Masters in FINANCE

RISKY DEBT - I
Merton's Model

Corporate Investment Appraisal

Fall 2017

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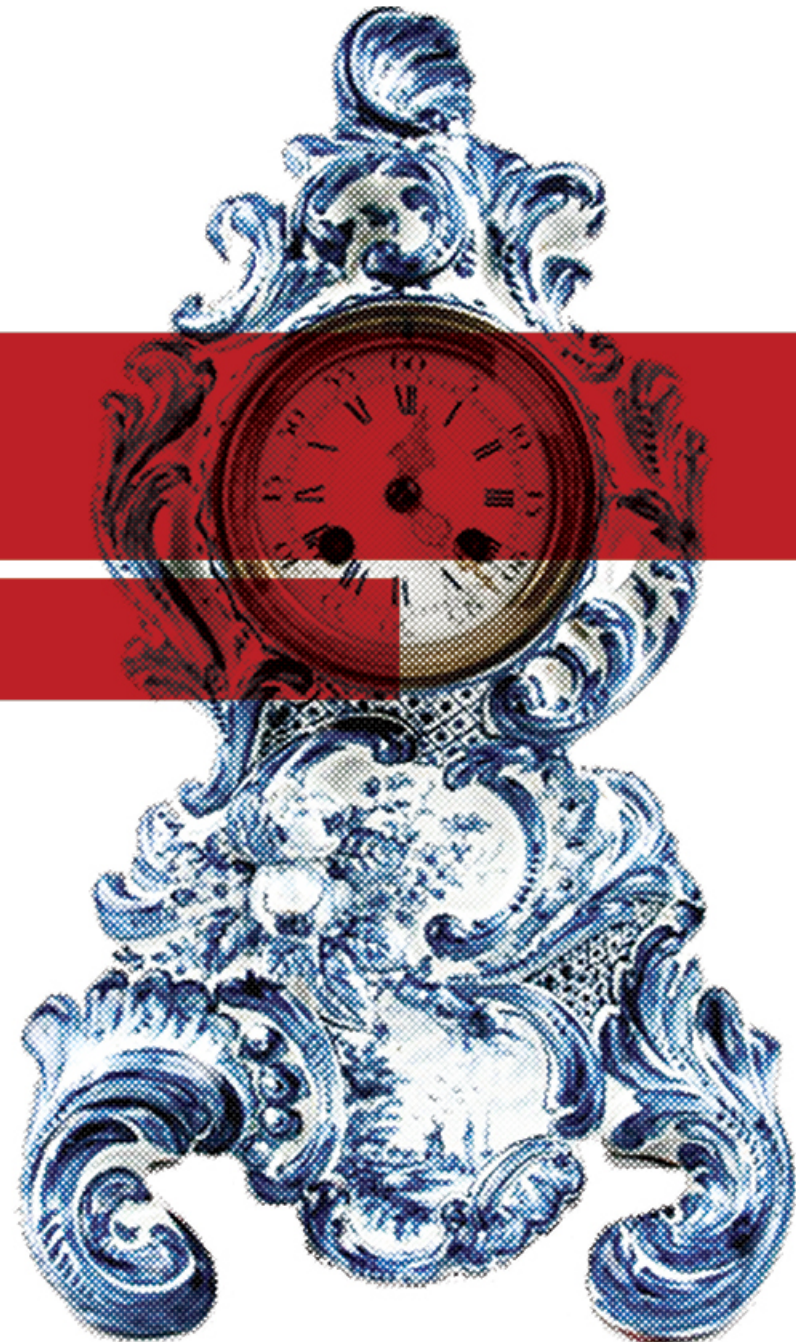
OUTLINE

The Option Theory approach;

The discrete version of Merton's model;

A possible practical implementation.

The "Option Pricing" approach

Suppose a company has debt with face value D.

Contingent on the value of the firm's assets at maturity of this debt, the payoffs to shareholders and debt-holders will be:

(note: disregarding "bankruptcy costs.")

