Capital Budgeting:
Investment Decision Rules

Gestão Financeira I
Gestão Financeira
Corporate Finance I
Corporate Finance

Licenciatura
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Outline

- Criteria for Accepting or Rejecting a Project:
    - The Payback Rule
    - Net Present Value (NPV)
    - Internal Rate of Return (IRR) and Modified Internal rate of return (MIRR)
- Choosing between mutually exclusive alternatives
- Evaluate projects with different lives
- Resourse Constraints: Rank projects when a company's resources are limited so that it cannot take all positive-NPV projects

Payback Period

- The Payback period is the amount of time it takes to recover or pay back the initial investment.

- If the payback period is less than a pre-specified length of time (which can be subjective), you accept the project. Otherwise, you reject the project.
  - The payback rule is used by many companies because of its simplicity.

- Example:

t	0	1	2	3	4
cash flow	-1000	250	330	500	610
Cumulative cash flow	-1000	-750	-420	80	690

$$\text{Payback Period} = 2 + \frac{420}{500} = 2.84 \text{ years}$$

- Limitation: Does not take into account the time value of money.

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Payback Period (discounted version)

- **Example**: Consider a discount rate (or cost of capital) r=11%.

t	0	1	2	3	4
cash flow	-1000	250	330	500	610
Discounted cash flow	-1000	225,2252	267,8354	365,5957	401,8259
Cumulative Discounted cash flow	-1000	-774,775	-506,939	-141,344	260,4822

$$\text{Discounted Payback Period} = 3 + \frac{141,344}{401,826} = 3.351754 \text{ years}$$

- **Limitations** of the Payback Rule:
  - Ignores what happens after the payback period (what if there were more cash flows?);
  - Difficult to apply when there are multiple investments over time.
  - Subjective definition of maximum acceptable period.

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Net Present Value

- Net Present Value (NPV) =

  Total PV of future CFs - Initial Investment

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}$$

- Minimum Acceptance Criteria: Accept if NPV >= 0

- Example: Consider the previous example, with discount rate r=11%

t	0	1	2	3	4
cash flow	-1000	250	330	500	610
Discounted cash flow	-1000	225,2252	267,8354	365,5957	401,8259

$$NPV = -1000 + \frac{250}{(1+0.11)} + \frac{330}{(1.11)^2} + \frac{500}{1.11^3} + \frac{610}{1.11^4} =$$

$$-1000 + 225.2 + 267.8 + 365.6 + 401.8 = 260.4822$$

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Properties of NPV:

- Additivity, Using All Cash Flows, and the Time Value of Money;
- Assumes intermediate cashflows are reinvested at the expected cost of capital r;
- Can deal with different discount rates during the life of a project ($r_{0,1}$; $r_{1,2}$; etc).

Limitations of NPV:

- It's an absolute measure, disregarding the scale of investment;
- It's indifferent to the length of the projects.
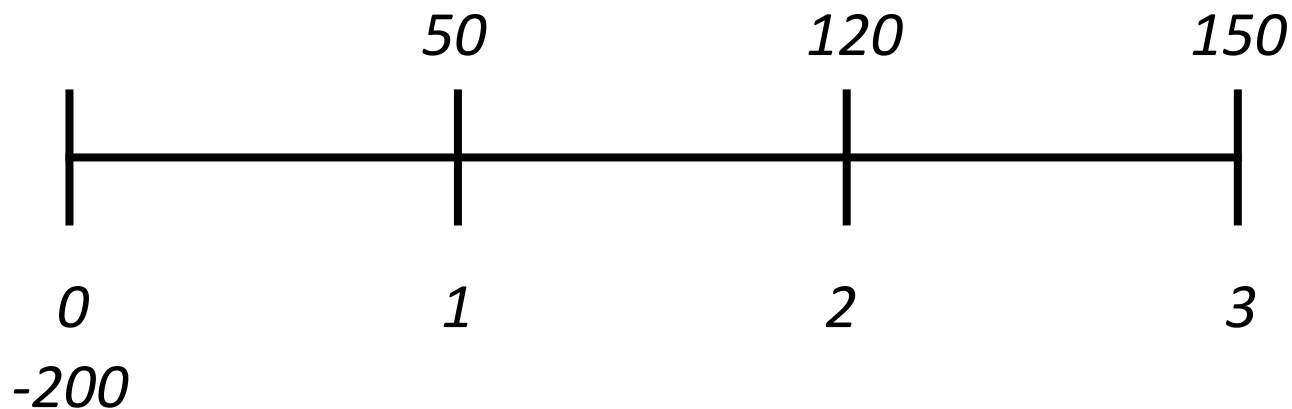
Computation in Excel

=npv(r;initialcell:finalcell)

Notes: (i) discounts immediately the first value; (ii) ignores empty cells.

