll}$ . We have

$$\frac{1}{2} \left( \frac{\frac{1}{2} \left( \frac{\frac{100+3.8}{(1+r_{ll} \exp(12/100))} + 3.8}{1+4.03\%} + \frac{\frac{100+3.8}{(1+r_{ll} \exp(6/100))} + 3.8}{1+4.03\%} \right) + 3.8}{1 + 3.5\%} + \frac{\frac{1}{2} \left( \frac{\frac{100+3.8}{(1+r_{ll} \exp(6/100))} + 3.8}{1+3.79\%} + \frac{\frac{100+3.8}{(1+r_{ll})} + 3.8}{1+3.79\%} \right) + 3.8}{1 + 3.5\%} \right) = 100$$

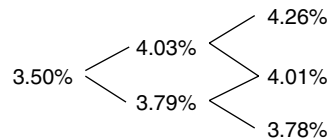
$$r_{ll} = 3.78\%$$

$$r_{ul} = r_{ll} \exp(6/100) = 4.01\%$$

and

$$r_{uu} = r_{ll} \exp(12/100) = 4.26\%$$

Hence, we get



*Exercise 12.2* What are the main drawbacks of the Vasicek (1977) model?

*Solution 12.2* This model generates zero-coupon rates that are not consistent with the observed yield curve.

The zero-coupon curve may only be flat, increasing or decreasing. But it does not allow inverted yield curves, a U-shaped curve or a hump-shaped curve.

$R(t, \infty)$  is a constant, which makes the right end of the yield curve fixed for a given choice of the parameters.

Besides, zero-coupon rates can take on negative values with positive probability.

*Exercise 12.11* The goal of this exercise is to construct the Hull and White's trinomial tree (see the Appendix 1 of Chapter 12). We consider the following parameter values:

$\lambda$	$\sigma$	$\Delta t$
0.15	0.01	1 year

1. Draw the graph of the zero-coupon rate volatilities.
2. Applying the normal scheme, construct the lattice of the short-term rate  $r^*$  (with an initial value equal to 0).
3. We now assume that the spot yield curve is as follows up to the 5-year maturity:

Maturity (years)	Spot rate (%)	Discount factor
1	4.00	0.96079
2	4.15	0.92035
3	4.25	0.88029
4	4.33	0.84097
5	4.40	0.80252

Construct the lattice of the short-term rate  $r$  by using the continuous-time relationship between  $r$  and  $r^*$

$$\alpha(t) = r(t) - r^*(t) = f(0, t) + \frac{\sigma^2}{2\lambda^2}(1 - e^{-\lambda t})^2$$

where  $f(0, t)$  is the instantaneous forward rate at date 0. Here, because  $\Delta t = 1$  year, it is the 1-year forward rate determined at date 0 and beginning at date  $t$ .

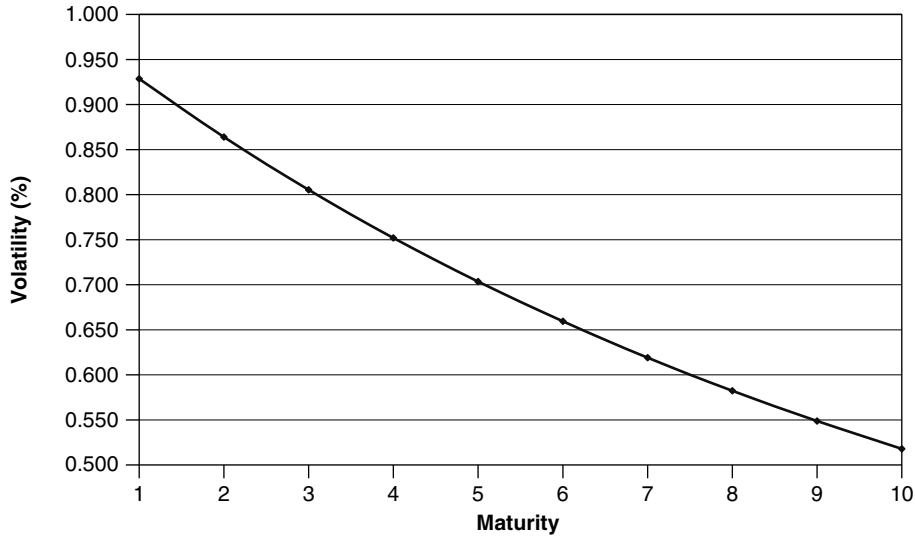
4. Same question as 3- but using the approach proposed by Hull and White.
5. Price the put option, which provides the following payoff at date  $t = 2$ :

$$\text{Max}[0; 10,000 \times (5\% - r)]$$

*Solution 12.11* 1. We draw below the graph of the zero-coupon rate volatilities  $V(t, T)$  given by the following formula:

$$V(t, T) = \sigma \left( \frac{1 - e^{-\lambda(T-t)}}{\lambda(T-t)} \right)$$

where  $T - t$  is the maturity of the zero-coupon rate.



