Master in Mathematical Finance

Interest Rate and Credit Risk Models

Exam – 9 January 2017

Time: 2:15h

1. Please consider the following information on the Euro area money market and Government debt yields for the 5th January 2017:

Interest rates (%)

Maturities	Euribor	Germany	Portugal
Overnight	-0,42		
1 week	-0,42		
1month	-0,33		
3 months	-0,31		
6 months	-0,23		
1 year	-0,115		
2 years		-0,773	0,085
5 years		-0,522	1,883
10 years		0,27	3,903

1.1. Compute the price of a futures contract for the 3-month Euribor, with expiry date in April 2017. (1,5/20)

Futures price = 100 – implied interest rate (%)

Implied interest rate = ${}_3f_3 = [(1+s_{m+n})^{m+n}/(1+s_m)^m]^{1/n} = [(1+s_{0,5})^{0,5}/(1+s_{0,25})^{0,25}]^{1/0,25}$ = -0,15% => Futures price =100,15

1.2. Considering that the 5 and 10 year maturities of the Portuguese Government debt are represented by bonds paying annual coupons, with a redemption value of 100 Euros and coupon rates of 2% and 3%, respectively, compute the number of 10 year bonds to use in a duration hedging strategy of a portfolio comprised by 1000 bonds representative of the 5-year maturity. (1,5/20)

q = - (P x Dp) / (H x Dh)

Portfolio value = 1000 5y bonds

Price of 5 year bond = $2/(1+0{,}01883) + 2/(1+0{,}01883)^2 + 2/(1+0{,}01883)^3 + 2/(1+0{,}01883)^4 +$

$102/(1+0{,}01883)^5 = 100{,}5534$ => Portfolio value (P) = 100,5534 x 1000 = 100553

5y bond Duration = 4,81

Price of 10 year bond = $3/(1+0{,}03903)+ 3/(1+0{,}03903)^2 + \dots + 3/(1+0{,}03903)^9 + 103/(1+0{,}03903)^{10} = 92{,}64$

10y Bond Duration = 8,73

q = - (P x Dp) / (H x Dh) = - (100553 x 4,81)/ (92,63 x 8,73) = -598,1

1.3. Assuming that the 2-year Portuguese Government bond has a redemption value of 100 Euros and an annual coupon rate of 1,5%, compute the 2-year spot rate using a bootstrapping methodology and identify the main conceptual differences to the yield to maturity. (1,5/20)

$P = 1{,}5/(1+0{,}00085)+101{,}5/(1+0{,}00085)^2 = 102{,}83$

$102{,}83 = 1{,}5/(1+s_1)+101{,}5/(1+s_2)^2$

$102{,}83 = 1{,}5/(1-0{,}00115)+101{,}5/(1+s_2)^2$

$102{,}83 - 1{,}5/(1-0{,}00115) = 101{,}5/(1+s_2)^2$

$(1+s_2)^2 = 101{,}5/(102{,}83-1{,}5/(1-0{,}00115))$

$s_2 = -0{,}085\%$

1.4. Considering the main explanatory theories of the term structure of interest rates, characterize the major differences between the German and the Portuguese yield curve regarding the expectations on the future behavior of short-term rates (2,5/20)

- According to the expectations theory, we could argue that expected short term interest rates are higher in Portugal. However, these countries share the same currency and the credit risk of their Government debt are different.
- According to the liquidity premium theory, the positive slope of the yield curves stems from the risk premium, which may be considered as higher in Portugal.
- According to the market segmentation theory, the difference between long term rates is motivated only by the demand and supply in long term. Therefore, the lower German yields may be explained only by the higher demand for long term German bonds
- The preferred habitat theory doesn't provide us a clear explanation of the meaning of long term yields and their differences.

1.5. How could you estimate the future path of Euribor interest rates, by using static and stochastic interest rate models? Please present briefly the main features of the models identified. (3/20)

(i) Static models:

- Polynomial methods
- Polynomial splines
- NS/NSS
- DPR
- Bjorn and Christensen
- Bliss

(ii) Stochastic models:

- Vasicek
- CIR
- Other short-term models
- Affine models

2. Please consider the following marginal probabilities of default for company CorpCo (in addition to the interest rate information on the Portuguese Government Debt provided in the previous Group):

Maturity (years)	Prob.Default
1	0,62%
2	1,31%

2.1. Compute the premium of a credit default swap with the following features: (2/20)

Maturity = 2 years

Notional = € 100.000

Payment in case of default = 60% of the notional

E1(po(2)|ND)= 0 x (1-0,0131)+,6*,0131 = 0,00786
E1(po(2)|D)= 0,6

E0(po(2))=E1(po(2)|ND)*(1-PD)+E1(po(2)|D)*PD = 0,00786*(1-0,0062)+0,6*0,0062 = 0,011531

V(0,2)=F(0,2) x E0(po(2))*nocional = 1,00170216995911*0,011531*100000 = 1155,063

1155 = p/(1-0,00115)+p(/1+0,00085)^2
⇔ p= 576,68 = 0,58%

2.2. Please explain how would you assess CorpCo credit risk from its share prices and identify the information required accordingly. (2/20)

- Merton model / Moody's KMV
- Information: Debt amount (and structure), share prices volatility, number of shares, risk-free interest rate, debt maturity

2.3. How could you model the PD of the company using reduced form models? (2/20)

- Gaussian models
- CIR models

2.4. Assuming that there are only 3 ratings classifications – investment grade (I), speculative grade (S) and default (D) – and CorpCo has a classification of "I", compute the 2-year probability of default, taking into consideration the rating transition matrix below and the potential rating migrations. (1,5/20)

	I	S	D
I	0,7	0,25	0,05
S	0,2	0,7	0,1
D	0	0	1

P(II)xP(ID)=	0,035
P(IS)xP(SD)=	0,025
P(ID)xP(DD)=	0,05
2y PD =	0,11

2.5. Please explain how does the correlation impact on the credit risk of a bond or loan portfolio (2,5).

- Binomial Expansion Technique by Moody's – higher correlation implies higher extreme losses