Internal Rate of Return

- IRR: discount rate that sets NPV to zero

- Minimum Acceptance Criteria:
  - Accept if the IRR exceeds the discount rate

- Reinvestment assumption:
  - All future cash flows assumed reinvested at the IRR

- When working properly, IRR's decision coincides with the NPV rule's decision:
  - When IRR>r (the cost of capital), NPV>0.

- But sometimes there are problems with the IRR rule (we'll see the limitations).

Internal Rate of Return

Example: Consider the project:
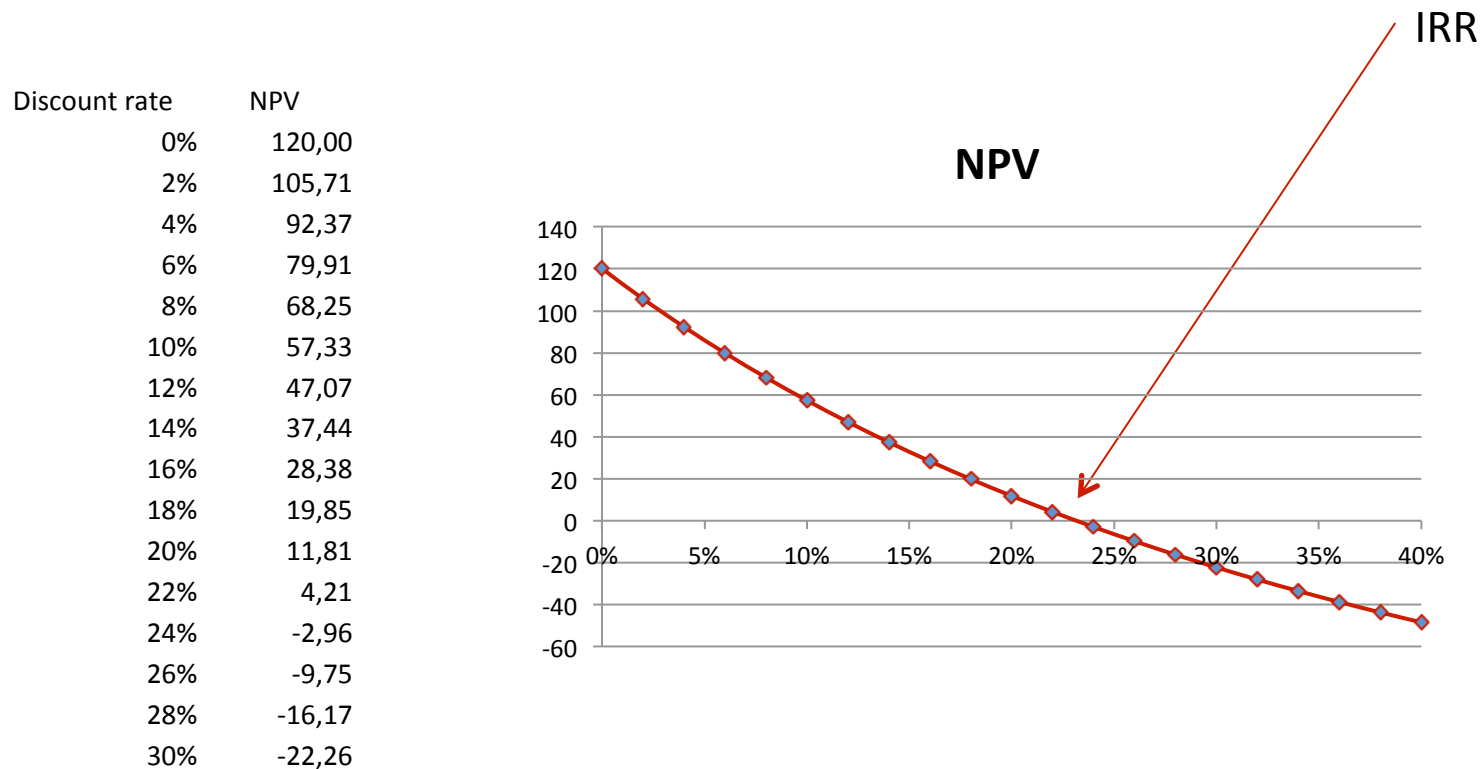

$$-200 + \frac{50}{(1+IRR)} + \frac{120}{(1+IRR)^2} + \frac{150}{(1+IRR)^3} = 0$$