2. We now build the lattice for  $r^*$  up to  $i = 2$ , applying the normal scheme.  $\Delta r = 1.73\%$ , and the results are shown below.

i=0		i=1		i=2	
					0.7617
				3.46%	0.1766
				$j=2$	0.0617
			0.1029		0.1029
		1.73%	0.6442	1.73%	0.6442
		$j=1$		$j=1$	0.2529
			0.2529		0.2529
	0.1667		0.1667		0.1667
0%	0.6666	0%	0.6666	0%	0.6666
$j=0$		$j=0$		$j=0$	
	0.1667		0.1667		0.1667
			0.2529		0.2529
		-1.73%	0.6442	-1.73%	0.6442
		$j=-1$		$j=-1$	
			0.1029		0.1029
					0.0617
				-3.46%	0.1766
				$j=-2$	
					0.7617

rates probabilities rates probabilities rates probabilities

3. We have to compute  $\alpha(0), \alpha(1)$  and  $\alpha(2)$ , which implies, to compute  $f(0, 0), f(0, 1)$  and  $f(0, 2)$ .

$$f(0, 0) = R^c(0, 1) = 4\%$$

$$f(0, 1) = \frac{B(0, 1)}{B(0, 2)} - 1 = \frac{0.96079}{0.92035} - 1 = 4.394\%$$

$$f(0, 2) = \frac{B(0, 2)}{B(0, 3)} - 1 = \frac{0.92035}{0.88029} - 1 = 4.550\%$$

Using the fact that

$$r(t) = r^*(t) + \alpha(t)$$

we finally obtain

$$r(0) = \alpha(0) = 4\%$$

$$r(1) = r^*(1) + 4.398\%$$

$$r(2) = r^*(2) + 4.565\%$$

so that we obtain the following lattice:

i=0	i=1	i=2	i=2
			0.7617
			8.030%
			$j=2$ 0.1766
			0.0617
		0.1029	0.1029
	6.130%	0.6442	6.297%
	$j=1$		$j=1$ 0.6442
		0.2529	0.2529
	0.1667	0.1667	0.1667
4%	0.6666	4.398%	4.565%
$j=0$		$j=0$	$j=0$ 0.6666
	0.1667	0.1667	0.1667
		0.2529	0.2529
	2.666%	0.6442	2.833%
	$j=-1$		$j=-1$ 0.6442
		0.1029	0.1029
			0.0617
			1.101%
			$j=-2$ 0.1766
			0.7617

**rates probabilities rates probabilities rates probabilities**

4. Using the approach from Hull and White, we obtain the following results:

- for  $\alpha(1)$

$$Q_{1,1} = 0.1601$$

$$Q_{1,0} = 0.6405$$

$$Q_{1,-1} = 0.1601$$

$$\alpha(1) = 4.305\%$$

- for  $\alpha(2)$

$$Q_{2,2} = 0.0155$$

$$Q_{2,1} = 0.1994$$

$$Q_{2,0} = 0.4866$$

$$Q_{2,-1} = 0.2028$$

$$Q_{2,-2} = 0.0161$$

$$\alpha(2) = 4.467\%$$

We finally obtain the following lattice:

i=0	i=1	i=2	
			0.7617
		7.931%	0.1766
		$j=2$	
			0.0617
		0.1029	0.1029
	6.037%	0.6442	6.199%
	$j=1$		$j=1$
		0.2529	0.2529
	0.1667	0.1667	0.1667
4%	0.6666	4.305%	0.6666
$j=0$	$j=0$	$j=0$	$j=0$
	0.1667	0.1667	0.1667
		0.2529	0.2529
	2.573%	0.6442	2.735%
	$j=-1$		$j=-1$
		0.1029	0.1029
			0.0617
		1.003%	0.1766
		$j=-2$	
			0.7617
<b>rates</b>	<b>probabilities</b>	<b>rates</b>	<b>probabilities</b>
		<b>rates</b>	<b>probabilities</b>

5. We now want to price the put option with the following payoff:

$$\text{Max}[0; 10,000 \times (5\% - r)]$$

From the lattice derived above, we can obtain the cash flow for each of the possible terminal nodes. For example, 53.29 corresponds to 10,000 (5% - 4.467%). The rate 4.467% below is the rate with 1-year maturity and with starting date  $i = 2$ , which corresponds to that node of the lattice (see lattice below).

