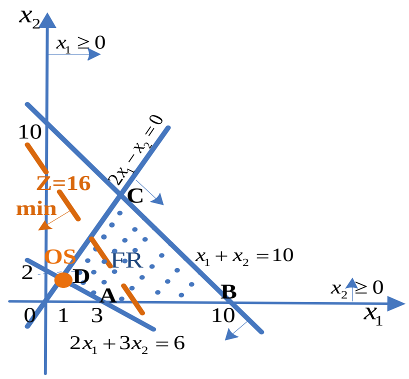
1.

a) Max $w = 6y_1 + 10y_2$

s.to: $\begin{cases} 2y_1 + y_2 + 2y_3 \le 4 \\ 3y_1 + y_2 - y_3 \le 2 \\ y_1, y_3 \ge 0,\ y_2 \le 0 \end{cases}$

b)



FR = [ABCD]

I) OS: $\begin{cases} 2x_1 - x_2 = 0 \\ 2x_1 + 3x_2 = 6 \end{cases} \Leftrightarrow \ldots \Leftrightarrow \begin{cases} x_1 = \frac{3}{4} \\ x_2 = \frac{3}{2} \end{cases} \Rightarrow$ Z* = 6

The solution is on the 1st and 3rd constraint lines, then the values of the slack variables x_3 and x_5 are zero, and

$x_1 + x_2 + x_4 = 10 \Leftrightarrow x_4 = \frac{31}{4}$.

b.II) Dual OS: from the complementary relationships: $x_1 \times y_4 = 0 \wedge x_2 \times y_5 = 0 \wedge x_4 \times y_2 = 0$.

Then, $\begin{cases} x_1 > 0 \Rightarrow 2y_1 + y_2 + 2y_3 = 4 \\ x_2 > 0 \Rightarrow 3y_1 + y_2 - y_3 = 2 \\ x_4 > 0 \Rightarrow y_2 = 0 \end{cases} \Leftrightarrow \begin{cases} y_1 = 1 \\ y_2 = 0 \\ y_3 = 1 \end{cases}$ and W* = Z* = 6

c) Graphically:

SI of b_1: $2x_1 + 3x_2 = b_1$. Critical points: B $\left(\frac{10}{3}, \frac{20}{3}\right)$ where $2x_1 + 3x_2 = \frac{80}{3}$ and (0,0) where $2x_1 + 3x_2 = 0$. Then $b_1 \in \left[0, \frac{80}{3}\right]$.

SI of b_2: $x_1 + x_2 = b_2$. Critical point D $\left(\frac{3}{4}, \frac{3}{2}\right)$ where $x_1 + x_2 = \frac{9}{4}$. Then $b_2 \in \left[\frac{9}{4}; +\infty\right[$.

d) If c_2=0 Z is parallel to x_2 axis. The OS remains unchanged and $Z^* = 4x_1 = 3$, decreases 3 units.

e) Let y_j=1 if x_j>0; 0 otherwise, with j=2,3 and let M be a constant with a value sufficiently high.

$Min\ \ Z = 4x_1 + 2x_2 + 2x_3$

s.to: $\begin{cases} 2x_1 + 3x_2 + x_3 \ge 6 \\ x_1 + x_2 + 2x_3 \le 10 \\ 2x_1 - x_2 \ge 0 \\ y_2 + y_3 \le 1 & \text{incompatible products} \\ x_j \le M\, y_j \quad j = 2,3 & \text{relationship between variables} \\ x_3 \ge 4\, y_3 & \text{if produced, at least 4 units should be considered} \\ x_1, x_2, x_3 \ge 0 \\ y_2,\ y_3 \in \{0,1\} \end{cases}$

2. **a)** **x**=(5, 0, 0, 6, 0, 4) BFS non optimal, since there is a negative coefficient in line Z.

