**ISEG Quantitative Finance – 1st year/Economics-Finance-Management, 1st sem., Exam 1 19. 01. 2018, 9:00**

**1st Part (70 marks) Total Time: 2h**

**1st Part: 70 Marks. All answers shall be given in the space available. All True/False questions have equal marking. During the test no comments or questions should be asked. Write your name and number on every sheet on the place available. No mobile phones, or any device with bluetooth or wifi, are allowed at any time.**

**Name:**____________________________________________________________**Number:**_________

***In the following group of questions, every right answer has 2.5 marks each, wrong answers have –2.5 each (2.5 penalty mark).*** *Each group of questions will have a mark between* **0** *(minimum) and* **10** *(maximum).*

Write True (**T**) or False (**F**), *with an "**X**" in the appropriate entry*.

**1.** Consider Simple and Compound Interest calculation:

| | **T** | **F** |
|---|---|---|
| In Compound Interest, the effective monthly interest rate is proportional to the (annual) nominal rate. | | |
| For a positive rate and same period, in Simple Interest discount rate and interest rate are the same. | | |
| Two rates of interest are said to be equivalent if they result in the same accumulated value at any, and the same, point in time, no matter the interest regime. | | |
| The Annual Nominal Rates should always be compounded annually. | | |

**2.** Consider discount and ordinary annuities:

| | **T** | **F** |
|---|---|---|
| *"The discounted value of an annuity certain should be calculated with the discount rate".* | | |
| Consider an annuity due and a deferred annuity with equal unit payments, same number of payments and the same constant non-negative interest rate. The value $\ddot{a}_{\overline{n}\|}$ corresponds to $a_{\overline{n}\|}$ accumulated one time period. | | |
| A sequence of periodic payments, located at equal time intervals, makes an annuity if and only if all payments are equal. | | |
| Let $i > 0$ and $n$ denote a constant interest rate and the term of an annuity certain, respectively. Then, $1 + i\, s_{\overline{n}\|8\%} = (1+i)^n$. | | |

**3.** Consider deferred annuities, perpetuities, variable payment annuities and debts:

| | **T** | **F** |
|---|---|---|
| $\ddot{a}_{\overline{10}\|5\%} > \ddot{a}_{\overline{10}\|1\%}$. | | |
| The formula $s_{\overline{n}\|i} = ((1+i)^n - 1)/i$ results from the sum of the payments of an annuity with variable payments in decreasing geometric progression. | | |
| Keeping $i$ and $n$ unchanged, the future value of an annuity, $s_{\overline{n}\|i}$, does not change if the annuity is deferred. | | |
| In a "constant amortization loan" both interest and principal payments are decreasing as payments are made. | | |

**4.** Consider bonds, leasing and shares:

| | **T** | **F** |
|---|---|---|
| A share grants its owner the benefit from future profits of the company the share refers to. | | |
| A Leasing is a rental contract used by companies and individuals for buying fixed assets. | | |
| Shares are equity securities. | | |
| A Zero Coupon Bond does not pay interest. | | |

***In the next group of questions, tick √ or write X in the box next to the answer you consider to be correct (<u>only one is</u>). In each group, a correct answer has <u>5 marks</u> and a wrong answer gets <u>-1.25</u> marks (penalty 1.25).***

**5.** In compound interest, Mr. Brad did a four year application of €10,000 at a nominal (annual) rate of 2% compounded semi-annually. Write the interest earned in the third year, approximately.

**a)** €105.10 ☐; **b)** €102.01 ☐; **c)** €209,16 ☐; **d)** None of the others ☐.

**6.** Consider compound interest and quarterly effective interest rate of 1%. Mr. Ed has to select one of the following receipt options:

i. Receive €10,100 in a year from now, and the same value within two years exactly;
ii. Receive €10,000 in half a year from now, and the same value within a year from then, exactly.