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$$IRR = 23.16\%$$

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NPV Profile and IRR

- If we graph NPV versus the discount rate, we can see the IRR as the x-axis intercept

Discount rate	NPV
0%	120,00
2%	105,71
4%	92,37
6%	79,91
8%	68,25
10%	57,33
12%	47,07
14%	37,44
16%	28,38
18%	19,85
20%	11,81
22%	4,21
24%	-2,96
26%	-9,75
28%	-16,17
30%	-22,26

IRR

NPV

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Limitations of the IRR rule

- Situations where the IRR rule and NPV rule may be in conflict:

  - Delayed Investments (*Investing or Lending?*)

  - Nonexistent IRR

  - Multiple IRRs

- Delayed Investment (i.e., positive cash flows come first, and negative cash flows come later).

- Example:
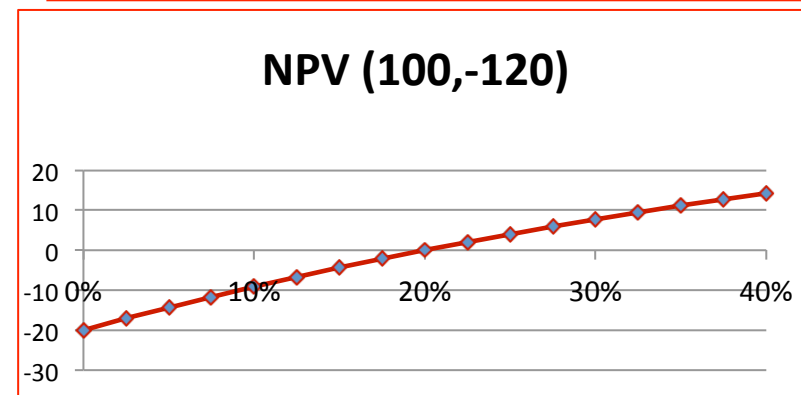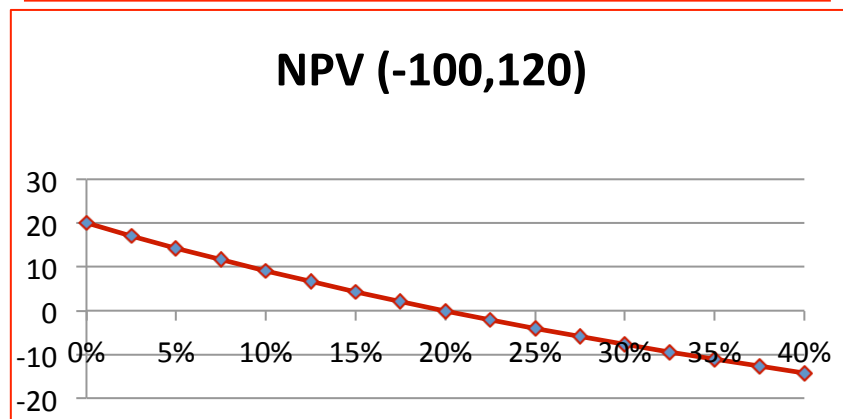 - Project A has cash flow stream: ($C_0 = -100$; $C_1 = +120$)
  - Project B has cash flow stream: ($C_0 = +100$; $C_1 = -120$)

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$$-100 + \frac{120}{1 + IRR_A} = 0 \Rightarrow IRR_A = 20\%$$

$$100 - \frac{120}{1 + IRR_B} = 0 \Rightarrow IRR_B = 20\%$$

NPV (-100,120)

NPV (100,-120)

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Limitations of the IRR: Nonexistent IRR

- **Nonexistent IRR**: For certain projects, no IRR exists; there is no discount rate that makes NPV equal to zero.

