ISEG Quantitative Finance – 1st year/Economics-Finance-Management, 1st sem., F2 19.01.2018, 9:00
1st Part (35 marks) Total Time: 1h

1st Part: 35 Marks. All answers shall be given in the space available. All True/False questions have equal marking. During the test no comments or questions should be asked. Write your name and number on every sheet on the place available. No mobile phones, or any device with bluetooth or wifi, are allowed at any time.

Name:______________________________ Number:________

In the following group of questions, every right answer has 2.5 marks each, wrong answers have -2.5 each (2.5 penalty mark). [*Each group of questions will have a mark between* **0** *(minimum) and* **10** *(maximum)*]
Write True (T) or False (F), *with an* **X** *in the appropriate entry.*

1. Consider deferred annuities, perpetuities, variable payment annuities and debts:

	T	F
$\ddot{a}_{\overline{10}\vert 5\%} > \ddot{a}_{\overline{10}\vert 1\%}$.		X
The formula $s_{\overline{n}\vert i} = ((1+i)^n - 1)/i$ results from the sum of the payments of an annuity with variable payments in decreasing geometric progression.		X
Keeping i and n unchanged, the future value of an annuity, $s_{\overline{n}\vert i}$, does not change if the annuity is deferred.	✓	
In a "constant amortization loan" both interest and principal payments are decreasing as payments are made.		X

2. Consider bonds, leasing and shares:

	V	F
A share grants its owner the benefit from future profits of the company the share refers to.	✓	
A Leasing is a rental contract used by companies and individuals for buying fixed assets.		X
Shares are equity securities.	✓	
A Zero Coupon Bond does not pay interest.	✓	

In the next group of questions, tick ✓ or write X in the box next to the answer you consider to be correct (only one is). In each group, a correct answer has 5 marks and a wrong answer gets -1.25 marks (penalty 1.25).

3. Mr. Brad faces an annuity whose value at some moment in time is $500\ s_{\overline{5}\vert 8\%}(1.08)^{-5}$. He is considering the calculations: (i) $500\ \ddot{a}_{\overline{5}\vert 8\%}(1.08)^{-1}$; (ii) $500\ a_{\overline{5}\vert 8\%}$; (iii) $250\ ({}_{3\vert}a_{\overline{2}\vert 8\%}) + 500\ a_{\overline{3}\vert 8\%} + 250 s_{\overline{2}\vert 8\%}(1.08)^{-5}$. The following calculations are equivalent to $500\ s_{\overline{5}\vert 8\%}(1.08)^{-5}$, choose the **best / more complete option**:

a) (i) ☐; **b)** (i) and (ii) ☐ **c)** (i), (ii) and (iii) ☑; **d)** None of others ☐ .

4. A laptop computer has a selling price of €1,957.60 (after a certain discount). Mr. Brad has been given the option to pay the computer in six monthly instalments of €332 each, being the first one within a year. Considering a annual interest rate of 6% compounded mothly, advise Mr. Ed the best option:

a) Pay €1,957.60 immediately ☐ ; **b)** Pay the annuity ☑; **c)** Indifferent ☐ ; **d)** Not enough information ☐ .

5. Consider the following information about a constant amortization loan:

Period	Debt at beginning of the period	Interest	Payment	Amortization	Accumulative Amortization	Debt at end of the period
11	121.878,92€	731,27€	3.871,08€	3.139,81	6.260,89€	118.739,12€
12					+ 3139,81	
					9.399,70	

The accumulative amortization at end of period 12 is :

a) €10,131.97 ☐; **b)** None of the others ☑ **c)** €12,576.42 ☐; **d)** € 9,419.54 ☐;

Quantitative Finance Formulas

Interest accumulation: $Fv = Pv + I$

Simple interest: $Fv = Pv\,(1 + i\cdot t)$

Compound interest: $Fv = Pv\,(1 + i)^t$

Simple discount: $D = Fv\cdot d\cdot t$

I=Interest; P=Principal; i=interest rate

t=number of periods

Effective rates conversion:

$i_L = (1 + i_S)^{L/S} - 1$; $i_s = (1 + i_L)^{s/L} - 1$

Relation between nominal and effective rates:
$i_A(m) = m[(1 + i_A)^{1/m} - 1]$

Continuous compounding:

Nominal rate: $\delta = \ln(1 + i_A)$

Future Value: $S=Pe^{\delta t}$

Present Value: $P=Se^{-\delta t}$

Present value of a n payment annuity immediate of 1 per period: $a_{\bar{n}|i} = \frac{1-(1+i)^{-n}}{i}$