		0
		7.931%
	12.69	0
	6.037%	6.199%
78.28	70.19	53.29
4%	4.305%	4.467%
	195.42	226.49
	2.573%	2.735%
		399.70
		1.003%

Cash flows at date  $i = 1$  are obtained in a classic way by discounting back to present the cash flows to be received at date  $i = 2$  under the risk-neutral probability, which actually is the probability under which the lattice has been built. We then obtain (see the lattice)

$$(0.2529 \times 53.29)e^{-6.037\%} = 12.69$$

$$(0.6667 \times 53.29 + 0.1666 \times 226.49)e^{-4.305\%} = 70.19$$

$$(0.2529 \times 53.29 + 0.6442 \times 226.49 + 0.1029 \times 399.70)e^{-2.573\%} = 195.42$$

Working backward through the lattice up to date  $i = 0$ , we finally obtain as value for the option \$78.28

$$(0.1666 \times 12.69 + 0.6667 \times 70.19 + 0.1666 \times 195.42)e^{-4\%} = 78.28$$

### 13 CHAPTER 13—Problems

Exercise 13.4 Consider the following credit transition matrix from S&P:

		TO							
		S&P	AAA (%)	AA (%)	A (%)	BBB (%)	BB (%)	B (%)	CCC (%)
F R O M	AAA	91.94	7.46	0.48	0.08	0.04	0.00	0.00	0.00
	AA	0.64	91.80	6.75	0.60	0.06	0.12	0.03	0.00
	A	0.07	2.27	91.69	5.11	0.56	0.25	0.01	0.04
	BBB	0.04	0.27	5.56	87.87	4.83	1.02	0.17	0.24
	BB	0.04	0.10	0.61	7.75	81.49	7.89	1.11	1.01
	B	0.00	0.10	0.28	0.46	6.95	82.80	3.96	5.45
	CCC	0.19	0.00	0.37	0.75	2.43	12.13	60.44	23.69
	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

1. What is the probability of going from the category AAA to CCC and from CCC to AAA?
2. What is the probability of a bond rated AAA being downgraded?



- Solution 13.4*
- From the table, the probability to go from AAA to CCC is 0.00%, while the probability to go from CCC to AAA is 0.19.
  - From the table, that probability is 8.06%

$$7.46\% + 0.48\% + 0.08\% + 0.04\% = 8.06\%$$

- Exercise 13.5* On September 1, these spot rates on US treasury and industrial corporates were reported by Bloomberg (type CURV [GO]):

Maturity	Treasuries (%)	Aaa Corporates (%)	Baa Corporates (%)
0.5	4.98	5.46	5.86
1	5.00	5.44	5.84
1.5	4.93	5.42	5.82
2	4.87	5.40	5.79

- Compute the implied prices of 2-year 6.5% Treasury, Aaa, and Baa bonds.
- Compute the YTM on these three bonds. Analyze the spreads.
- Which factors account for them?

- Solution 13.6*
- The implied prices are

$$P_{\text{Treasury}} = \frac{3.25}{(1 + 4.98)^{\frac{1}{2}}} + \frac{3.25}{(1 + 5.00)^1} + \frac{3.25}{(1 + 4.93)^{\frac{3}{2}}} + \frac{103.25}{(1 + 4.87)^2}$$

$$= 105.7765$$

$$P_{\text{Aaa}} = \frac{3.25}{(1 + 5.46)^{\frac{1}{2}}} + \frac{3.25}{(1 + 5.44)^1} + \frac{3.25}{(1 + 5.42)^{\frac{3}{2}}} + \frac{103.25}{(1 + 5.4)^2}$$

$$= 104.7360$$

$$P_{\text{Baa}} = \frac{3.25}{(1 + 5.86)^{\frac{1}{2}}} + \frac{3.25}{(1 + 5.84)^1} + \frac{3.25}{(1 + 5.82)^{\frac{3}{2}}} + \frac{103.25}{(1 + 5.79)^2}$$

$$= 103.9716$$

- The implied YTMs are

$$\text{YTM}_{\text{Treasury}} = 4.88\%$$

$$\text{YTM}_{\text{Aaa}} = 5.40\%$$

$$\text{YTM}_{\text{Baa}} = 5.79\%$$

The spreads are

$$\text{Spread}_{\text{Aaa}} = 52 \text{ basis points}$$

$$\text{Spread}_{\text{Baa}} = 91 \text{ basis points}$$

- The factors are

- default factor: probability and magnitude of default
- liquidity factor: more likely than not, the two corporate bonds are not traded in a market as liquid as the Treasury market. This commands a liquidity premium from the part of an investor.

It is fairly difficult to disentangle the respective magnitude of default and liquidity factors.

## 14 CHAPTER 14—Problems

*Exercise 14.4* Consider a callable bond with semiannual coupon 4%, maturity 5 years and market price 100.5.

1. Compute its yield-to-worst if it is callable at 104 at the end of year 1, at 103 at the end of year 2, at 102 at the end of year 3 and at 101 at the end of year 4.
2. Same question if it is callable at 100 after 1 year.