- **Example**: A famous writer, Star, is able to get her publisher to increase her advance to $750,000, in addition to the $1 million when the book is published in four years. In years 1, 2, and 3 her annual cash flow is negative (-$500,000). No rate would make this project have a negative NPV.
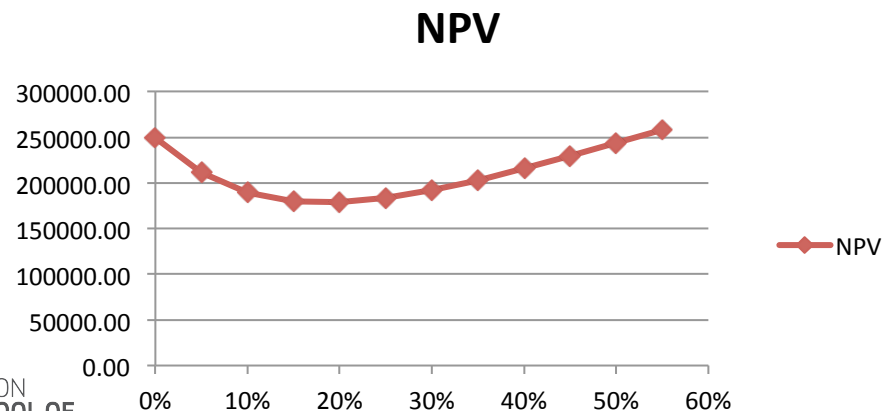
NPV

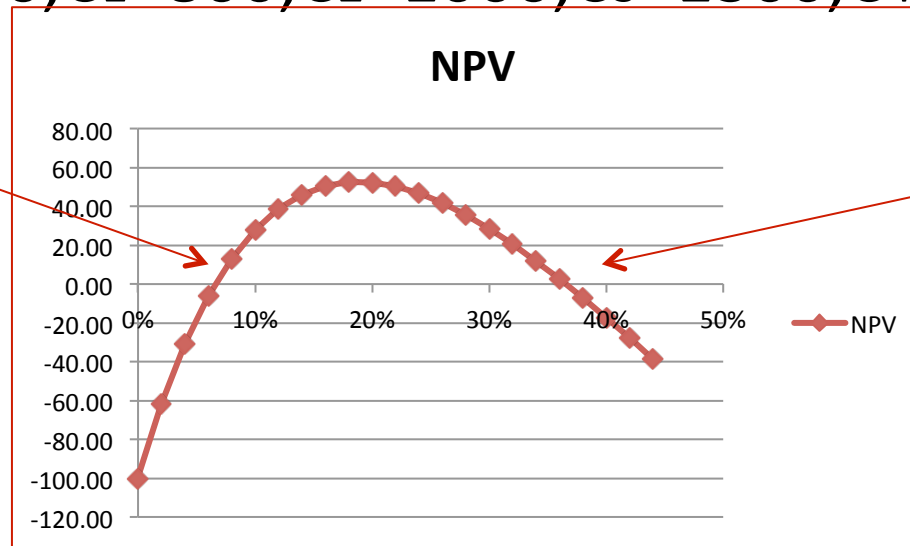- So, the IRR rule can't be used in this case.

- **Multiplicity of IRR**: when there's more than one change of sign of the cash flows, there might be more than one rate for which NPV=0.
  - In such cases you cannot apply the IRR rule.

- **Example**: Consider a project with cash flows:

$(C_0 = -1000, C_1 = 800, C_2 = 1000, C_3 = 1300, C_4 = -2200)$

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IRR1=6.6%

NPV

IRR2=36.545%

- So, the IRR rule can't be used in this case.

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Choosing between projects

- If projects are independent, it's easy: choose the ones that have non-negative NPV (and have IRR>discount rate, if IRR can be applied).

- If projects are Mutually Exclusive:
  - We must choose the projects that have the highest NPV.
  - The IRR rule could be misleading
    - Compared to NPV, IRR favors projects of smaller scale;
    - Compared to NPV, IRR favors projects of short duration.

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Choosing between projects: Use NPV Rule

- **Example:** You own a small piece of commercial land near a university. You are considering what to do with it. You have been approached recently with an offer to buy it for $220,000. You are also considering three alternative uses yourself: a bar, a coffee shop, and an apparel store. You assume that you would operate your choice indefinitely, eventually leaving the business to your children. You have collected the following information about the uses. What should you do?