Accumulated value of a n payment annuity immediate of 1 per period:

$$s_{\bar{n}|i} = \frac{(1+i)^n-1}{i} = a_{\bar{n}|i}(1+i)^n$$

Present value of annuity due:

$$\ddot{a}_{\bar{n}|i} = 1 + a_{\overline{n-1}|i} = a_{\bar{n}|i}(1+i)$$

Accumulated value of annuity due:

$$\ddot{s}_{\bar{n}|i} = s_{\bar{n}|i}(1+i)$$

Present value of deferred annuity:

$$_{k|}a_{\bar{n}|i} = a_{\bar{n}|i}(1+i)^{-k}$$

Accumulated value of deferred annuity:
$_{k|}s_{\bar{n}|i} = s_{\bar{n}|i}$

Forborne annuities

$FV=R.S_{n|i}(1+i)^p$

p- number of intervals between the last payment and FV.

Present value of perpetuity immediate: $a_{\bar{\infty}|i} = \frac{1}{i}$

Increasing arithmetic progression:

$(C-h)a_{\bar{n}|i} + h(Ia)_{\bar{n}|i}$; $(Ia)_{\bar{n}|i} = \frac{\ddot{a}_{\bar{n}|i} - n(1+i)^{-n}}{i}$

Decreasing arithmetic progression:

$(D-h)a_{\bar{n}|i} + h(Da)_{\bar{n}|i}$; $(Da)_{\bar{n}|i} = \frac{n-a_{\bar{n}|i}}{i}$

Geometric progression: $C\,\frac{1-r^n(1+i)^{-n}}{1+i-r}$

M^{thly} payable annuity:

$a^{(m)}_{\bar{n}|i} = a_{\bar{n}|i}\frac{i}{i(m)}$; $s^{(m)}_{\bar{n}|i} = s_{\bar{n}|i}\frac{i}{i(m)}$

Leasing:

Lease payment=PMT + I

Pv=PMT $a_{\bar{n}|i}$, I=RV $\cdot\, i$

Leasing (for an annuity immediate):

$Vc = E + Ra_{\bar{n}|i} + RV(1+i)^{-n}$, where

Vc: value of the contract; E: entry value

RV = residual value; PMT = periodic payment

Linear Interpolation:

Rn=R1+[(R2-R1)/(t2-t1)].(tn-t1)

Rn - unknown rate
R1 and R2 - two known

2[nd] Part (65 marks)

In this group write your calculations in the space below the question and write the final answer in the box provided. Do not forget to present all formulae and intermediate calculations needed.

Name:____ **Number:**_________

1. (45 marks)

Mr. Brad's sharehold company, Omega, issued a bond loan with the following terms:

- Date of issue: 01/01/yy.
- Nominal Value: €10.00.
- N.º of bonds issued: 20,000.
- Issue value at par;
- Loan term: 3 years.
- Semi-annual coupon rate: 3.0%.
- Payment of semi-annual interest. The first payment will occur one semester after issuance.
- Mode of Redemption (above the par): Repayments semi-annually of equal number of bonds, starting one year after the issuance date;

Redemption premium: €0.50 per bond during the first two repayments and €1.00 per bond after that.

a) Compute the total value of the bond loan and the redemption premium to be paid.

Loan: 10(20.000) = 200,000

Premium: 2(2000)+3(4000)= 16 000

R: € 200,000

b) Fill out the bond amortization table (Euros).

Semester	Debt at beginning of the period	Interest	Nº de bonds repaid	Amortization	Premium	Total Payment
0	—	—	—	—	—	—
1	200,000	6000	—	—	—	6000
2	200,000	6000	4000	40,000	2,000	48,000
3	160,000	4800	4000	40,000	2,000	46,800
4	120,000	3600	4000	40,000	4,000	47,600
5	80,000	2400	4000	40,000	4,000	46,400
6	40,000	1200	4000	40,000	4,000	45,200

2. (20 marks)

Mr. Brad financed a car aquisition with a leasing contract that has the following conditions:

- Term of leasing contract: 5 years;
- Quarterly effective rate of 3.0%, the first payment to be paid 3 months after the date of the contract;
- Down payment of 7% of the car value;
- Residual value with last payment and equal to 10% of the car value, assume €2,000 for the residual value;
- Constant payments every quarter (at the end of each period).

Compute the payments that are associated with the leasing contract.

Residual value, $RV = 0.1\,P = 2000 \Leftrightarrow P = 20{,}000$

Term, $n = 20$, $i_T = 3\%$, Down pay. $= 0.07(20000) = 1{,}400$

$\Rightarrow 20{,}000 = 1{,}400 + R\, a_{\overline{20}|3\%} + 2{,}000\,(1.03)^{-20}$

$R \simeq 1{,}175.78$

R: $\sim 1{,}175{,}78$