*Solution 14.4* 1. We have:

Year	Yield-to-call (%)
1	7.40
2	5.18
3	4.45
4	4.10
	Yield-to-maturity (%)
5	3.89

The yield-to-worst of this callable bond is equal to the lowest of its yields-to-call and yield to maturity. So, it is equal to its yield to maturity, that is 3.89%.

2. We have

Year	Yield-to-call (%)
1	3.49
2	3.74
3	3.82
4	3.86
	Yield-to-maturity (%)
5	3.89

This time, the yield-to-worst of the callable bond is equal to its yield to the next call date, that is 3.49%.

*Exercise 14.11* If a bond can be converted into 40.57 shares of the company's common stock, and that common stock is currently selling for \$30, then calculate the conversion value.

*Solution 14.11* The conversion value is  $40.57 \times \$30 = \$1,217.10$ .

*Exercise 14.16* Consider the following interest-rate tree (four 1-year periods):

				0.338464
			0.262012	
		0.196941		0.245776
	0.14318		0.186493	
0.1		0.137401		0.17847
	0.097916		0.13274	
		0.095862		0.129596
			0.09448	
				0.094106

1. Calculate the value at each node of the tree of a 5-year pure discount bond with face value 100. What is the YTM on that bond?
2. Calculate the price of an identical bond that would be callable at \$75. Calculate the YTM on that bond and the spread over the straight bond. What is the value of the embedded call option according to the model?
3. Assume that the market price of the callable bond is \$44. What is the OAS of the bond, that is, what is the constant spread that must be added to all rates in the tree to make the model price equal to the market price?
4. Redo question 2 for a call price equal to 80.
5. Assuming that the market price is 50, calculate the OAS for that bond.

*Solution 14.16* 1. The straight bond-price tree is

					100
				74.71251	
			61.40344		100
		54.7185		80.27125	
	52.02685		69.58618		100
51.9368		64.23362		84.85579	
	62.23422		76.53257		100
		72.42227		88.52723	
			82.19705		100
				91.39882	
					100

where you have, for example,

$$69.58618 = \frac{1}{1 + 0.186493} [0.5 \times 80.27125 + 0.5 \times 84.85579]$$

The YTM is given by

$$YTM = \left( \frac{100}{51.9368} \right)^{\frac{1}{5}} - 1 = 14\%$$

2. The callable bond-price tree is

				100
			74.71251	
		59.315101		100
		51.18319	75	
	47.27042		63.2115	100
45.9894		56.89402	75	
	53.90635		66.21113	100
		61.47527	75	
			68.52569	100
			75	
				100

where we have, for example,

$$74.71251 = \min \left( \frac{1}{1 + 0.262012} [0.5 \times 100 + 0.5 \times 100], 75 \right)$$

The YTM is given by

$$YTM = \left( \frac{100}{45.9894} \right)^{\frac{1}{5}} - 1 = 16.81\%$$

so that the spread is 2.81%. The value of the embedded call option is  $51.9368 - 45.9894 = 5.9474$ .

3. The OAS is 1.24040357783673%, or a little more than 124 basis points. You may indeed check that adding 1.24% to all rates in the tree will give you a value for the callable bond equal to \$44.
4. The new callable bond-price tree is

				100
			74.71251	
		61.40344		100
		54.7185	80.27125	
	51.7275		69.58618	100
51.0838		63.54918	84.85579	
	60.65679		74.97563	100
		69.64293	85	
			77.66245	100
			85	
				100

The YTM is now equal to 14.38%, and the spread is 38 basis points. The value of the embedded call option is  $51.9368 - 51.0838 = 0.853082784$ .

5. The OAS for the bond is 0.525801293207843%, a little more than 52.58 basis points.

## 15 CHAPTER 15—Problems

*Exercise 15.3* Let us consider a put on a  $T$ -bond futures contracted at date  $t = 10/24/02$  with nominal amount \$10,000,000, strike price  $E = 116\%$ , which expires on  $T = 12/13/02$ .

1. What is the price formula of this put using the Black (1976) model?
2. Assuming that the  $T$ -bond futures price is 115.47% at date  $t$ , its volatility 8% and that the zero-coupon rate  $R^c(t, T - t)$  is equal to 5%, give the price of this put.
3. Compute the put Greeks.
4. We consider that the futures price increases by 0.10%. Compute the actual put price variation. Compare it with the price variation estimated by using the delta, and by using the delta and the gamma.
5. We consider that the volatility increases by 1%. Compute the actual put price variation. Compare it with the price variation estimated by using the vega.
6. We consider that the interest rate increases by 1%. Compute the actual put price variation. Compare it with the price variation estimated by using the rho.
7. We reprice the put one day later. Compute the actual put price variation. Compare it with the price variation estimated by using the theta.

*Solution 15.3* 1. The put price at date  $t$  in Black (1976) model is given by

$$P_t = N \cdot B(t, T) \cdot [-F_t \Phi(-d_f) + E \Phi(-d_f + \sigma \sqrt{T - t})]$$

where

$N$  is the nominal amount.