	$V \leq D$	$V > D$
Shareholders	0	V-D
Bondholders	V	D
Value	V	V

Shareholders have a call option, with payoffs at maturity:

$V_E = \max(0, V-D)$

Exercise Price = D

Underlying Asset = V

Debtholders wrote a put option, together with having riskless debt:

$V_D = D + \min(0, V-D)$

Exercise Price = D

Underlying Asset V

If we intended to apply, for example, the Black-Scholes formula, we would obtain:

$$VE = V \cdot N(d_1) - e^{-R_f T} \cdot D \cdot N(d_2)$$

$$d_1 = \frac{\ln\left(V/D\right) + R_f T}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

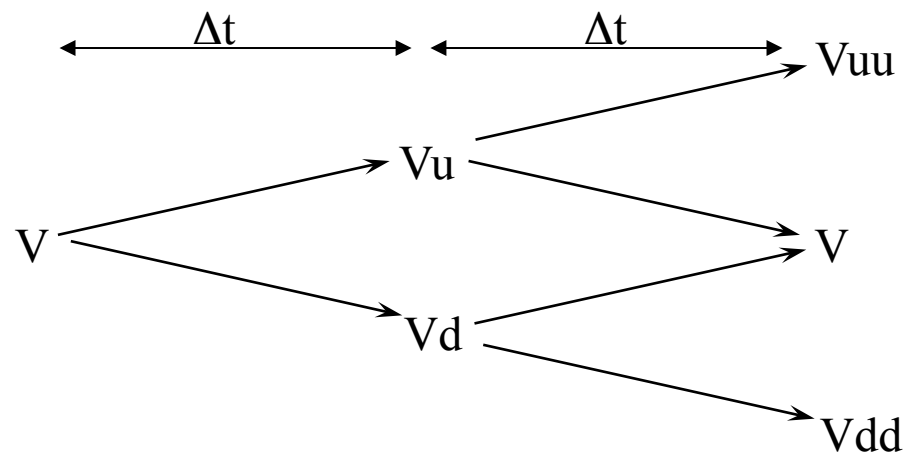We could then determine the value of debt via put-call parity.

Discrete Version of Merton's Model

- Merton used a formalization in continuous time:

$$\frac{dV}{V} = \mu dt + \sigma dW$$

- Possibly the easiest way to implement this type of analysis is using the binomial tree.

Tree to Value the Assets

$$u = e^{\sigma\sqrt{\Delta t}} \text{ and } d = e^{-\sigma\sqrt{\Delta t}}, \text{ where } \sigma \text{ is the annualized volatility.}$$

$$p = \frac{(1 + r\Delta t) - d}{u - d}$$

u and d incorporate a term of volatility, taking into account the ways in which the price can evolve.

r is the riskless interest rate.

p is a *pseudo-probability (*risk-neutral)*.*

In the limit, for a very small t, this tree converges to a Geometric Brownian Motion.

Value of Firm Equity = 3

Volatility of Equity = 25%

Term Structure of Interest Rates flat at 7%

The firm has a loan

Maturity within six months;
Face value 10;
There are no coupons until maturity.

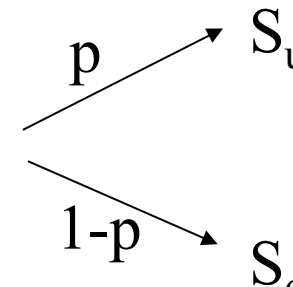There will be default if the assets are worth less than the face value of debt.

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At debt maturity, the value of the shares = the value of the assets after

payments to debt-holders

$$S_{0.5} = \max(V - 10, 0)$$

We use the standard binomial approach under risk neutrality in order to

determine prices:

$$\frac{pS_u + (1-p)S_d}{1 + r\Delta t}$$

$p \quad\nearrow\quad S_u$

$1\text{-}p \quad\searrow\quad S_d$

We need values for V and sigma in order to build the tree for S.

