Financial Markets and Instruments

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Suggested Solutions

. . . .

GROUP I (45 points)

1. Define the various forms of market efficiency. Assuming inside trading is forbidden because it is possible to obtain abnormal returns based upon private information, explain what form(s) of market efficiency if any is(are) consistent with the existence of that restriction. . . . . [15p]
   Answer:
   There are three forms of market efficiency: weak, semi-strong and strong. In any case market efficiency has to do with the hypothesis that prices reflect all available information in an appropriate way. When we consider as "available information" only historical information concerning financial time series and it is impossible to obtain abnormal returns based upon that information alone, we say the market is weak efficient.
   If to the historical information we add all possible public information and still it is not possible to obtain abnormal returns we say a market is semi-strong efficient.
   Finally if we consider all historical, public and private information and still it is not possible to obtain abnormal returns, then we say a market is strong efficient.
   The existence of laws against inside trading implies governments believe inside traders can exploit the common investor using private information they may hold. Thus, the existence of such laws is not consistent with the hypothesis of strong efficiency. However, as it only concerns the use of private information, it is consistent with both weak and semi-strong efficiency.

2. Establish a connection between the risk measure *value-at-risk* and Kataoka's safety criterion. For the special case of normally distributed returns, derive their mean-variance representation and sketch it in $(\sigma, \bar{R})$ space. . . . . [15p]
   Answer:
   The safest portfolio according to Kataoka safety criterion is the one with the highest R_L verifying the condition $\Pr(R_p < R_L) \leq \alpha$. Note R_L for each investment is nothing but the α - quantile of the return distribution.
   On the other hand the Value-at-risk with parameter α, $VaR_{(1-\alpha)}$, can be defined in terms of the final wealth W and initial investment W_0, and can be interpreted as the amount required to cover $(1-\alpha)\%$ of the losses, i.e. $\Pr\left(W_0 - W \geq VaR_{(1-\alpha)}\right) \leq \alpha$. Using $W = W_0(1+R_p)$

$$\begin{aligned}
\Pr\left(W_0 - W \geq VaR_{(1-\alpha)}\right) &= \Pr\left(W \leq W_0 - VaR_{(1-\alpha)}\right) \\
&= \Pr\left(W_0(1+R_p) \leq W_0 - VaR_{(1-\alpha)}\right) \\
&= \Pr\left(1 + R_p \leq 1 - \frac{VaR_{(1-\alpha)}}{W_0}\right) \\
&= \Pr\left(R_p \leq -\frac{VaR_{(1-\alpha)}}{W_0}\right) \quad \Longrightarrow \quad R_L = -\frac{VaR_{(1-\alpha)}}{W_0}
\end{aligned}$$

which can be rewritten as $VaR_{(1-\alpha)} = -W_0 R_L$, and we can conclude that, for a given α, the portfolio with the highest R_L is also the one with the lowest VaR.
For Gaussian returns we have continuous distributions, so the maximum R_L for each investment is guaranteed at equality

$$\Pr\left(R_p < R_L\right) = \alpha \quad \Leftrightarrow \quad \Pr\left(\frac{R_p - \bar{R}_p}{\sigma_p} < \frac{R_L - \bar{R}_P}{\sigma_p}\right) = \alpha \quad \Leftrightarrow \quad \Phi\left(\frac{R_L - \bar{R}_P}{\sigma_p}\right) = \alpha$$

and portfolios with the same R_L will be on the same straight line $\bar{R}_p = R_L - \Phi^{-1}(\alpha)\ \sigma_p$ in the $\left(\sigma_p, \bar{R}_p\right)$ space. The safest portfolio according to Kataoka, and also the one with the smallest VaR, is the one with the highest possible R_L for a fixed slope $-\Phi^{-1}(\alpha)$.

...Missing Sketch ...

3. Choose <u>ONE</u> of the following statements and discuss whether they are true or false. ..... [15p]

   I. *The reason why there is equilibrium in financial markets is because the majority of investors are risk averse.*
   Comment: TRUE
   Investors can be classified into risk lovers, risk neutral and risk averse investors.
   The first type of investors – the risk lovers – appreciate risk so much they are be willing to accept a decrease in expected return provided risk increases. These investors have decreasing indifference curves in $\left(\sigma, \bar{R}\right)$ space.Risk neutral investors take decisions based upon expected return only, as they are immune with respect to risk levels. Thus, their indifference curves are horizontal lines in $\left(\sigma, \bar{R}\right)$ space.Finally risk averse investors, which constitutes the market's majority, require higher expected return to incur in additional risk, so the have increasing indifference curves in $\left(\sigma, \bar{R}\right)$ space.
   Both risk neutral investors and risk lovers will choose to maximize expected return. In particular, this means that whenever it is possible to take a loan to invest in financial markets, they take on the largest possible loan, as this would imply higher risk and, thus, higher expected return. If all investors would be risk neutral or risk lovers there would be no equilibrium in financial markets as no one would deposit money and, consequently no loans would exist.
   The reason why there is equilibrium in financial markets is because most investors are risk averse and some will choose to deposit (at least part of) their money, while others will decide to take on (moderate) loans.