$B(t, T)$  is the discount factor equal to

$$B(t, T) = e^{-(T-t) \cdot R^c(t, T-t)}$$

$\Phi$  is the cumulative distribution function of the standard Gaussian distribution.

$F_t$  is the futures price at date  $t$ .

$d_f$  is given by

$$d_f = \frac{\ln\left(\frac{F_t}{E}\right) + 0.5\sigma^2(T - t)}{\sigma \sqrt{T - t}}$$

$\sigma$  is the volatility of the underlying rate  $F$ .

2.  $N = \$10,000,000$ .

The discount factor is equal to

$$B(t, T) = e^{-\frac{50}{365} \cdot 5\%} = 0.993174088$$

We obtain the following values for  $-d_f$  and  $-d_f + \sigma\sqrt{T-t}$

$$-d_f = 0.139857241$$

$$-d_f + \sigma\sqrt{T-t} = 0.169466569$$

so that  $\Phi(-d_f)$  and  $\Phi(-d_f + \sigma\sqrt{T-t})$  are equal to

$$\Phi(-d_f) = 0.555613635$$

$$\Phi(-d_f + \sigma\sqrt{T-t}) = 0.567285169$$

Note that  $\Phi$ , the cumulative distribution function of the standard Gaussian law is already preprogrammed in Excel (see Excel functions).

We finally obtain the put price

$$P_t = \$163,712 = 1.637\% \text{ of the nominal amount}$$

3. We obtain the following Greeks:

$$\Delta = -5,518,211$$

$$\gamma = 114,759,940$$

$$\nu = 1,676,856$$

$$\rho = -22,426$$

$$\theta = -481,456$$

4. When the futures price changes for 115.57%, the actual price variation is  $-\$5,461$  as the price variation approximated by the delta is  $-\$5,518$

$$\begin{aligned} \text{Price Variation} &= -5,461 \simeq \Delta \times 0.1\% \\ &= -5,518 \end{aligned}$$

For a best approximation of the price variation, we must take into account the convexity of the option price with the gamma so that

$$\begin{aligned} \text{Price Variation} &= -5,461 \simeq \Delta \times 0.1\% + \delta \times \frac{(0.1\%)^2}{2} \\ &= -5,461 \end{aligned}$$

5. When the volatility changes for 9%, the actual price variation is  $\$16,791$  as the price variation approximated by the vega is  $\$16,769$

$$\begin{aligned} \text{Price Variation} &= 16,791 \simeq \nu \times 1\% \\ &= 16,769 \end{aligned}$$

6. When the zero-coupon rate  $R(t, T-t)$  changes for 6%, the actual price variation is  $-\$224$  as the price variation approximated by the rho is  $-\$224$

$$\text{Price Variation} = -224 \simeq \rho \times 1\% = -224$$

7. One day later, when we reprice the futures put, the actual price variation is  $-\$1,326$  as the price variation due to time approximated by the theta is  $-\$1,319$

$$\text{Price Variation} = -1,326 \simeq \theta \times \frac{1}{365} = -1,319$$

*Exercise 15.7* Let us consider a caplet contracted at date  $t = 05/13/02$  with nominal amount  $\$10,000,000$ , exercise rate  $E = 5\%$ , based upon the 6-month Libor  $R^L\left(t, \frac{1}{2}\right)$  and with the following schedule:

$t = 05/13/02$	$T_0 = 06/03/02$	$T_1 = 12/03/02$
When the caplet is contracted	When the caplet starts	Payoff payment

At date  $T_1 = 12/03/02$ , the cap holder receives the cash flow  $C$

$$C = \$10,000,000 \times \text{Max} \left[ 0; \delta \cdot \left( R^L \left( T_0, \frac{1}{2} \right) - 5\% \right) \right]$$

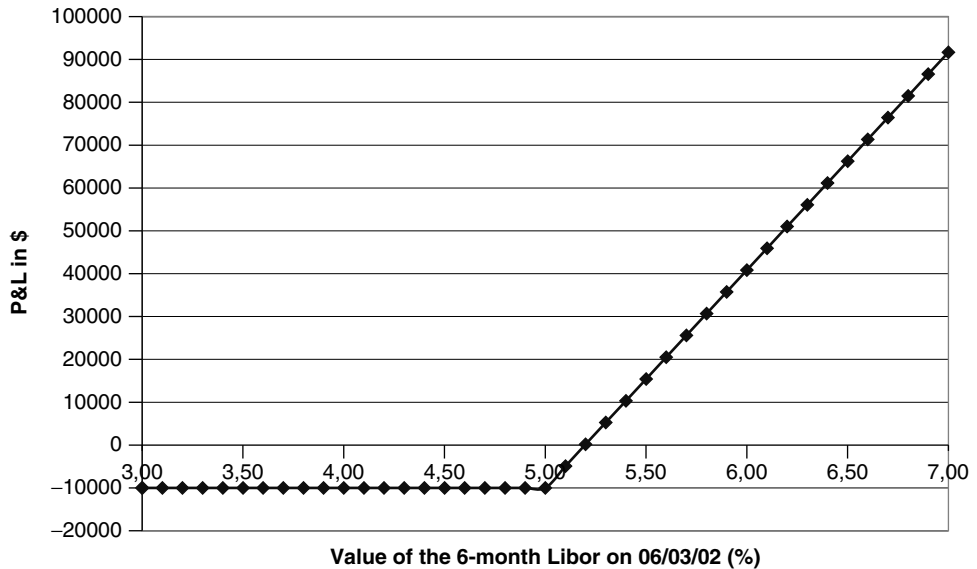
where  $\delta = T_1 - T_0$  (expressed in fraction of years using the Actual/360 day-count basis).