We try to guess sigma=25%,

$$u = e^{0.25\sqrt{0.25}} = 1.1331$$

$$d = e^{-0.25\sqrt{0.25}} = 0.8825$$

$$p = \frac{(1 + 0.07/4) - d}{u - d} = 0.5386$$

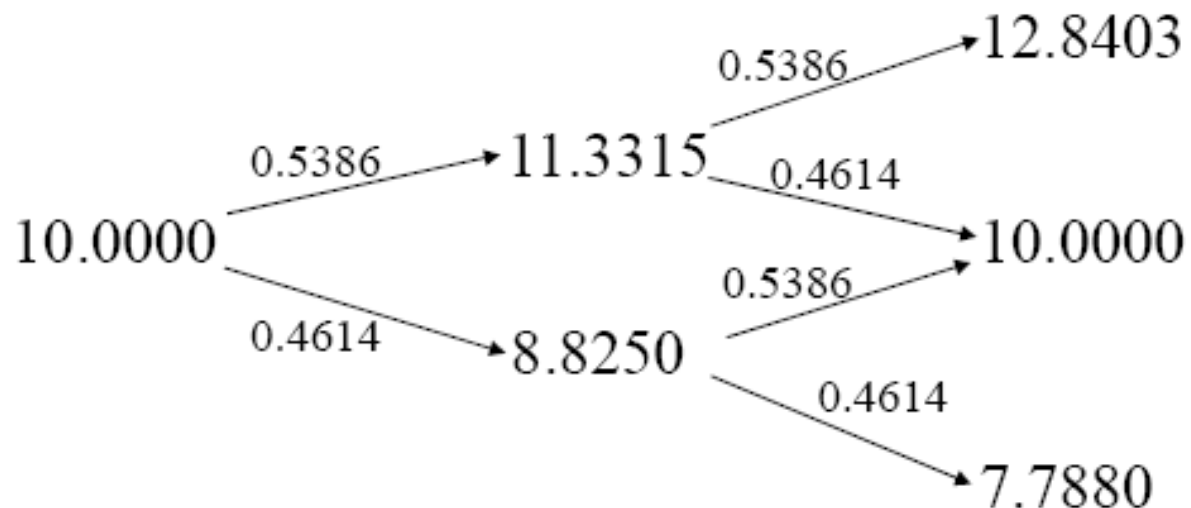We try to guess value of assets equal to 10;

We use these values to build the tree.

Using the parameters suggested before:

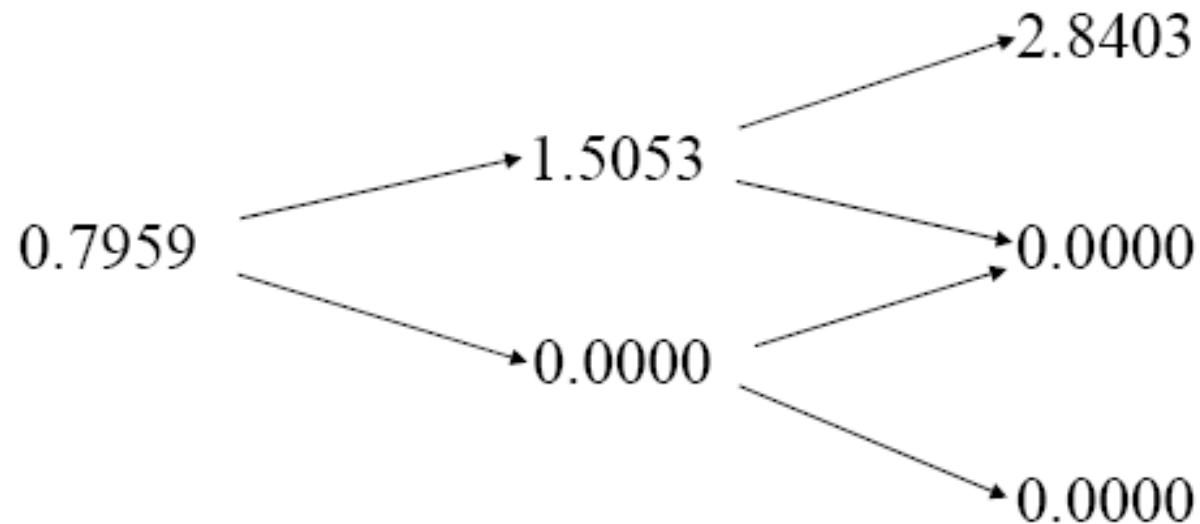We work recursively from here…

With the parameters we guessed, equity would be worth 0.7959;

Not exactly what we were looking for...