2.b)

				E.C						
VB	z	x_1	$x_2 \downarrow$	x_3	x_4	x_5	x_6	RHS		
z	1	0	-3	0	0	2	0	20	L.C.	
x_1	0	1	-1	2	0	1	0	5		
$\leftarrow x_4$	0	0	(2)	0	1	0	0	6	6/2	Min
x_6	0	0	1	1	0	2	1	4	4/1	
z	1	0	0	0	3/2	2	0	29		
x_1	0	1	0	2	1/2	1	0	8		
x_2	0	0	1	0	1/2	0	0	3		
x_6	0	0	0	1	-1/2	2	1	1		

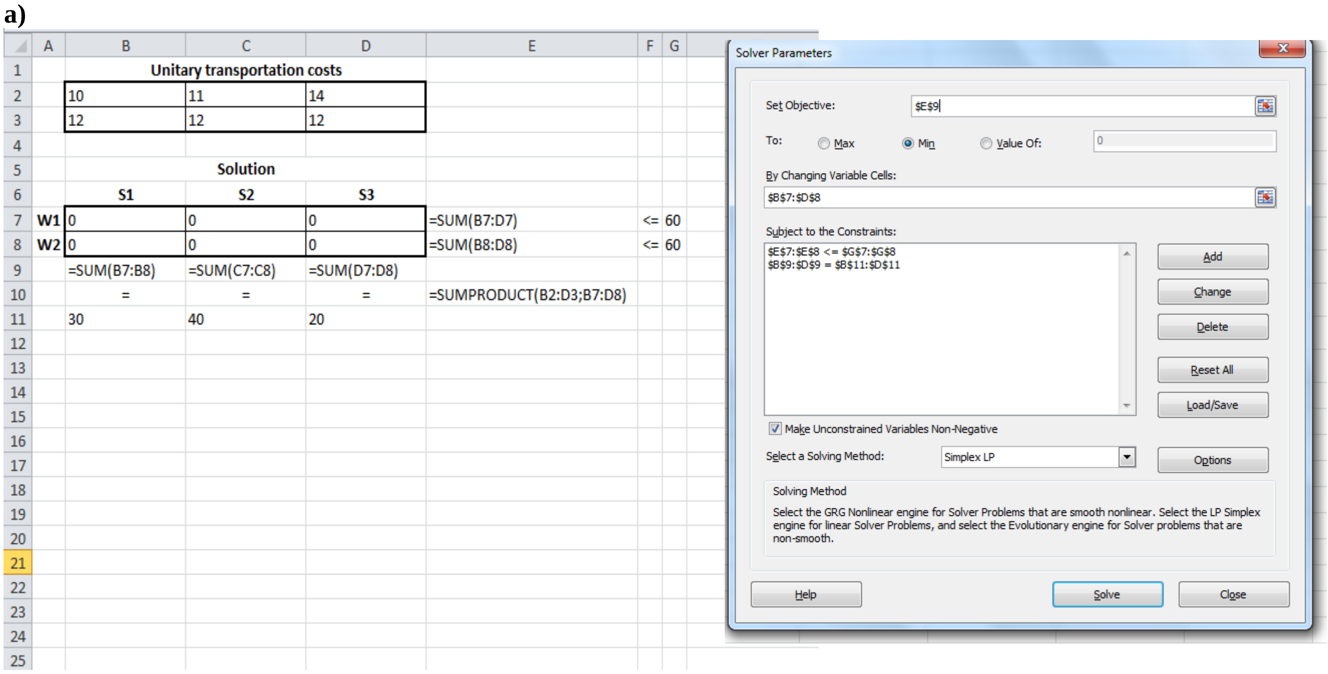
OS since all the coefficients at line Z are ≥0. **x***=(8, 3, 0, 0, 0, 1), Z*=29=W*, y_1=3/2; y_2=2; y_3=0 (coefficients of the slack variables at line Z).

c) y_1=3/2; x_4=0. The work hours available in the 1st section represent a scarce resource (x_4=0). For each additional w.h. in this section, within its sensitivity interval, the total gross margin will increase 3/2 m.u.

d.I) There will be an alternative BFS , since the objective function value remains equal 20.

d.II) An infeasible BS will be obtained, since the pivot is a negative number (-1) the corresponding BV will be negative.

3. a)



	A	B	C	D	E	F	G
1			Unitary transportation costs				
2		10	11	14			
3		12	12	12			
4							
5			Solution				
6		S1	S2	S3			
7	W1	0	0	0	=SUM(B7:D7)	<=	60
8	W2	0	0	0	=SUM(B8:D8)	<=	60
9		=SUM(B7:B8)	=SUM(C7:C8)	=SUM(D7:D8)			
10		=	=	=	=SUMPRODUCT(B2:D3;B7:D8)		
11		30	40	20			

b) The restriction associated to the warehouse **W2** should be replace by an equality: $x_{21} + x_{22} + x_{23} = 60$.