1. Draw the P&L of this caplet considering that the premium paid by the buyer is equal to  $0.1\%$  of the nominal amount.
2. What is the price formula of this caplet using the Black (1976) model?
3. Assuming that the 6-month Libor forward is  $5.17\%$  at date  $t$ , its volatility  $15\%$  and that the zero-coupon rate  $R(t, T_1 - t)$  is equal to  $5.25\%$ , give the price of this caplet.
4. Find the margin taken by the seller of this caplet.
5. Compute the caplet Greeks.
6. Assuming that the 6-month forward goes from  $5.17\%$  to  $5.18\%$ , compute the actual caplet price variation and compare it with the approximated price variation using first the delta, and then the delta and the gamma.
7. Assuming that the volatility goes from  $15\%$  to  $16\%$ , compute the actual caplet price variation and compare it with the approximated price variation using the vega.
8. Assuming that the interest rate  $R^c(t, T_1 - t)$  goes from  $5.25\%$  to  $5.35\%$ , compute the actual caplet price variation and compare it with the approximated price variation using the rho.
9. Compute the caplet price one day later. Compare the actual caplet price variation with the approximated price variation using the theta.

*Solution 15.7* 1. The P&L of the caplet depends on the value of the 6-month Libor at date  $T_0$ , and is given by the following formula (considering the buyer position; of course the seller position is the opposite one)

$$\text{P\&L} = \$10,000,000 \times \left\{ \text{Max} \left[ 0; \delta \cdot \left( R^L \left( T_0, \frac{1}{2} \right) - 5\% \right) \right] - 0.1\% \right\}$$

It appears on the following graph:



2. The caplet price at date  $t$  in Black (1976) model is given by

$$\text{Caplet}_t = N \cdot \delta \cdot B(t, T_1) \cdot [F(t, T_0, T_1)\Phi(d) - E\Phi(d - \sigma\sqrt{T_0 - t})]$$

where  $N$  is the nominal amount.

$B(t, T_1)$  is the discount factor equal to

$$B(t, T_1) = e^{-(T_1-t) \cdot R(t, T_1-t)}$$

$\Phi$  is the cumulative distribution function of the standard Gaussian distribution.

$F(t, T_0, T_1)$  is the 6-month Libor forward as seen from date  $t$ , starting at date  $T_0$  and finishing at date  $T_1$ . Note in particular that

$$F(T_0, T_0, T_1) = R^L\left(T_0, \frac{1}{2}\right)$$

$d$  is given by

$$d = \frac{\ln\left(\frac{F(t, T_0, T_1)}{E}\right) + 0.5\sigma^2(T_0 - t)}{\sigma\sqrt{T_0 - t}}$$

$\sigma$  is the volatility of the underlying rate  $F(t, T_0, T_1)$ , which is usually referred to as the *caplet volatility*.



3.  $N = \$10,000,000$  and  $\delta = \frac{183}{360}$ .

The discount factor is equal to

$$B(t, T_1) = e^{-\frac{204}{365} \cdot 5.25\%} = 0.97108384$$

We obtain the following values for  $d$  and  $d - \sigma\sqrt{T_0 - t}$

$$d = 0.94100172$$

$$d - \sigma\sqrt{T_0 - t} = 0.90477328$$

so that  $\Phi(d)$  and  $\Phi(d - \sigma\sqrt{T_0 - t})$  are equal to

$$\Phi(d) = 0.82664803$$

$$\Phi(d - \sigma\sqrt{T_0 - t}) = 0.81720728$$

Note that  $\Phi$  the cumulative distribution function of the standard Gaussian law is already preprogrammed in Excel (see Excel functions).