   II. *To an investor who does not verify the Von-Neuman-Morgensten axioms, one should recommend safe portfolios according to criteria such as Roy, Kataoka or Telser.*
   Comment: FALSE
   If an investor does not satisfy the Von-Neuman-Morgenstern axioms, it means one cannot apply the principle of maximizing expected utility when choosing her optimal portfolio. However, this does not mean alternative portfolio proposals should necessarily be based upon safety criteria.
   Portfolios based upon the safety criteria such as Roy, Kataoka or Telser should only be proposed to investors particularly worried about bad outcomes and for whom volatility is not a good measure of risk. The choice of the safety criteria to apply depends on the particular way investors express their worries, for instance, in terms of minimizing the probability of portfolio returns below a given level, choosing the portfolio with a higher quantile return or maximizing expected returns given a probability condition of returns below a given level is satisfied.

GROUP II (30 points)

Any second order Taylor approximation of a generic utility function $U(W)$ around the initial wealth W_0, is quadratic in W, i.e. we always get $U(W) \approx aW^2 + bW + c$.

(i) Derive the parameters a, b and c in terms of the initial wealth's utility and its derivatives. . . [10p]

(ii) Show the quadratic approximation of U is *equivalent* to $V(W) = (1 + RRA_0)\, W - \frac{1}{2} ARA_0 W^2$, where RRA_0 and ARA_0 stand for the U relative and absolute risk aversions evaluated at W_0. . . [10p]

(iii) Consider investments A, B and C represented by the points $A = (0\%, 5\%)$, $B = (10\%, 10\%)$ and $C = (15\%, 25\%)$ in the space $(\sigma, \bar{R})$. For and investor investor with $U(W) = -e^{-0.002W}$ and initial wealth $W_0 = 1\ 000$, use the generic risk tolerance function, $f(\sigma_p, \bar{R}_p) = \bar{R}_p - \frac{1}{2} RRA_0(\bar{R}_p^2 + \sigma_p^2)$, to rank A, B and C according to his preferences. Analyze his risk profile and find the smallest certainty return to his preferred investment. . . . . [10p]

Solution:

(i) The second Taylor expansion of $U(W)$ around the initial wealth W_0 is

$$U(W) \approx U(W_0) + U'(W_0)(W - W_0) + \frac{1}{2}U''(W_0)(W - W_0)^2$$
$$\approx \underbrace{U(W_0) - U'(W_0)W_0 + \frac{1}{2}W_0^2 U''(W_0)}_{c} + \underbrace{\big(U'(W_0) - U''(W_0)W_0\big)}_{b}\, W + \underbrace{\frac{1}{2}U''(W_0)}_{a}\, W^2$$

(ii) Two utility functions are *equivalent* if they represent the same preferences. Any linear transformation of an utility function leads to an equivalent utility function $U(W) = \alpha + \beta V(W)$, provided $\beta > 0$. In our case

$$\begin{aligned} U(W) &\approx c + \big(U'(W_0) - U''(W_0)W_0\big)\, W + \frac{1}{2}U''(W_0)W^2 \\ &\approx c + \frac{U'(W_0) - U''(W_0)W_0}{U'(W_0)} U'(W_0)W + \frac{1}{2}\frac{U''(W_0)}{U'(W_0)} U'(W_0)W^2 \\ &\approx c + (1 + RRA_0)\, U'(W_0)W - \frac{1}{2}ARA_0 U'(W_0)W^2 \\ &\approx c + U'(W_0)\left[(1 + RRA_0)\, W - \frac{1}{2}ARA_0 W^2\right] \\ &\approx c + U'(W_0)V(W) \quad \prec\!\approx\!\succ \quad V(W)\ , \end{aligned}$$

where $\alpha = c$, $\beta = U'(W_0) > 0$, and we use the definitions $ARA_0 = -U''(W_0)/U'(W_0)$ and $RRA_0 = W_0 ARA_0$.

