# Instituto Superior de Economia e Gestão <br> Operacional Research -Semester 1 <br> Época de Recurso 

Note: Justify all your answers and present the calculations

1. Consider the following LP problem:
$\operatorname{Min} Z=4 x_{1}+2 x_{2}$
s.to: $\left\{\begin{aligned} 2 x_{1}+3 x_{2} & \geq 6 \\ x_{1}+x_{2} & \leq 10 \\ 2 x_{1}-x_{2} & \geq 0 \\ x_{1}, x_{2} & \geq 0\end{aligned}\right.$
a) $(1,5$ points) Formulate the dual of the given problem.
b) ( 4,5 points) Solve the given LP graphically, indicate the points that form the Feasible Region and write:
I) The optimal solution of the augmented LP.
II) The optimal solution (decision variables) and optimal value of the dual.
c) (2 points) Determine the sensitivity intervals of the first and the second constraints right hand sides.
d) (1 point) If the coefficient of $x_{2}$ in the objective function turns to be null, what is the impact in the values of $x_{1}, x_{2}$ and in the optimal value?
e) ( 1,5 points) Using an ILP model, formulate the problem that results from the previous by introducing a third variable that, to be considered, must have a minimum value of 4 is incompatible with $x_{2}$, and have coefficient 2 in the objective function and coefficients 1 , 2,0 in the three given constraints.
2. The following simplex tableaux corresponds to a problem of maximizing the gross margin achieved by the production of three products, whose quantities are represented by decision variables, $x_{1}, x_{2}$ and $x_{3}$. The conditions for the functioning of the system are due to limited capacity of the processing into three sections (in wh) and written in three functional constraints ( $\leq$ ).

| VB | z | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ | $x_{6}$ | T. I. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| z | 1 | 0 | -3 | 0 | 0 | 2 | 0 | 20 |
| $x_{1}$ | 0 | 1 | -1 | 2 | 0 | 1 | 0 | 5 |
| $x_{4}$ | 0 | 0 | 2 | 0 | 1 | 0 | 0 | 6 |
| $x_{6}$ | 0 | 0 | 1 | 1 | 0 | 2 | 1 | 4 |

a) (1 point) Write and classify the primal solution associated with the given tableaux.
b) (3 points) Get and write the primal optimal solutions (decision and slack variables) and dual (only decision variables), and the value of the respective objective functions.
c) (1 point) Interpret the meaning of the first shadow price relating it, if possible, with the value of the slack variable of the first primal constraint.
d) (1 point) Considering the given simplex tableaux and without performing calculations explain:
I) What are the implications of introducing variable $x_{3}$ in the set of basic variables?
II) What are the implications of replacing $x_{1}$ by $x_{2}$ in the set of basic variables?
3. The company AEI has two warehouses (W1 and W2) where it stores the products for its shops. The transport of a certain type of perishable products requires speed. Daily, in each warehouse, 60 units of the product can be stored, and the demands are 30,40 and 20 units in each of three shops ( $\mathbf{S 1}, \mathbf{S} 2$ and $\mathbf{S 3}$ ). The unit transportation costs between warehouses and shops are displayed in the following table.

|  | S1 | S2 | S3 |
| :---: | :---: | :---: | :---: |
| W1 | 10 | 11 | 14 |
| W2 | 12 | 12 | 12 |

a) ( 2,5 points) In the Excel sheet attached formulate the problem to identify the quantities that each shop should order to the warehouses daily to satisfy the demand while minimizing the total cost associated with transportation.
b) (1 point) What changes should be done in the model to ensure that the warehouse $\mathbf{W} \mathbf{2}$ will order the maximum units allowed and they will be fully disposed.


#### Abstract

NAME:


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