We finally obtain the Caplet price

$$\text{Caplet} = \$9,267 = 0.09267 \text{ of the nominal amount}$$

4. The seller of this caplet takes a margin equal to 7.91%

$$\frac{10,000}{9,267} - 1 = 7.91\%$$

5. We obtain the following caplet Greeks:

$$\Delta = 4,080,618$$

$$\gamma = 675,296,853$$

$$v = 15,794$$

$$\rho = -5,251$$

$$\theta = -19,820$$

6. The actual caplet price variation is equal to \$411

$$\text{Price Variation} = \$9,678 - \$9,267 = \$411$$

very near from the quantity “delta  $\times$  change of the forward rate” equal to \$408

$$\text{delta} \times \text{change of the forward rate} = 4,080,618 \times 0.01\% = \$408$$

The price variation is very well explained by the quantity “delta  $\times$  change of the forward rate + gamma  $\times$  (change of the forward rate)<sup>2</sup>/2”

$$\begin{aligned} &\text{delta} \times \text{change of the forward rate} + \text{gamma} \\ &\quad \times (\text{change of the forward rate})^2 / 2 \\ &= \$408 + \$3 = \$411 \end{aligned}$$

7. The actual price variation is equal to \$162

$$\text{Price Variation} = \$9,429 - \$9,267 = \$162$$

very near from the quantity “vega  $\times$  change of volatility” equal to \$157.94

$$\text{vega} \times \text{change of rate} = 15,794 \times 1\% = \$157.94$$

8. The actual price variation is equal to  $-\$5$

$$\text{Price Difference} = \$9,262 - \$9,267 = -\$5$$

very near from the quantity “rho  $\times$  change of rate” equal to  $-\$5.25$

$$\text{rho} \times \text{change of rate} = -5,251 \times 0.1\% = -\$5.25$$

9. One day later, on 05/14/02, the caplet price becomes \$9,212. The actual price variation is equal to  $-\$55$

$$\text{Price Difference} = \$9,212 - \$9,267 = -\$55$$

very near from the quantity “theta  $\times \frac{1}{365}$ ” equal to  $-\$54.3$

$$\text{theta} \times \frac{1}{365} = -19,820 \times \frac{1}{365} = -\$54.3$$

## 16 CHAPTER 16—Problems

- Exercise 16.2*
1. Prove that the value of a European Caplet Up-and-In is equal to the value of a European Caplet minus the European Caplet Up-and-Out.
  2. In the same spirit, prove that the value of a European Floorlet Down-and-In is equal to the value of a European Floorlet minus the European Floorlet Down-and-Out.

- Solution 16.2*
1. The strike and the barrier of the caplet Up-and-In are denoted by  $E$  and  $H$  respectively, with  $E < H$ . For simplicity purposes, we consider the tenor and the nominal amount equal to 1.

The payoff of this product depends on the value of the exercise rate  $X$  at expiry. It is  $\text{Max}[0, X - E] \cdot \mathbf{1}_{X \geq H}$ , which can be decomposed into

- if  $X \geq H$ : Payoff =  $X - E$
- if  $X < H$ : Payoff = 0

The payoff of the caplet Up-and-Out is  $\text{Max}[0, X - E] \cdot \mathbf{1}_{X < H}$ , which can be decomposed into

- if  $X \geq H$ : Payoff = 0
- if  $E < X < H$ : Payoff =  $X - E$
- if  $X \leq E$ : Payoff = 0

When we sum the two payoffs, we obtain the following results (what we call payoff total):

- if  $X \geq H$ : Payoff Total =  $X - E$
- if  $E < X < H$ : Payoff Total =  $X - E$
- if  $X \leq E$ : Payoff Total = 0

The payoff total is in fact the payoff of a standard cap, which concludes the proof.

2. We have necessary  $H < E$ .

The payoff of a floor Down-and-In is  $\text{Max}[0, E - X] \cdot \mathbf{1}_{X \leq H}$

- if  $X \leq H$ : Payoff =  $E - X$
- if  $X > H$ : Payoff = 0

The payoff of the floor Down-and-Out is  $\text{Max}[0, E - X] \cdot \mathbf{1}_{X > H}$ , which can be decomposed into

- if  $X \leq H$ : Payoff = 0
- if  $H < X < E$ : Payoff =  $E - X$
- if  $X \geq E$ : Payoff = 0

When we sum the two payoffs, we obtain the following results (what we call payoff total):

- if  $X \leq H$ : Payoff Total =  $E - X$
- if  $H < X < E$ : Payoff Total =  $E - X$
- if  $X \geq E$ : Payoff Total = 0

The payoff total is, in fact, the payoff of a standard floor, which concludes the proof.

*Exercise 16.4* On 05/13/02, a firm buys a barrier caplet Up-and-In whose features are the following:

- notional amount: \$10,000,000
  - reference rate: 3-month Libor
  - strike rate: 5%
  - starting date: 06/03/02
  - barrier: 6%
  - day-count: Actual/360
1. What is the payoff of this option for the buyer on 09/03/02?
  2. Draw the P&L of this caplet considering that the premium paid by the buyer is equal to 0.08% of the nominal amount.
  3. What is the advantage and the drawback of this option compared to a classical caplet?