	Initial Investment	Cash flow in the First Year	Growth rate	Cost of capital
Bar	$400,000	$60,000	3.5%	12%
Coffee shop	$200,000	$40,000	3%	10%
Apparel Store	$500,000	$85,000	3%	13%

- The NPVs are:

$$\text{Bar} \qquad \frac{\$60,000}{0.12 - 0.035} - \$400,000 = \qquad \$305,882$$

$$\text{Coffee Shop:} \quad \frac{\$40,000}{0.10 - 0.03} - \$200,000 = \qquad \$371,429$$

$$\text{Apparel Store:} \quad \frac{\$75,000}{0.13 - 0.03} - \$500,000 = \qquad \$250,000$$

Alternative	NPV
Coffee Shop	$371,429
Bar	$305,882
Apparel Store	$250,000
Sell the Land	$220,000

- You should choose the Coffee Shop alternative.

Choosing between projects with the IRR: Scale **Problem**

- **Example**: Consider two mutually exclusive projects (r = 10%):

  – Small: ($CF_0 = -1000, CF_1 = 2000$);

  IRR = 100%; NPV = 818

  – Large: ($CF_0 = -1500, CF_1 = 2750$);

  IRR = 83%; NPV = 1000

- IRR and NPV give different answers:

  – IRR favors small scale project, which has lower NPV; but we should pick large scale project – with highest NPV.
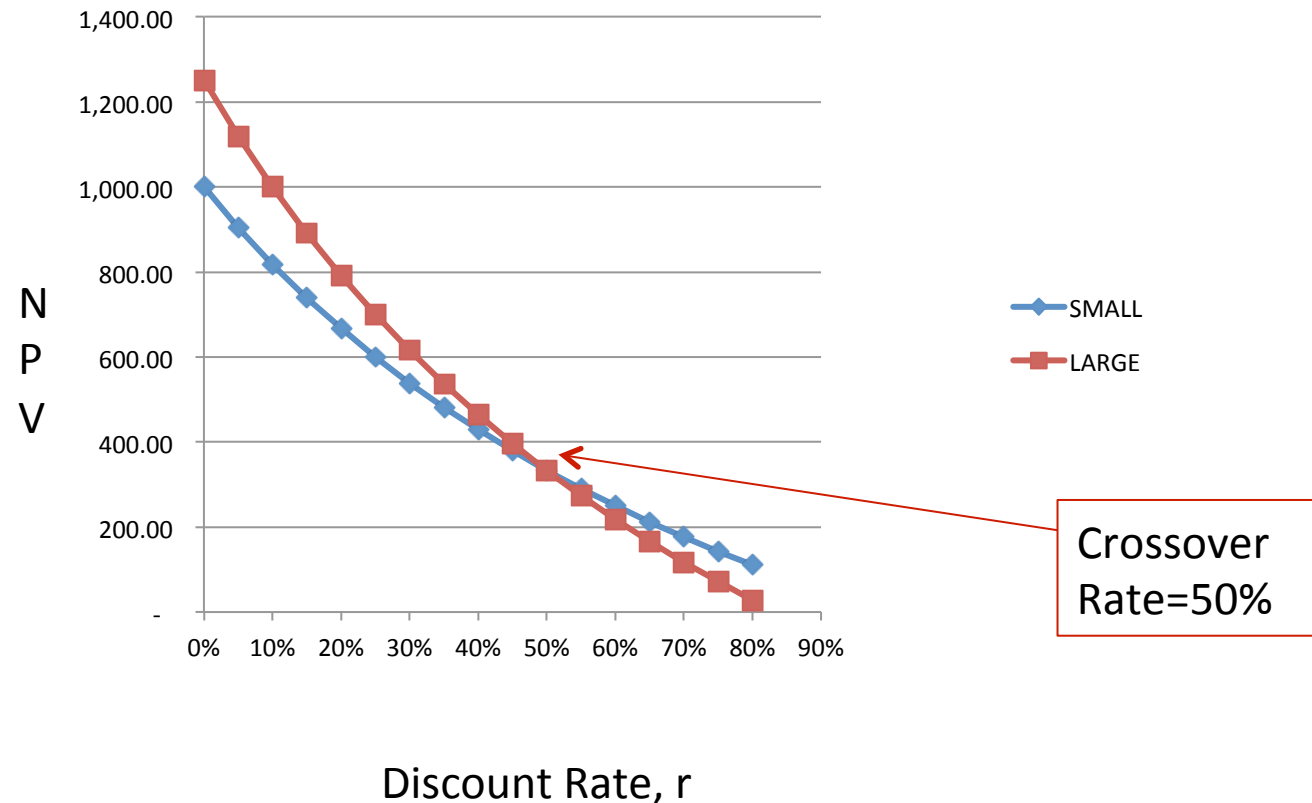
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Choosing between projects with Different Scales of Investment: Crossover point

- You can identify at which rate the two projects would be indifferent in terms of NPV. That is the crossover point.

$$-1000 + \frac{2000}{1+r_{cross}} = -1500 + \frac{2750}{1+r_{cross}}$$

$$500 = \frac{750}{1+r_{cross}}$$

$$r_{cross} = 0.5 = 50\%$$

Choosing between projects with Different Scales of Investment: Crossover point

- Project Large better as long as discount rate r<50%;
- Project Small better as long as discount rate r>50%.