To reach compatible values for V and S, we can keep sigma constant and change
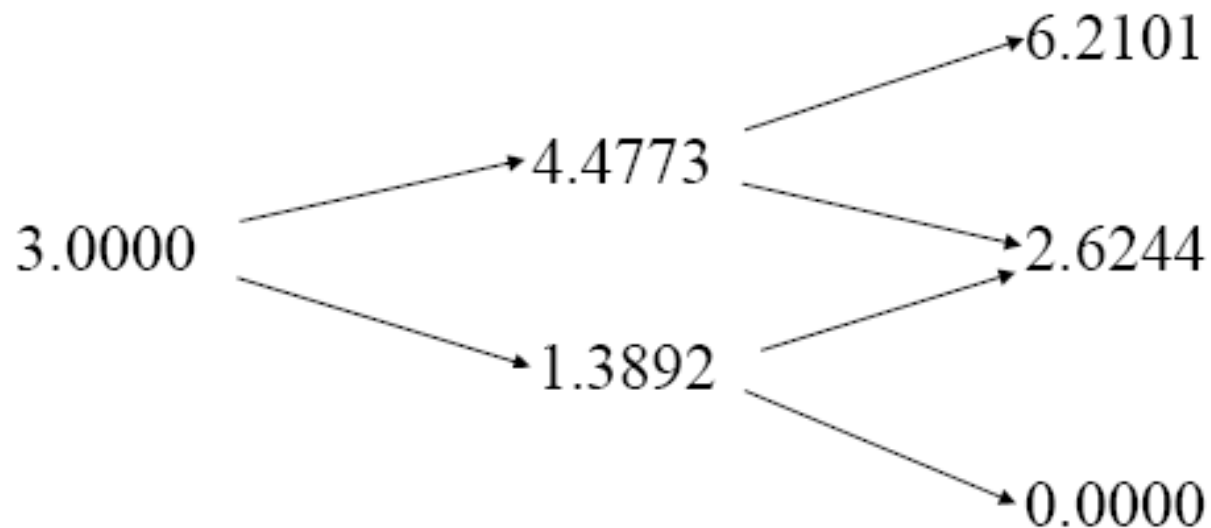
V until we get S=3.

Goalseek in Excel can be used for this.

Seems better: stock price resembles the real world's.

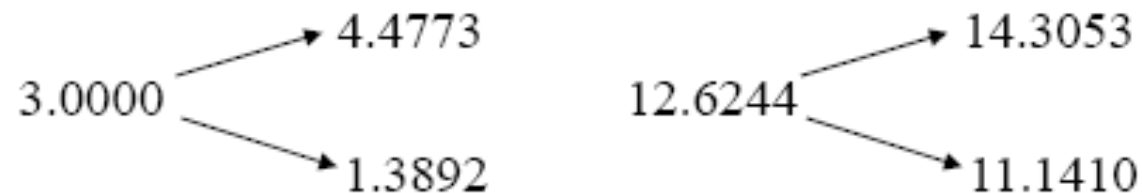Is this sufficiently good to compute the value of Debt?

We know from Itô's Lemma that:

$$\sigma_S = \frac{\partial S}{\partial V} \frac{V}{S} \sigma$$

We expect the volatility of the shares to be higher or lower than the volatility of the assets?

We should check if we have sensible values for σ$_S$.