*Solution 16.4* 1. The payoff of this option for the buyer on 09/03/02 is

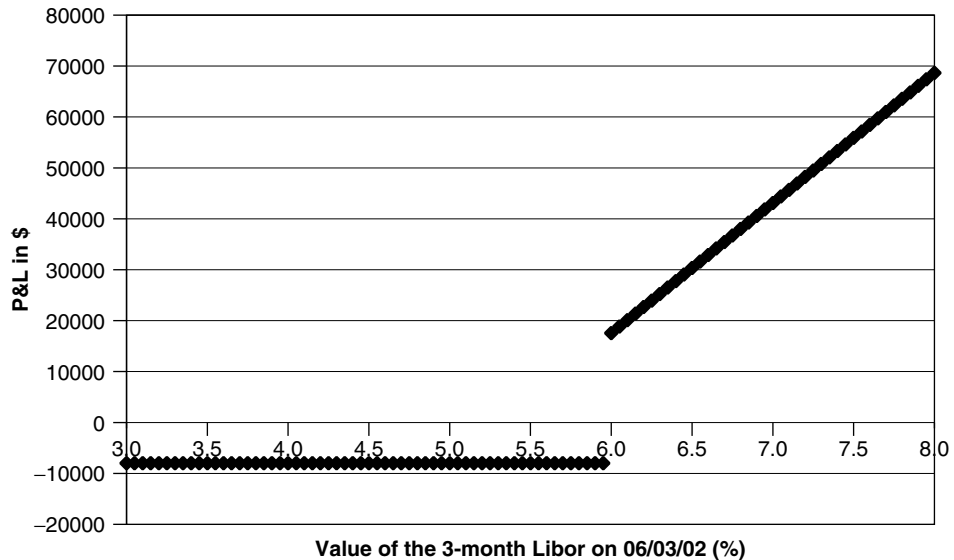
$$\text{Payoff} = \$10,000,000 \times \frac{92}{360} \times \text{Max} \left[ 0; R \left( 06/03/02, \frac{1}{4} \right) - 5\% \right] \\ \times \mathbf{1}_{R(06/03/02, \frac{1}{4}) \geq 6\%}$$

where  $R\left(06/03/02, \frac{1}{4}\right)$  is the 3-month Libor rate observed on 06/03/02, 92 is the number of days between 06/03/02 and 09/03/02, and  $\mathbf{1}_A = 1$  if event A occurs and 0 otherwise.

2. The P&L is given by the following formula:

$$\text{P\&L} = \text{Payoff} - \$10,000,000 \times 0.08\%$$

It appears in the following graph.



3. The advantage of this option compared to a classical caplet is that the buyer will pay a lower premium. The drawback is that he will gain only if the reference rate is equal or above the barrier.

*Exercise 16.13* Give at least three reasons why a bank may use credit derivatives.

*Solution 16.13* There are a variety of reasons why a bank may use credit derivatives. Some of those include the following:

- reducing the capital required to support assets on the balance sheet;
- reducing credit risk concentrations by assuming a risk position in a market that it may otherwise not have access to;
- improving earnings by assuming credit risk in a specifically targeted risk;
- managing credit risk at the account level while not negatively affecting the customer relationship; and
- creating new assets and synthetic assets to meet wider investor demand and/or filling maturity and credit quality gaps.

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## 17 CHAPTER 17—Problems

*Exercise 17.1* What does debt securitization consist in?

*Solution 17.1* Debt securitization consists in transforming the illiquid assets of a financial company into tradable securities backed by these assets. When the resulting securities are backed by mortgage loans, they are called *Mortgage-Backed Securities*; when they are backed by other types of loans, essentially installment loans and revolving loans, they are called *Asset-Backed Securities*. The secured status of these bonds comes from the fact that they are priority claims on the pool of the underlying assets, which are isolated from the general business assets of the financial company. Hence, the holders of these securities are protected against a default of the company.

*Exercise 17.3* To what extent can an MBS be considered a callable bond?

*Solution 17.3* An MBS can be considered a callable bond in that it is subject to prepayment risk. Indeed, the mortgages in the underlying pool may be prepaid at any time. In other words, the lender has granted the householder an American call option to pay back the mortgage at its face value.

## 18 CHAPTER 18—Problems

*Exercise 18.2* What is credit enhancement?

*Solution 18.2* Credit enhancement is used to improve the credit quality of an ABS compared to that of the underlying loans. This mechanism aims at protecting security holders against adverse credit events such as defaults. Credit enhancement can take several forms.

*Exercise 18.5* What are the three most frequent types of ABSs?

*Solution 18.5* The three most frequent types of ABSs are:

- automobile ABSs,
- credit card ABSs,
- home equity ABSs.