CHAPTER 3: THEORY OF THE FIRM

Exercise 1

Solve the long run profit maximization problem for a production function given by $f(x) = x^a$, $a > 0$.

Exercise 2

Solve the long run profit maximization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $a + b < 1$, $a > 0$, $b > 0$.

Exercise 3

Solve the short run profit maximization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $1 > a > 0$, $b > 0$, where x_2 represents the quantity of the fixed input.

Exercise 4

Solve the long run cost minimization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $a > 0$, $b > 0$.

Exercise 5

Solve the short run cost minimization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $1 > a > 0$, $b > 0$, where x_2 represents the quantity of the fixed input.

Exercise 6

Derive the input demand, the output supply, and the profit functions for the following production functions:

    a. $f(x_1, x_2) = x_1 + x_2$.
    b. $f(x_1, x_2) = \min\{x_1, x_2\}$.

Exercise 7

Let $f(x)=10x-x^2/2$. Determine the input demand, the output supply, and the profit functions.

Exercise 8

Derive the conditional input demand functions and the cost function for the technologies given by:

   a. $f(x)= x_1+ x_2$.
   b. $f(x)= \min\{x_1, x_2\}$.
   c. $f(x)= (x_1{}^a+ x_2{}^a)^{1/a}$, for $a < 1$.

Exercise 9

Let $f(x_1, x_2, x_3, x_4)=\min\{x_1, x_2\}+ \min\{x_3, x_4\}$ and let $g(x_1, x_2, x_3, x_4)=\min\{x_1+x_2, x_3+x_4\}$.

   a. Determine the cost functions and the conditional input demands for both production functions.
   b. What kind of returns to scale does each of these technologies exhibit?

Exercise 10

Show that if the production function is homogeneous of degree 1, the marginal rate of technical substitution is independent of the scale of production.