INSTITUTO SUPERIOR DE ECONOMIA E GESTÃO

OPERATIONAL RESEARCH – 1ST Semester

Date: 10/01/2018 **Época Normal – Block 2** Duration: **1 hour**

Note: Justify all your answers and present all the calculations

1. A group of ISEG students offered to help the Mayor of a municipality affected by the fires and droughts that occurred in Portugal with the knowledge acquired in IO. As is well known, the funds are scarce to cope with the various situations caused by these scourges. One of the pressing issues is to provide food for animals. The animals require a minimum daily amount of two types of nutrients that could be provided by pasture or through two forages, **A** and **B**. To identify the funds to be assigned the following LP problem has been formulated:

$$\min Z = 3x_A + 4x_B + x_P$$

$$\text{s.t.}: \begin{cases} x_A + 2x_B + x_P \geq 6 \\ 3x_A + 2x_B + x_P \geq 8 \\ x_P \leq 3 \\ x_A,\ x_B,\ x_P \geq 0 \end{cases}$$

where, x_j is the quantity (in tones, t.) of forage j needed per day, $j =$ **A**, **B** and x_P the time (in hours) of grazing per day. The objective function minimizes the daily cost of the animals feeding (in monetary units, m.u.).

Solving the LP problem by Solver/Excel the following "Sensitivity Report" was obtained:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B6	forage A t.	1	0	3	3	1
C6	forage B t.	1	0	4	2	2
D6	Hours of grazing	3	0	1	1	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E2	nutrient 1	6	1,5	6	2	1,333
E3	nutrient 2	8	0,5	8	4	2
E4	Max hours of pasture	3	-1	3	2	3

Answer all the following questions based only on the "Sensitivity Report" and consider them independent.

a) (1,5 points) The Mayor may negotiate the price of forage **A** to be supplied at 2,5 m.u. a tone. What is the change in the daily feed cost of the animals?

b) (1,5 points) Usually, before the fires and drought, the maximum pasture time was 5 hours per day. Knowing that, calculate the effect on the cost of feeding the animals caused by the scourges.

c) (1,0 point) The budget for the first ten days is 105 m.u.. Is that amount enough or this information should be included in the model?

d) (2,0 points) Rewrite the model in order to consider also the transportation cost. The transport is made in trucks with a capacity of half a ton being the cost each trip 150 m.u. whether the load is total or partial, in addition, the forages cannot be mixed.

(please turn)

2. (2,0 points) The failure of the communications network was one of the important problems identified. To prevent this from happening again, the Mayor received a donation of € 20,000 to build a supplementary network to ensure the connection of strategic sites. The sites, as well as the respective distances (in km) by paths in which the passage of fiber optic is feasible, are shown in the following table:

	A	B	C	D	E	F
A		65		30	50	
B	65		40	50		
C		40				20
D	30	50				30
E	50					60
F			20	30	60	

Knowing that the price of fiber optics is 200 € per km, use a network optimization problem studied to determine how much the municipality should spend in addition to the donation.

3. (2,0 points) As a result of the drought, the two reservoirs, **R1** and **R2**, which usually supply water to the population, are at very low levels. In this way, it was decided that each one should receive 4,000 m^3 of water that can be collected in three nearby reservoirs **R3**, **R4** and **R5**, with a maximum water capture of 3 000 m^3 each. The transport cost (in u.m.) of one cubic meter of water between the reservoirs is given in the following table:

	R1	**R2**
R3	5	3
R4	2	5
R5	2	3

Formulate the problem to decide how to transfer water between reservoirs at a minimum total cost using an LP model.