A standard way of computing $\frac{\partial S}{\partial V}$ based on the binomial tree is by looking at S and V in the first branches:

Approximately $\dfrac{\partial S}{\partial V} = \dfrac{4.4773 - 1.3892}{14.3053 - 11.1410} = 0.9759$

Our current parameters produce:

$$\sigma_S = \frac{\partial S}{\partial V}\frac{V}{S}\sigma = 0.9759 \times \frac{12.6244}{3} \times 0.25 = 1.0267$$

The volatility of the shares when using these parameters is almost 103%.
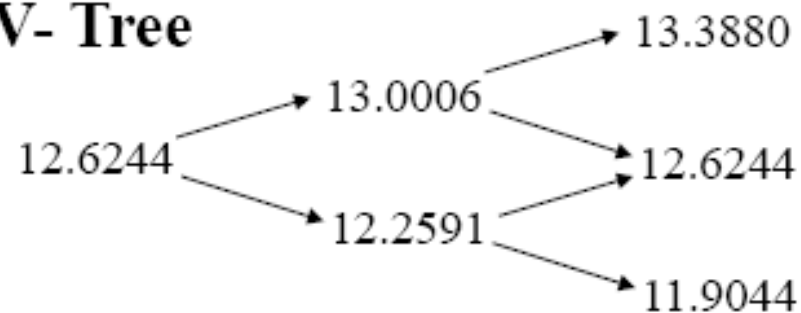
Not exactly ideal yet...

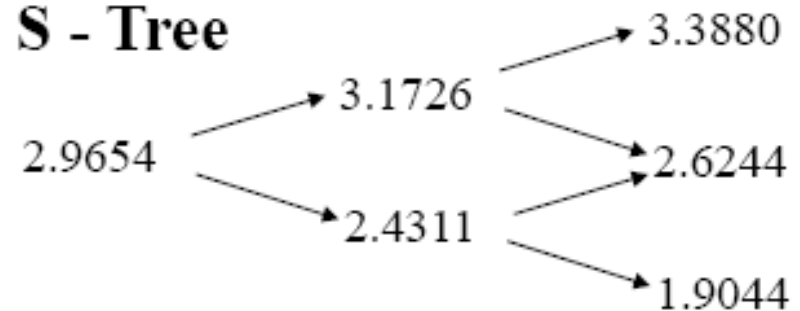We may keep V=12.6244 constant and alter σ until σ_S=25%.

Trees for V and S:
V=12.6244, σ=5.87%

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V- Tree

$$12.6244 \rightarrow 13.0006 \rightarrow 13.3880$$
$$13.0006 \rightarrow 12.6244$$
$$12.6244 \rightarrow 12.2591 \rightarrow 12.6244$$
$$12.2591 \rightarrow 11.9044$$

S - Tree

$$2.9654 \rightarrow 3.1726 \rightarrow 3.3880$$
$$3.1726 \rightarrow 2.6244$$
$$2.9654 \rightarrow 2.4311 \rightarrow 2.6244$$
$$2.4311 \rightarrow 1.9044$$

With these trees,

$$\sigma_S = \frac{\partial S}{\partial V}\frac{V}{S}\sigma = \frac{3.1726 - 2.4311}{13.0006 - 12.2591} \times \frac{12.6244}{2.2591} \times 0.0587 = 25\%$$

S is now 2.9654, but we're getting there...

Repeatedly we can iterate until being satisfied with both parameters.

This can be done with repeated use of Excel Goalseek, or using Excel Solver once.

