ISEG **Quantitative Finance – 1st year/Economics-Finance-Management, 1st sem.,** **F1** **08. 11. 2017, 9h**
1st Part **(35 marks)** **Total Time: 1h**

1st Part: 35 Marks. All answers shall be given in the space available. All True/False questions have equal marking. During the test there won't be any comments or questions given. Write your name and number on every sheet. No mobile phones are allowed at any time.

Name: XXX **Number:** XXXX

In the following group of questions, every right answer has 2.5 marks each, wrong answers have –2.5 each (2.5 penalty mark). *[Each group of questions will have a mark between **0** (minimum) and **10** (maximum)]*
Write True (T) or False (F), *with an **X** in the appropriate entry.*

1. Consider Simple and Compound Interest calculation:

	T	F
In Compound Interest the effective semiannual interest rate is always proportional to the corresponding annual nominal rate compounded quaterly.		X
Annual discount factor v is the same as the simple discount interest rate d.		X
Money accumulated under simple or compound interest always lead to different values, no matter is the accumulation period, as long as the interest rate is positive.		X
In simple interest, the interest amount in each accumulation period is always constant (Principal and rate stay unchanged).	✓	

2. Consider Ordinary or Deferred annuities:

	T	F
The names *Annuity-Immediate* and *Annuity-Due* refer to the same.		X
A life insurance, with monthly paid premiums, is an *annuity-certain*.		X
$295.55({}_{5\|}\ddot{a}_{\overline{2}\|5\%}) = 295.55({}_{4\|}a_{\overline{2}\|5\%})$.	✓	
$1/a_{\overline{2}\|5\%} - 1/s_{\overline{2}\|5\%} = 0.05$.	✓	

In the next group of questions, tick ✓ or write X in the box next to the answer you consider to be correct (only one is). In each group, a correct answer has 5 marks and a wrong answer gets -1.25 marks (penalty 1.25).

3. In compound interest, compute the nominal anual rate compounded semiannualy equivalent to an effective quarterly rate of 2%:

a) 8.08% [X]; **b)** 4.00% [ ]; **c)** 8.00% [ ]; **d)** 4.04% [ ].

4. Consider compound interest. Mr. Ed invested €50,000 for 5.5 years with an effective semiannual rate of 3%. The total interest produced in this application is:

a) €8,826.73 [ ]; **b)** €19.211,69 [X]; **c)** €18.889,38 [ ]; **d)** None of the previous [ ].

5. Mr. Ed has to choose one of the following receipts:

(i) Receive €10,000 in one year and the same amount in two years time, exactly;
(ii) Receive €10.250 within six months and the same amount within nine months.
Assuming a quaterly effective rate of 1% and compound interest, should Mr. Ed choose:

a) (i) [ ]; **b)** (ii) [X]; **c)** indifferent [ ]; **d)** Not enough information [ ];

Quantitative Finance Formulas

Interest accumulation: $Fv = Pv + I$

Simple interest: $Fv = Pv(1 + i \cdot t)$

Compound interest: $Fv = Pv(1 + i)^t$

Simple discount: $D = Fv \cdot d \cdot t$

I=Interest; P=Principal; i=interest rate

t=number of periods

Effective rates conversion:

$i_L = (1 + i_S)^{L/S} - 1;\ i_S = (1 + i_L)^{S/L} - 1$

Relation between nominal and effective rates:
$i_A(m) = m[(1 + i_A)^{1/m} - 1]$

Continuous compounding:

Nominal rate: $\delta = \ln(1 + i_A)$

Future Value: $S = Pe^{\delta t}$

Present Value: $P = Se^{-\delta t}$

Present value of a n payment annuity immediate of 1 per period: $a_{\bar{n}|i} = \frac{1-(1+i)^{-n}}{i}$

Accumulated value of a n payment annuity immediate of 1 per period:

$$s_{\bar{n}|i} = \frac{(1+i)^n - 1}{i} = a_{\bar{n}|i}(1 + i)^n$$

Present value of annuity due:

$$\ddot{a}_{\bar{n}|i} = 1 + a_{\overline{n-1}|i} = a_{\bar{n}|i}(1 + i)$$

Accumulated value of annuity due:

$$\ddot{s}_{\bar{n}|i} = s_{\bar{n}|i}(1 + i)$$

Present value of deferred annuity:

$$_{k|}a_{\bar{n}|i} = a_{\bar{n}|i}(1 + i)^{-k}$$

Accumulated value of deferred annuity:

$$_{k|}s_{\bar{n}|i} = s_{\bar{n}|i}$$

Forborne annuities

$FV = R.S_{n|i}(1+i)^p$

p- number of intervals between the last payment and FV.