(iii) Our investor has

$$U(W) = -e^{-0.002W}\ , \qquad U'(W) = +0.002e^{-W} > 0\ , \qquad U''(W) = -(0.002)^2 e^{-W} < 0$$
$$RRA(W) = -\frac{U''(W)}{U'(W)}W = -\frac{-(0.002)^2 e^{-W}}{0.002e^{-W}}W = 0.001W\ , \quad RRA'(W) = 0.002 > 0\ .$$

So, the investor preferes more to less ($U' > 0$), is risk averse ($U'' < 0$) and has increasing relative risk aversion ($RRA' > 0$). His relative risk aversion at W_0 is $RRA_0 = 0.002W_0 = 0.002 \times 1000 = 2$. Investments A has no risk but B and C are risky, the investor ranks them as follows

$$f(A) = f(0\%, 5\%) = 5\% - (5\%)^2 = 0.0475 \quad f(B) = f(10\%, 10\%) = 10\% - ((10\%)^2 + (10\%)^2) = 0.08$$
$$f(C) = f(15\%, 25\%) = 25\% - ((25\%)^2 + (15\%)^2) = 0.065 \qquad \Longrightarrow \qquad B \succ C \succ A\ .$$

To find the certain equivalent return to investment B we need to solve

$$f(certain) = f(B) = 0.08 \quad \Leftrightarrow \quad R_{certain} - R_{certain}^2 = 0.08 \quad \Leftrightarrow \quad R_{certain} = 8.769\%\ .$$

GROUP III

Problem 1 (75 points)

There is a financial market where all return correlations can be explained by one common-return factor R_m with volatility 20%. The table below shows information about three risky assets existent in that market.

Asset	$\bar{R}_i$	$\sigma_i^{\text{systematic}}$	$\sigma_i^{\text{specific}}$
A	6%	10%	6.64%
B	12%	30%	18.03%
Z	4 %	0 %	4%

1. Identify the type return generating model appropriate to use in this setup? Why? .... [5p]
   Answer:
   The most appropriate return generating model for the described situation is a single-factor model. The only assumption underlying that type of models is that all returns correlations can be modelled by one common-factor. Since this is exactly the case here, there would be no model risk.

2. Determine the β_A, β_B and β_Z implicit in the above information and show that the mean-variance inputs are

$$\bar{R} = \begin{pmatrix} 6\% \\ 12\% \\ 4\% \end{pmatrix} \qquad \text{and} \qquad V = \begin{pmatrix} 0.0144 & 0.03 & 0 \\ 0.03 & 0.1225 & 0 \\ 0 & 0 & 0.0016 \end{pmatrix}$$

   .... [10p]
   Solution:
   The vector of expected returns $\bar{R}$ comes directly from the information table. To determine the variance-covariance matrix V, recall that under the assumption of a single index model each individual asset variance can be written as

$$\sigma_i^2 = \underbrace{\beta_i^2 \sigma_m^2}_{\text{systematic variance}} + \underbrace{\sigma_{ei}^2}_{\text{specific variance}} ,$$

   and, for each of our assets we have

$$\sigma_A^{\text{systematic}} = \beta_A \sigma_m \quad \Leftrightarrow \quad 10\% = \beta_A 20\% \quad \Leftrightarrow \quad \beta_A = 0.5 \ ,$$
$$\sigma_B^{\text{systematic}} = \beta_B \sigma_m \quad \Leftrightarrow \quad 30\% = \beta_B 20\% \quad \Leftrightarrow \quad \beta_B = 1.5 \ ,$$
$$\sigma_Z^{\text{systematic}} = \beta_Z \sigma_m \quad \Leftrightarrow \quad 0\% = \beta_Z 20\% \quad \Leftrightarrow \quad \beta_Z = 0 \ .$$

   The total variances $\sigma_i^2 = \left(\sigma_i^{\text{systematic}}\right)^2 + \left(\sigma_i^{\text{specific}}\right)^2$, and covariances $\sigma_{ij} = \beta_i \beta_j \sigma_m^2$ are:

$$\sigma_A^2 = \left(\sigma_A^{\text{systematic}}\right)^2 + \left(\sigma_A^{\text{specific}}\right)^2 \quad \Leftrightarrow \quad \sigma_A^2 = (10\%)^2 + (2\%)^2 = 0.0144 \quad \Rightarrow \quad \sigma_A = 12\% \ ,$$
$$\sigma_B^2 = \left(\sigma_B^{\text{systematic}}\right)^2 + \left(\sigma_B^{\text{specific}}\right)^2 \quad \Leftrightarrow \quad \sigma_B^2 = (30\%)^2 + (5\%)^2 = 0.1225 \quad \Rightarrow \quad \sigma_B = 35\% \ ,$$
$$\sigma_Z^2 = \left(\sigma_Z^{\text{systematic}}\right)^2 + \left(\sigma_Z^{\text{specific}}\right)^2 \quad \Leftrightarrow \quad \sigma_Z^2 = (0\%)^2 + (4\%)^2 = 0.0016 \quad \Rightarrow \quad \sigma_Z = 4\% \ ,$$
$$\sigma_{AB} = \beta_A \beta_B \sigma_m^2 = 0.5 \times 1.5 \times (20\%)^2 = 0.03 \qquad \sigma_{AZ} = \beta_A \beta_Z \sigma_m^2 = 0 \qquad \sigma_{BZ} = \beta_B \beta_Z \sigma_m^2 = 0 \ .$$

