FORMULAE Fixed Income Markets and Products

- Clean price vs Dirty price: $Dirty\ Price = Clean\ Price + AI$

  $AI = N \times Coupon\ rate \times Factor$

- Annual Effective Equivalent rate (r^a)

  Using a rate (r) compounded n times per year : $r^a = \left(1 + \frac{r}{n}\right)^n - 1$

  Using a rate (r^c) with continuous compounding: $r^a = e^{r^c} - 1$

- Current Yield: $y_c = \frac{c}{P}$

- Relation between spot and forward rates (discrete cap.): $F(t,T,S) = \left[\frac{\left[1 + r(t,T+S)\right]^{(T+S-t)}}{\left[1 + r(t,T)\right]^{(T-t)}}\right]^{\frac{1}{S}} - 1$

- Pure Discount Bond (zero coupon bonds - ZCB):

$$p(t,T) = \frac{1}{\left[1 + r(t,T)\right]^{(T-t)}} \Leftrightarrow r(t,T) = \left(\frac{1}{p(t,T)}\right)^{\frac{1}{T-t}} - 1$$

- Relation between ZCB prices and continuous capitalization spot rates:

$$p(t,T) = e^{-r(t,T)[T-t]} \quad r(t,T) = -\frac{1}{T-t}\ln\ p(t,T) \quad F(t,T,S) = -\frac{1}{S}\ln\left(\frac{p(t,T+S)}{p(t,T)}\right)$$

- Par yield (c) : $c(T-t) = \left[1 - \frac{1}{\left[1 + r(t,T)\right]^{(T-t)}}\right] \Bigg/ \left[\sum_{i=t+1}^{T} \frac{1}{\left[1 + r(t,i)\right]^{(i-t)}}\right]$

- Value of a Fixed Coupon Bond

  Using the (discrete) YTM $y(t,T)$:

$$P(t) = \sum_{i=1}^{T} \frac{CF_i}{\left[1 + y(t,T)\right]^{(i-t)}} = \frac{c}{y(t,T)}\left(1 - \frac{1}{\left[1 + y(t,T)\right]^{(T-t)}}\right) + \frac{N}{\left[1 + y(t,T)\right]^{(T-t)}}$$

  Using the (continuous) YTM $y^c(t,T)$: $P(t) = \sum_{i=1}^{T} CF_i e^{-y^c(t,T)\times[i-t]}$

  Using discrete *spot* rates ($r(t,T)$): $P(t) = \sum_{i=1}^{T} \frac{CF_i}{\left[1 + r(t,i)\right]^{(i-t)}}$

  Using discount factors ($p(t,T)$): $P(t) = \sum_{i=1}^{T} p(t,i) CF_i$

  Using discrete 1 year *forward* rates ($F(t,x,1)$): $P(t) = \sum_{i=1}^{T} \frac{CF_i}{\prod_{k=t+1}^{i}(1 + F(t,k-1,1))}$

- Hedging

Relation between price and yield of a bond (Taylor series expansion)

$$\Delta P = \frac{\partial P}{\partial y}\Delta y + \frac{1}{2}\frac{\partial^2 P}{\partial^2 y}\ \Delta y\ ^2 + \cdots$$

Discrete annual compounding

- $Duration ($\$Dur$): $$\$Dur = -\frac{1}{1+y}\sum_{i=1}^{n}\frac{iCF_i}{(1+y)^i}$$
- Modified Duration ($MDur$): $$MDur = -\frac{\$Dur}{P}$$
- Macaulay Duration (D): $$D = \frac{\sum_{i=1}^{n}\frac{iCF_i}{(1+y)^i}}{P}$$
- 1st order measures relations: $$MDur = \frac{D}{(1+y)} \qquad \$Dur = -MDur \times P$$
- $Convexity ($\$Conv$): $$\$Conv = \frac{1}{(1+y)^2}\sum_{i=1}^{n}\frac{i(i+1)CF_i}{(1+y)^i}$$
- Modified Convexity ($MConv$): $$MConv = \frac{\$Conv}{P}$$
- Convexity Macaulay (C): $$C = \frac{\sum_{i=1}^{n}\frac{i(i+1)CF_i}{(1+y)^i}}{P}$$
- 2nd order measures relations: $$MConv = \frac{C}{(1+y)^2} \qquad \$Conv = MConv \times P$$

- Swaps: $$SWAP_{T_0} = N \times \left\{\sum_{i=1}^{n} F\left(\frac{T_{ki} - T_{k(i-1)}}{360}\right) B\ T_0, T_{ki}\ + B\ T_0, T_m\ \right\} - N$$

- Futures on short interest rates: $$forward\ rate = \left[\frac{1 + R_L\frac{D_L}{360}}{1 + R_C\frac{D_C}{360}} - 1\right] \times \frac{360}{D_L - D_C}$$

- Futures on Bonds: $$P_{prazo} = P_{vista} + JD_0 + P_{vista} + JD_0\left(r_0\frac{T}{360}\right) - JD_T - X_1 C\left(1 + r_1\frac{T - T_1}{360}\right)$$