Present value of perpetuity immediate: $a_{\bar{\infty}|i} = \frac{1}{i}$

Increasing arithmetic progression:

$$(C - h)a_{\bar{n}|i} + h(Ia)_{\bar{n}|i};\qquad (Ia)_{\bar{n}|i} = \frac{\ddot{a}_{\bar{n}|i} - n(1+i)^{-n}}{i}$$

Decreasing arithmetic progression:

$$(D - h)a_{\bar{n}|i} + h(Da)_{\bar{n}|i};\quad (Da)_{\bar{n}|i} = \frac{n - a_{\bar{n}|i}}{i}$$

Geometric progression: $C\frac{1-r^n(1+i)^{-n}}{1+i-r}$

M^{thly} payable annuity:

$$a^{(m)}_{\bar{n}|i} = a_{\bar{n}|i}\frac{i}{i^{(m)}};\ s^{(m)}_{\bar{n}|i} = s_{\bar{n}|i}\frac{i}{i^{(m)}}$$

Leasing:

Lease payment=PMT + I

Pv=PMT $a_{\bar{n}|i}$, I=RV $\cdot\ i$

Leasing (for an annuity immediate):

$Vc = E + Ra_{\bar{n}|i} + RV(1 + i)^{-n}$, where

Vc: value of the contract; E: entry value

RV = residual value; PMT = periodic payment

Linear Interpolation:

Rn=R1+[(R2-R1)/(t2-t1)].(tn-t1)

Rn - unknown rate
R1 and R2 - two known

2nd Part (65 marks)

In this group write your calculations in the space below the question and write the final answer in the box provided. Do not forget to present all formulae and intermediate calculations needed.

1. (20 marks)

Mr. Ed borrowed a certain amount of money at a semiannual rate of 5%. He agreed to pay back the debt (money+interest) in the following terms:

€3,000.00 after 3 months, €5,000.00 after 9 months, and €10,000.00 Euros after 15 months.

Find the amount of money Mr. Ed borrowed considering simple interest.

$i_5 = 0.05$

(20)

$$PV = \frac{3,000}{(1+0.5(0.05)} + \frac{5000}{1+1.5(0.05)} + \frac{10,000}{1+2.5(0.05)}$$

$$PV = \frac{3,000}{1.025} + \frac{5000}{1.075} + \frac{10000}{1.125} = 16,466.88$$

R: 16,466.88

2. (20 marks)

Find the future value, S, of 20 semianually payments of €3,000.00 each that Mr. Ed has to face, at $i_S = 4\%$, using compound interest.

$$S = 3000\, s_{\overline{20}|0.04} = 3000\, \frac{1.04^{20} - 1}{0.04} \simeq 30.000(29,778\,08)$$

(20)

$$\simeq 89,334.24$$

R: $S = 89,334.24$

Name:_____ Number:_____

3. (25 marks)

Mr. Ed borrowed money to buy a new equipment. He will pay back that money in 4 years, through quarterly payments of €450 each. The nominal annual interst rate is 8,0% compounded quarterly. Find how much the equipment costs, if:

a) The first payment is due three months after the loan;

b) The first payment is due six months after the loan.

a)

$n = 4 \times 4 = 16$, $i^{(4)}_4 = 0.08 \longrightarrow i_1 = 0.02$

$P = 450\, a_{\overline{16}|0.02} = 450\, \dfrac{1-1.02^{-16}}{0.02} \simeq 450\,(13.577\,71)$

$\simeq 6,109.97$

(20)

R: $\simeq 6,109.97$

b)

$6,109.97\,(1.02)^{-1} \simeq 5,990.17$

(5)

R: $\simeq 5,990.17$