Corporate Investment Appraisal

Masters in Finance

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Problem Set Nº 1: Guideline to Solutions

Problem 1: What are the Nash equilibria of the following game, after elimination of dominated strategies? Explain the steps followed in order to reach your results.

		Player B		
		Left	Center	Right
Player A	Top	4,0	1,2	1,2
	Middle	4,5	0,4	0,1
	Bottom	0,1	2,0	2,1

This is a possible sequence to determine a DS equilibrium:

- From player A's perspective, since 4≥4, 1≥0, and 1≥0, strategy T weakly dominates M regardless of the other player's action). Thus, we eliminate row "Middle".
- Since 2≥0 and 1≥1, for player B strategy Right weakly dominates Left. We can eliminate "Left".
- Because 2≥1 and 2≥1, for Player A strategy Bottom weakly dominates Top. So, we eliminate "Top".
- Finally, as 1≥0, for Player B strategy Right dominates CEnter. We can eliminate "Center".
- We are left with (Bottom,Right), which is the only equilibrium in dominated strategies (DSE).

Problem 2: Two Californian teenagers, Bill and Ted, are playing a game with the following pay-offs matrix:

		Ted	
		Left	Right
Bill	Top	-4,-4	4,2
	Bottom	2,4	-2,-2

(a) Determine all equilibria in pure strategies. Explain.
(b) Determine all equilibria in mixed strategies. Explain.
(c) What's the probability of both players having positive pay-offs? Explain.

(a) NE in Pure strategies: (B,L) and (T,R). Explain...

(b) NE in mixed strategies: Bill chooses Top with probability 1/2 and Ted chooses Left with probability 1/2.

Why?

$$E(Top) = -4\pi_L + 4(1-\pi_L)$$
$$E(Bottom) = 2\pi_L - 2(1-\pi_L)$$
$$-4\pi_L + 4(1-\pi_L) = 2\pi_L - 2(1-\pi_L)$$
$$\pi_L = \frac{1}{2}$$
$$E(Left) = -4\pi_T + 4(1-\pi_T)$$
$$E(Right) = 2\pi_T - 2(1-\pi_T)$$
$$-4\pi_T + 4(1-\pi_T) = 2\pi_T - 2(1-\pi_T)$$
$$\pi_T = \frac{1}{2}$$

(c) When the solutions are (B,L) and (T,R) both players have strictly positive payoffs.

In the case of pure strategies, the outcomes (B,L) and (T,R) are NE and we can't "really" say the probabilities of each being chosen by the players.

If they play the mixed strategy equilibrium, the probability of (B,L) happening is ½*½=1/4, and the probability of (T,R) happening is ½*½=1/4.

Problem 3: Consider the following coordination game:

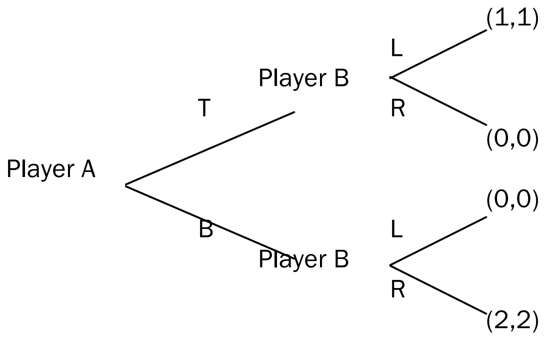
		Player B	
		Left	Right
Player A	Top	1,1	0,0
	Bottom	0,0	2,2

(a) Compute all pure strategy equilibria of this game. Explain.
(b) Do any of these strategies dominate any of the others? Explain.
(c) Now suppose that Player A plays first, committing to choose either Top or Bottom. Are the strategies of question (a) still Nash equilibria?
(d) What are the "subgame perfect" equilibria of this game?

(a) NE in pure strategies: (T,L), (B,R). Explain…

(b) No strategy dominates any other. Explain…

(c) and (d)



The sub-game perfect equilibrium of this game is (B,R). Why?

A plays first.

If A plays T, player B will choose L (1>0). Hence, Player A would get 1.

If A plays B, then player B will choose R (because 2>0). Hence A would get 2.

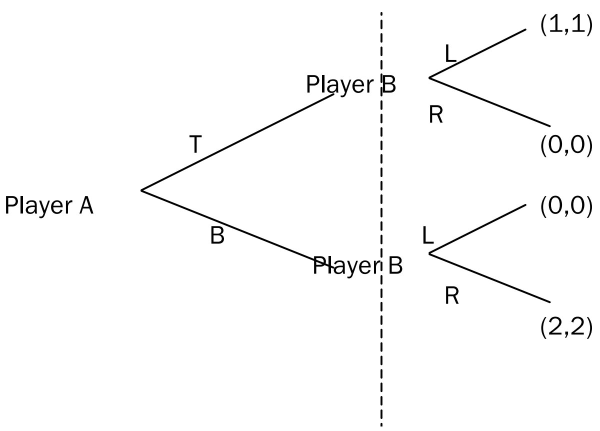
Therefore, player A chooses Bottom, then player B chooses Right, and the SPE is (B,R), with payoffs (2,2).

Problem 4: Consider the previous question's game, in which the players choose their strategies simultaneously.

(a) Represent the game in extensive form.

(b) Describe the perfect Bayesian equilibria (PBE) of this game.

(a) Extensive Form (assuming that player A plays first):



(b) Analysis of the perfect Bayesian equilibria (PBE):

(i) If player A believes that player B plays R with probability q and plays L with probability (1-q), player A knows that:

- If she plays B her expected payoff is 2q+0(1-q)=2q
- If she plays T her expected payoff is 0q+(1-q)=1-q

(ii) In equilibrium player A should choose (let's say p is the probability of player A choosing Bottom):

- p=1 if 2q > 1-q
- p in [0,1] if q=1/3
- p=0 if q<1/3

(iii) If player B believes that player A chooses B with probability p, then he knows that:

- If he plays R his expected payoff is: 2p+0(1-p)=2p
- If he plays L his expected is 0p+1(1-p)=1-p

(iv) Hence, Player B should choose according to (where q is the probability with which he plays R):

- q=1 if p>1/3
- q in [0,1] if p=1/3
- q=0 if p < 1/3

(v) Finally what will characterize an equilibrium, taking into account that a condition for equilibrium is that the beliefs of each player about its oponent's behavior must coincide with the equilibrium strategies:

- Start with the case in which A chooses p>1/3. If B guesses this correctly, B chooses q=1. But if q=1, and A guesses this correctly, then A would choose p=1, which is compatible with the initial conjecture of p>1/3. We found a PBE equilibrium in which (p=1, q=1).
- If Player A chooses p=1/3, and B guesses this correctly, B is indifferent between L and R. He may choose any q in the interval [0,1]. In case B chooses q=1/3, that would be compatible with A "replying" p=1/3, since A would also be indifferent between strategies. We found another PBE with (p=1/3,q=1/3).
- Finally, if A chooses p<1/3, and player B guesses this correctly, player B chooses q=0. But if B chooses q=0, and player A guesses this correctly, then player A should respond with p=0 (which is compatible with the conjecture that p<1/3). We found the third PBE of this game, with (p=0,q=0).