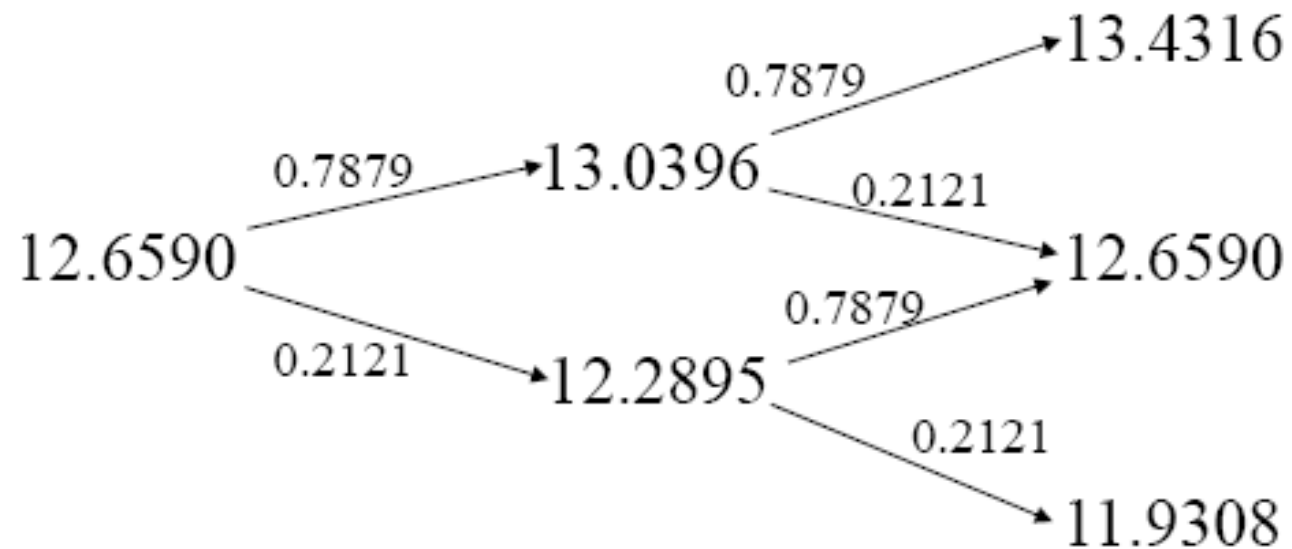In this case, the "correct" parameters are:

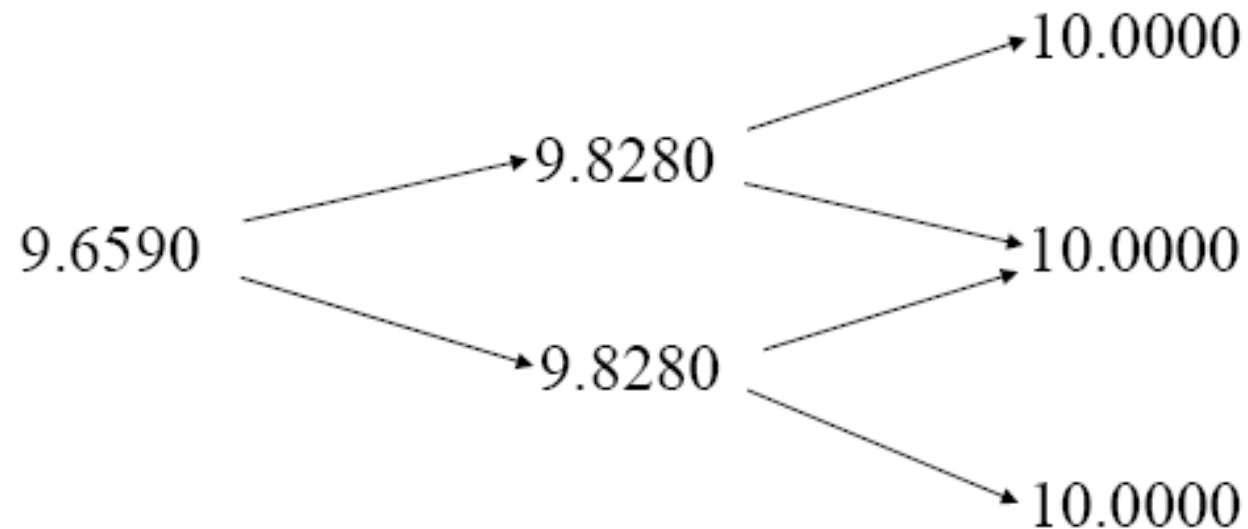$$V = 12.6590$$
$$\sigma = 5.92\%$$
$$u = 1.0301$$
$$p = 0.7879$$

We can now use the tree for V in order to value debt.

In this example debt does not involve credit risk.

Exercise

The spreadsheet MertonExercise.xls contains a 6-step

implementation of the model with the following parameters:

A debt security, with 6 months maturity;

Par Value 100;

Flat Interest Rates 8%;

Equity Value 30;

Volatility of Shares 40%;

Six steps to maturity, each 1 month long.