- Example:

t	0	1	2	3	NPV	IRR
Proj A: CFt	-10000	5000	4000	6100	1.383,25 €	22,92%
Proj B: CFt	-10000	3000	3500	9500	1.501,60 €	22,23%
r	0,15				B	A

- NPV rule and IRR rule disagree in ranking the projects.
- IRR tends to favor projects that are faster in returning cash. But it is project B that generates more value (given r=15%).
- We could compute the crossover point and determine for which discount rates it's better to choose A (because it has the highest NPV) and for which discount rates it's better to choose B (because it is the one with the highest NPV).

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$$-10000 + \frac{5000}{1+r_{cross}} + \frac{4000}{\left(1+r_{cross}\right)^2} + \frac{6100}{\left(1+r_{cross}\right)^3} = -10000 + \frac{3000}{1+r_{cross}} + \frac{3500}{\left(1+r_{cross}\right)^2} + \frac{9500}{\left(1+r_{cross}\right)^3}$$

$$\frac{2000}{1+r_{cross}} + \frac{500}{\left(1+r_{cross}\right)^2} - \frac{3400}{\left(1+r_{cross}\right)^3} = 0$$

$$r_{cross} = 0.18482 = 18.482\%$$
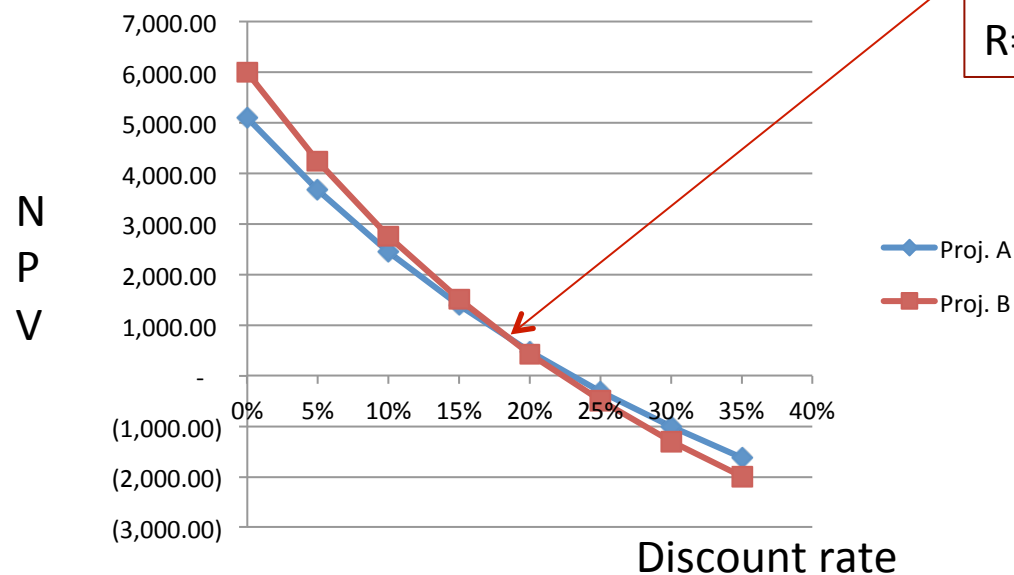
Crossover Point
R=18.48%

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With r=15%, B adds more value than A. Indeed, for any rate below 18% B would be preferable.

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Choosing between projects with the Modified Internal Rate of Return (MIRR)

- – Used to overcome problem of multiple IRRs

- – Computes the discount rate that sets the NPV of modified cash flows to zero

- – Possible modifications
  - Bring all negative cash flows to the present and incorporate into the initial cash outflow
  - Leave the initial cash flow alone and compound all of the remaining cash flows to the final period of the project

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Choosing between projects with the Modified Internal Rate of Return (MIRR)

- Consider the following example:

Project	t	0	1	2	3	NPV	IRR
C	CFt	-1000	830	580	-200	61,09	17,82%
D	CFt	-1000	400	1000	-100	83,16	17,73%
r	12%					D	C

- NPV and IRR agree in saying that both projects are good (NPV>0 and IRR>12%, which is the discount rate.