3. Consider all possible combinations of A, B and Z, assume that there are no restrictions to shortsell and that all returns are normally distributed.

(a) Sketch the investment opportunity set, identifying A, B, Z and highlight the efficient frontier. Explain. .... [10p]

Solution:

Since we only have risky assets, with $n \geq 3$, and there is no restrictions to shortselling, we know the investment opportunity set is the open area bounded by the envelop hyperbola and all the three assets A, B, and Z are in the interior of the investment opportunity set, so they are not efficient. The efficient frontier is the upper part of the envelop hyperbola from its minimum variance point upwards.

... Missing Sketch ...

(b) Find the safest portfolio according to Roy's criterion, for $R_L = 0\%$. Motivate you computations and argue this is an efficient portfolio. .... [12.5p]

Solution:

The safest combination according to Roy and with $R_L = 0\%$ is the the one that solves $\min_p \Pr(R_p < 0\%)$. For Guassian returns we have

$$\Pr(R_p < 0\%) = \Pr\left(\frac{R_p - \bar{R}_p}{\sigma_p} < \frac{0\% - \bar{R}_p}{\sigma_p}\right) = \Phi\left(\frac{0\% - \bar{R}_p}{\sigma_p}\right)$$

and thus

$$\min_p \Pr(R_p < 0\%) \quad \Leftrightarrow \quad \min_p \frac{0\% - \bar{R}_p}{\sigma_p} \quad \Leftrightarrow \quad \max_p \frac{\bar{R}_p}{\sigma_p}$$

We know that, when maximising such ratios, solving the FOC is equivalent to solve a linear system of equations, or a linear matrix equation:

$$\left[\bar{R} - 0\%\right] = \begin{pmatrix} 6\% \\ 12\% \\ 4\% \end{pmatrix} \qquad \text{and} \qquad V = \begin{pmatrix} 0.0144 & 0.03 & 0 \\ 0.03 & 0.1225 & 0 \\ 0 & 0 & 0.0016 \end{pmatrix}$$

$$Z = V^{-1}\bar{R} = \begin{pmatrix} 141.78 & -34.72 & 0 \\ -34.72 & 16.67 & 0 \\ 0 & 0 & 625 \end{pmatrix} \begin{pmatrix} 6\% \\ 12\% \\ 4\% \end{pmatrix} = \begin{pmatrix} 4.340 \\ -0.083 \\ 25 \end{pmatrix} \Rightarrow \quad X^{Roy}_{R_L=0\%} = \begin{pmatrix} 14.84\% \\ -0.28\% \\ 85.45\% \end{pmatrix}.$$

This portfolio can be interpreted as a *tangent portfolio* – the point where a straight line starting at the point $(0\%, 0\%)$ touches the investment opportunity set – in a world where there would be a riskless asset paying zero return. Since all tangent portfolios are efficient, even when we take away the riskless assets, we can conclude the above portfolio must be efficient.

(c) Find another efficient portfolio. Explain. .... [7.5p]

Solution:

To find another efficient portfolio, we just need to consider another R_L and determine another Roy portfolio. If we take $R_L = 4\%$ we get

$$Z = V^{-1}\left[\bar{R} - 4\%\right] = \begin{pmatrix} 141.78 & -34.72 & 0 \\ -34.72 & 16.67 & 0 \\ 0 & 0 & 625 \end{pmatrix} \begin{pmatrix} 2\% \\ 8\% \\ 0\% \end{pmatrix} = \begin{pmatrix} 0.058 \\ 0.639 \\ 0 \end{pmatrix} \Rightarrow X^{Roy}_{R_L=4\%} = \begin{pmatrix} 8.31\% \\ 91.69\% \\ 0.00\% \end{pmatrix}.$$

OBS: you could consider *any* efficient portfolio including the minimum variance (MV) one.

