Notes: - Justify all answers and display the calculations performed.

- All answers should be given by using methodologies studied in Operational Research.

1. $(4,5)$ Consider the following LP problem.

$$
\begin{aligned}
& \operatorname{Max} Z=2 x_{1}+4 x_{2} \\
& \text { s.t.: } \\
& \qquad\left\{\begin{array}{c}
2 x_{1}+x_{2} \leq 20 \\
x_{1}+2 x_{2} \leq 18 \\
x_{1}-2 x_{2} \geq 0 \\
x_{1}, x_{2} \geq 0
\end{array}\right.
\end{aligned}
$$

a) Perform one iteration of simplex method.
b) Write and classify the solution obtained in a).
c) Write the dual of the given problem.
2. $(1,0)$ Justify in what situation may the following output be obtained while solving an LP problem by Solver/Excel.

3. An investor has 1200 monetary units (m.u.) that can be invested in funds. The information gathered about the three funds available is displayed in the following table:

| fund | return rate | aquisition value (in m.u.) <br> of one participation unit |
| :---: | :---: | :---: |
| A | $9 \%$ | 1,20 |
| B | $8 \%$ | 1,80 |
| C | $6 \%$ | 1,90 |

In order to control the portfolio risk the following rules should be observed:
rule 1 - At least 110 participation units of fund $C$ should be acquired
rule 2 - The minimum investment in fund $B$ is 400 m.u.
a) $(2,0)$ Formulate the investor problem by a linear programming model, assuming that any fraction of a participation unit can be acquired, and that the aim is to maximize the total return.

An LP formulation of the described problem was solved by Solver/Excel and the reports are on this page. Based on them answer to the following questions.
b) $(1,5)$ Indicate and interpret the optimal solution of the problem (decision and slack variables).
c) $(1,0)$ Indicate and interpret the optimal value of the dual variable associated with the constraint called "budget" in the Solver / Excel output.

## Answer Report

Objective Cell (Max)

| Cell | Name | Original Value | Final Value |
| ---: | :--- | :---: | :---: |
| $\$ E \$ 8$ | OF | 0 | 97,73 |

Variable Cells

| Cell | Name | Original Value | Final Value | Integer |
| ---: | :--- | :---: | :---: | :---: |
| $\$ B \$ 9$ | Fund A | 0 | 492,5 | Contin |
| $\$ C \$ 9$ | Fund B | 0 | 222,22 | Contin |
| $\$ D \$ 9$ | Fund C | 0 | 110 | Contin |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
| :---: | :--- | :---: | :---: | :---: | :---: |
| $\$$ E $\$ 5$ | rule 1 | 110 | $\$ E \$ 5>=\$ G \$ 5$ | Binding | 0 |
| $\$ E \$ 6$ | rule 2 | 400 | $\$ E \$ 6>=\$ G \$ 6$ | Binding | 0 |
| $\$ E \$ 7$ | budget | 1200 | $\$ E \$ 7<=\$ G \$ 7$ | Binding | 0 |

## Sensitivity Report

Variable Cells

| Cell | Name | Final <br> Value | Reduced <br> Cost | Objective <br> Coefficient | Allowable <br> Increase | Allowable <br> Decrease |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: |
| $\$ B \$ 9$ | Fund A | 492,5 | 0 | 0,108 | $1 E+30$ | 0,012 |
| $\$ C \$ 9$ | Fund B | 222,22 | 0 | 0,144 | 0,018 | $1 E+30$ |
| $\$ D \$ 9$ | Fund C | 110 | 0 | 0,114 | 0,057 | $1 E+30$ |

## Constraints

| Cell | Name | Final <br> Value | Shadow <br> Price | Constraint <br> R.H. Side | Allowable <br> Increase | Allowable <br> Decrease |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| $\$ E \$ 5$ | rule 1 | 110 | $-0,057$ | 110 | 311,05 | 110 |
| $\$ E \$ 6$ | rule 2 | 400 | $-0,01$ | 400 | 591 | 400 |
| $\$ E \$ 7$ | budget | 1200 | 0,09 | 1200 | $1 E+30$ | 591 |