**a)** Choose i) ☐; **b)** Choose ii) ☐; **c)** Indifferent ☐; **d)** Not enough information ☐.

**7.** Mr. Brad applied €2,000 for two years at an annual rate of 5% and after during a certain period at a yearly rate of 4%, having accumulated €2,432.16 in the end. For compound interest, what was the complete application period?

**a)** 4 years ☐; **b)** 4.5 years ☐ **c)** 5 years ☐; **d)** None of the others ☐.

**8.** A laptop computer has a selling price of €1,957.60 (after a certain discount). Mr. Brad has been given the option to pay the computer in six monthly instalments of €332 (approximately) each, being the first one within six months. Considering a annual interest rate of 6% compounded monthly, advise Mr. Ed the best option:

**a)** Pay €1,957.60 immediately ☐; **b)** Pay the annuity ☐; **c)** Indifferent ☐; **d)** Not enough information ☐.

**9.** Mr. Brad faces an annuity whose value at some moment in time is $500\ s_{\overline{5}|8\%}(1.08)^{-5}$. He is considering the calculations: (i) $500\ \ddot{a}_{\overline{5}|8\%}(1.08)^{-1}$; (ii) $500\ a_{\overline{5}|8\%}$; (iii) $250\left({}_{3|}a_{\overline{2}|8\%}\right) + 500\ a_{\overline{3}|8\%} + 250 s_{\overline{2}|8\%}(1.08)^{-5}$. The following calculations are equivalent to $500\ s_{\overline{5}|8\%}(1.08)^{-5}$, choose the **best / more complete option**:

**a)** (i) ☐; **b)** (i) and (ii) ☐ **c)** (i), (ii) and (iii) ☐; **d)** None of others ☐.
.

**10.** Consider Mr. Brad's Incomplete statement: "For a given positive interest rate, it is better to make an application of an amount of money in compound interest then in simple interest, …" . Complete with the best correct option:

**a)** "… if the application period is less than a year." ☐; **b)** "… depending on the interest rate size." ☐;

**c)** "… if the application period is greater than a year." ☐; **d)** "… depending on the reference period of the interest rate." ☐ .

**Name:______________________________________________________________Student No.:_________**

**2nd Part (130 marks)**

**In this group write your calculations in the space below the question and write the final answer in the box provided.** Do not forget to present all formulae and intermediate calculations needed**.**

## 1. (45 marks)

Mr. Brad intends to contract a 15 year mortgage loan whose amount is €125,000. The bank offered a mortgage loan contract with the following conditions:

- Nominal annual interest rate of 6.0%;
- The loan is payable with equal monthly payments (principal plus interest) at the end of each month, with no deferment;

**a)** Compute the value of each monthly payment.

R:

**b)** Compute the first four lines of the following amortization table:

| Month | Debt at beginning of the period | Interest | Payment | Amortization | Accumulative Amortization | Debt at end of the period |
|---|---|---|---|---|---|---|
| 1 | 125,000 | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |

## 2. (40 marks)

Mr. Brad financed a car aquisition with a leasing contract that has the following conditions:

- Term of leasing contract: 5 years;
- Quarterly effective rate of 3.0%, the first payment to be paid 3 months after the date of the contract;
- Down payment of 7% of the car value;
- Residual value with last payment and equal to 10% of the car value, assume €2,000 for the residual value;
- Constant payments every quarter (at the end of each period).

**a)** Compute the payments that are associated with the leasing contract.

| R: |
|---|

**b)** Compute the debt amount at the end of the first year, immediately after the corresponding payment.

| R: |
|---|

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**3. (45 marks)**

Mr. Brad's sharehold company, Omega, issued a bond loan with the following terms:

- Date of issue: 01/01/yy.
- Nominal Value: €10.00.
- N.º of bonds issued: 20,000.
- Issue value at par;
- Loan term: 3 years.
- Semi-annual coupon rate: 3.0%.
- Payment of semi-annual interest. The first payment will occur one semester after issuance.
- Mode of Redemption (above the par): Repayments semi-annually of equal number of bonds, starting one year after the issuance date;

Redemption premium: €0.50 per bond during the first two repayments and €1.00 per bond after that.

