

$$4(b) \quad dX_t = 2X_t dt + \cancel{X_t} dB_t + \int_{|x| \geq 1} \underbrace{(e^{\frac{x}{2}} - 1)}_{K(t,x)} N(dt, dx) + \int_{|x| < 1} \underbrace{0}_{\tilde{K}(t,x)} \tilde{N}(dt, dx)$$

$$Y(t) = \ln X(t)$$

or  $Y(t) = f(X(t))$  with  $f(x) = \ln x$

By Ito's formula (3) for slides

$$\textcircled{0.5} \quad dY(t) = \frac{\partial f}{\partial x}(X(t)) dX(t) + \frac{1}{2} \frac{\partial^2 f}{\partial x^2}(X(t)) d[X, X](t) + \int_{|x| \geq 1} \ln(X(t) + K(t,x) - \ln(X(t))) N(dt, dx)$$

$$= \frac{1}{X(t)} [2X_t dt + \cancel{X_t} dB_t] - \frac{1}{2X(t)^2} (\cancel{X_t})^2 dt$$

$$+ \int_{|x| \geq 1} \ln(\cancel{1} + e^{\frac{x}{2}} - 1) N(dt, dx)$$

$$\Leftarrow dY(t) = 2 dt + \cancel{dB_t} - \frac{(\cancel{X_t})^2}{2} dt + \int_{|x| \geq 1} \left(\frac{x}{2}\right) N(dt, dx)$$

$$\textcircled{0.5} \Rightarrow Y(t) = Y(0) + (2 - \frac{1}{2})t + \cancel{B_t} + \int_0^t \int_{|x| \geq 1} \left(\frac{x}{2}\right) N(ds, dx)$$