PROBABILITY AND STOCHASTIC PROCESSES

1. PRINCIPLES OF ACTUARIAL MODELLING
   - 1.1. The model–based approach: why and how models are used
   - 1.2. Benefits and limitations of modelling
   - 1.3. Stochastic versus deterministic models; static random phenomena versus stochastic processes
   - 1.4. Suitability of a model; analysing the output of a model; sensitivity testing
   - 1.5. Communicating the results of a model

2. DISTRIBUTIONS AND BASIC DISTRIBUTIONAL QUANTITIES
   - 2.1. Overview of some concepts: experiment, outcome, sample space, event, event space, algebra of events, probability function, probability space, conditional probability, theorem of total probability, Bayes' formula, multiplication rule, independent events
   - 2.2. Random variable; distribution function; continuous, discrete and mixed random variables; hazard rate
   - 2.3. Multivariate random variables; independent random variables
   - 2.4. Moments and related quantities
   - 2.5. Some well-known discrete and continuous random variables
   - 2.6. Residual life; left censored and shifted random variable; limit loss variable
   - 2.7. Quantiles
   - 2.8. Moment generating function, probability generating function and cumulant generating function
   - 2.9. Sum of independent random variables; central limit theorem
   - 2.10. Tails of distributions: comparison of the tail based on moments, on the limiting tail behaviour, on the hazard rate function and on the mean excess loss function; The equilibrium distribution and the tail behaviour

3. CHARACTERISTICS OF ACTUARIAL MODELS
   - 3.1. Parametric and scale distributions: scale distribution and scale parameter; location and shape parameters; the exponential family and the linear exponential family
   - 3.2. Mixed distributions: discrete and continuous mixtures

4. SEVERITY MODELS (CONTINUOUS MODELS)
   - 4.1. Creating new distributions: sums of distributions; transformation of random variables; mixing of distributions
   - 4.2. Recognition of families of distributions and their relations

5. GENERAL NOTIONS OF STOCHASTIC PROCESSES
   - 5.1. Some definitions
   - 5.2. Specification of a stochastic process
   - 5.3. Classification of a stochastic process

6. DISCRETE TIME MARKOV CHAINS
    6.1. Definitions
    6.2. Transition probability matrices
    6.3. First step analysis
    6.4. Classification of states
    6.5. Limit Behaviour
    6.6. Applications to no claim discount and *bonus-malus* systems

7. INTRODUCTION TO COUNTING PROCESSES
    7.1. Some definitions: counting process, Markov counting process, birth process – the homogeneous and the non-homogeneous processes
    7.2. The homogeneous Poisson process: its genesis, discussion of the postulates, some related distributions –the exponential, the gamma, the binomial and the uniform
    7.3. The non-homogeneous Poisson process
    7.4. The mixed Poisson process; the Polya process

8. CONTINUOUS TIME HOMOGENEOUS MARKOV CHAINS
    8.1. Introduction: time homogeneous Markov process; Chapman-Kolmogorov equations
    8.2. The transition probability matrix
    8.3. The forward and backward differential equations
    8.4. The embedded Markov chain
    8.5. Stationary and limiting distributions

9. TIME INHOMOGENEOUS MARKOV CHAINS
    9.1. Introduction; Chapman-Kolmogorov equations
    9.2. Kolmogorov's forward differential equations
    9.3. Probabilities of remaining in states for given time periods
    9.4. Kolmogorov's backward differential equations
    9.5. Applications in insurance