**a)** Compute the total value of the bond loan.

R:

**b)** Fill out the bond amortization table (Euros).

| Semester | Debt at beginning of the period | Interest | Nº de bonds repaid | Amortization | Premium | Total Payment |
|---|---|---|---|---|---|---|
| **0** | | | | | | |
| **1** | | | | | | |
| **2** | | | | | | |
| **3** | | | | | | |
| **4** | | | | | | |
| **5** | | | | | | |
| **6** | | | | | | |

**Quantitative Finance Formulas**

Interest accumulation: $Fv = Pv + I$

Simple interest: $Fv = Pv\,(1 + i \cdot t)$

Compound interest: $Fv = Pv\,(1 + i)^t$

Simple discount: $D = Fv \cdot d \cdot t$

I=Interest; P=Principal; i=interest rate

t=number of periods

Effective rates conversion:

$i_L = (1 + i_S)^{L/S} - 1$; $i_s = (1 + i_L)^{s/L} - 1$

Relation between nominal and effective rates:
$i_A(m) = m[(1 + i_A)^{1/m} - 1]$

Continuous compounding:

Nominal rate: $\delta = \ln(1 + i_A)$

Future Value: $S = Pe^{\delta t}$

Present Value: $P = Se^{-\delta t}$

Present value of a n payment annuity immediate of 1 per period: $a_{\bar{n}|i} = \frac{1-(1+i)^{-n}}{i}$

Accumulated value of a n payment annuity immediate of 1 per period:

$$s_{\bar{n}|i} = \frac{(1+i)^n - 1}{i} = a_{\bar{n}|i}(1+i)^n$$

Present value of annuity due:

$$\ddot{a}_{\bar{n}|i} = 1 + a_{\overline{n-1}|i} = a_{\bar{n}|i}(1+i)$$

Accumulated value of annuity due:

$$\ddot{s}_{\bar{n}|i} = s_{\bar{n}|i}(1+i)$$

Present value of deferred annuity:

$$_{k|}a_{\bar{n}|i} = a_{\bar{n}|i}(1+i)^{-k}$$

Accumulated value of deferred annuity:
$$_{k|}s_{\bar{n}|i} = s_{\bar{n}|i}$$

Forborne annuities

$FV = R.S_{n|i}(1+i)^p$

p- number of intervals between the last payment and FV.

Present value of perpetuity immediate: $a_{\bar{\infty}|i} = \frac{1}{i}$

Increasing arithmetic progression:

$(C-h)a_{\bar{n}|i} + h(Ia)_{\bar{n}|i}$; $(Ia)_{\bar{n}|i} = \frac{\ddot{a}_{\bar{n}|i} - \mathrm{n}(1+i)^{-n}}{i}$

Decreasing arithmetic progression:

$(D-h)a_{\bar{n}|i} + h(Da)_{\bar{n}|i}$; $(Da)_{\bar{n}|i} = \frac{n - a_{\bar{n}|i}}{i}$

Geometric progression: $C\,\frac{1-r^n(1+i)^{-n}}{1+i-r}$

M[thly] payable annuity:

$a_{\bar{n}|i}^{(m)} = a_{\bar{n}|i}\,\frac{i}{i(m)}$; $s_{\bar{n}|i}^{(m)} = s_{\bar{n}|i}\,\frac{i}{i(m)}$

Leasing:

Lease payment=PMT + I

Pv=PMT $a_{\bar{n}|i}$, I=RV $\cdot\, i$

Leasing (for an annuity immediate):

$Vc = E + Ra_{\bar{n}|i} + RV(1+i)^{-n}$, where

Vc: value of the contract; E: entry value

RV = residual value; PMT = periodic payment

Linear Interpolation:

Rn=R1+[(R2-R1)/(t2-t1)].(tn-t1)

Rn - unknown rate
R1 and R2 - two known