- But NPV ranks project D above project C

- And IRR is higher for project C.

- Solution: Always choose project with highest NPV

- But me may also compute a MIRR…

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Choosing between projects with the **Modified Internal Rate of Return (MIRR)**

- When we compute the MIRR for the projects, we get the same ranking as with the NPV criterion.

Project	t	0	1	2	3	NPV	MIRR
C	Modified CFt	-1142,4	0	0	1691	61,09	13,96%
D	Modified CFt	-1071,2	0	0	1622	83,16	14,83%
r		12%				D	D

- Let's exemplify for project A:

$$\text{Modified } CF_0 = -1000 + \frac{-200}{(1+0.12)^3} = -1142.4$$

$$\text{Modified } CF_3 = 830(1+0.12)^{3-1} + 580(1+0.12)^{3-2} = 1690.75$$

$$NPV = -1142.4 + \frac{1690.75}{(1+0.12)^3} = 61.09$$

$$-1142.4 + \frac{1690.75}{(1+MIRR)^3} = 0$$

$$MIRR = 13.96\%$$

Evaluating Projects with Different Lives

- Often, a company will need to choose between two solutions to the same problem.

- Example: Cash Flows ($ Thousands) for Network Server Options

Year	PV at 10%	0	1	2	3
A	−12.49	−10	−1	−1	−1
B	−10.47	−7	−2	−2	

- The costlier options seems to be A.

- But it also lasts for 3 years (longer than equipment B).

- We can compare them based on an equivalent constant annual cost.

- In this example we can find annuities for each project that are equivalent to the true cash flows in terms of generating the same total present value.

$$EA = \frac{PV}{\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]}$$

- Example: Cash Flows ($ Thousands) for Network Server Options, Expressed as Equivalent Annual Annuities

Year	PV at 10%	0	1	2	3
A	−12.49	0	−5.02	−5.02	−5.02
B	−10.47	0	−6.03	−6.03	

$$EA_A = \frac{12.49}{\frac{1}{0.1}\left[1 - \frac{1}{(1+0.1)^3}\right]} = 5.02$$

$$EA_B = \frac{10.47}{\frac{1}{0.1}\left[1 - \frac{1}{(1+0.1)^2}\right]} = 6.03$$

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Evaluating Projects with Different Lives: Equivalent Annual Annuity

- In the end, the equivalent annuity cost is lower for project A.

- Note: it only makes sense to use the Equivalent annuities if it is credible that projects will be repeated over time, and if it's reasonable that the values will be similar in the future.

Choosing Among Projects when Resources are Limited

- When the company has resource constraints it may not be able to invest in all projects that have positive NPV.

- How to choose, then?

- In the end, it will be necessary to really choose exhaustively the combination of projects (amongst those that satisfy the constraint) that maximizes total NPV.

- But it's also common to compute the Profitability Index of the Projects – helps, but not always…

Profitability Index

- In its simplest version the Profitability Index is the ratio between the project's NPV and the resource it consumes:

$$\text{Profitability Index } = \frac{\text{Value Created}}{\text{Resource Consumed}} = \frac{\text{NPV}}{\text{Resource Consumed}}$$

- The Resource Consumed can be anything: e.g., investment capital, human resources, a rare raw material, space.

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Profitability Index

- **Example**: Consider a company with the following investment opportunities (projects A to G), but with limited capital of 100 to finance the new projects:

Project	Investment	NPV	PI	Ranking
A	25	10	40%	4
B	60	30	50%	3
C	5	5	100%	1
D	100	25	25%	7
E	50	15	30%	5
F	70	20	29%	6
G	35	20	57%	2

- Projects C, G and B should be accepted since, by following the PI ranking, we completely use the budget of 100, without any slack. Total NPV is 55.

- The Profitability Index rule works well in this context.

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Limitations of the Profitability Index

- If, when following the PI ranking, the budget is not fully spent, then we cannot be sure that the PI rule is maximizing total value.
  - Example: Going back to the previous example. If the budget were 150:
    - Combination C, G, B and E generates a higher NPV than does portfolio C, G, B and A (if we followed the PI ranking).

- With multiple resource constraints, the profitability index can break down completely.