(d) Consider an investor that wishes to invest 50 000 euros and with an optimal expected return $\bar{R}^* = 10\%$. How much should he invest in A, B and Z? .... [10p]

Solution:

All points the the envelop hyperbola can be seen as combinations of any two efficient portfolios. From (b) and (c) we already have two efficient portfolios: $E1$ and $E2$.

$$X_{E1} = \begin{pmatrix} 14.84\% \\ -0.28\% \\ 85.45\% \end{pmatrix} \qquad X_{E2} = \begin{pmatrix} 8.31\% \\ 91.69\% \\ 0.00\% \end{pmatrix} .$$

Their expected returns are $\bar{R}_{E1} = 4.27\%$ and $\bar{R}_{E2} = 11.5\%$. A combination that invests x in $E1$ and $(1-x)$ in $E2$ has expected return $\bar{R}^* = x\bar{R}_{E1} + (1-x)\bar{R}_{E2}$.

The combination with $\bar{R}^* = 10\%$ is, thus,

$$10\% = x4.27\% + (1-x)11.5\% \quad \Leftrightarrow \quad x = \frac{10\% - 11.5\%}{4.27\% - 11.5\%} \quad \Leftrightarrow \quad x = 20,78\% .$$

The investor should allocate 20.78% of his money in $E1$ and the remaining 79.22% in $E2$. In terms of the basic assets A, B and X we have

$$X^* = 0.20778 \times \begin{pmatrix} 14.84\% \\ -0.28\% \\ 85.45\% \end{pmatrix} + 0.7922 \times \begin{pmatrix} 8.31\% \\ 91.69\% \\ 0.00\% \end{pmatrix} = \begin{pmatrix} 9,66\% \\ 72,58\% \\ 17,75\% \end{pmatrix} \Longrightarrow W_0 = \begin{pmatrix} 4\ 831.11 \\ 36\ 292.22 \\ 8\ 876.67 \end{pmatrix} .$$

4. An analyst believes that some form of CAPM holds for this market, that assets A and B are in *equilibrium*, but there is no risk-free asset.

(a) Derive and interpret the parameters of the equilibrium relationship in this setup. ... [10p]

Solution:

The equilibrium is this case is describe by the line $\bar{R}_i^e = \bar{R}_Z^e + \beta_i \left(\bar{R}_m - \bar{R}_Z^e\right)$, where $\bar{R}_Z^e$ is the equilibrium expected returns of zero beta assets, that is, assets with no systematic risk. From the data in the problem we have:

$$\begin{cases} \bar{R}_A^e = \bar{R}_Z^e + \beta_A \left(\bar{R}_m - \bar{R}_Z^e\right) \\ \bar{R}_B^e = \bar{R}_Z^e + \beta_B \left(\bar{R}_m - \bar{R}_Z^e\right) \end{cases} \Leftrightarrow \begin{cases} 0.06 = \bar{R}_Z^e + 0.5 \left(\bar{R}_m - \bar{R}_Z^e\right) \\ 0.12 = \bar{R}_Z^e + 1.5 \left(\bar{R}_m - \bar{R}_Z^e\right) \end{cases} \Leftrightarrow \begin{cases} \bar{R}_m = 0.09 \\ \bar{R}_Z^e = 0.03 \end{cases}$$

The equilibrium equation is $\bar{R}_i = 0.03 + 0.06\beta_i$.

(b) Check if asset Z is also in equilibrium or not.Verify if there are arbitrage opportunities and if they exist, design a strategy to exploit it. .... [10p]

Solution:

Asset Z is a zero beta asset with a return higher than $\bar{R}_Z^e = 0.03$, so it is not in equilibrium. It is underpriced and there is an arbitrage opportunity. To exploit an arbitrage strategy we need to find a portfolio of A and B also with zero systematic risk ($\beta_p = 0$), but with a different return. Since the β of a portfolio is the weighted average of each security β and the weights of asset A and asset B must sum 1, it comes

$$\begin{cases} x_A + x_B = 1 \\ \beta_p = x_A\beta_A + x_B\beta_B \end{cases} \Leftrightarrow \begin{cases} x_B = 1 - x_A \\ 0 = 0.5x_A + 1.5(1 - x_A) \end{cases} \Leftrightarrow \begin{cases} x_A = 1.5 \\ x_B = -0.5 \end{cases}$$

The return of this replication portfolio is $R_p = 1.5 \times 0.06 - 0.5 \times 0.12 = 0.03$, the equilibrium expected return for any zero beta asset. Therefore, we should buy asset Z and sell the replica portfolio, earning a 1% profit.

Problem 2 (45 points)

Consider two assets whose returns have *perfect positive correlation*, and can be represented, in the space $(\sigma, \bar{R})$, as two points $A = (15\%, 15\%)$ and $B = (30\%, 25\%)$. Assume shortselling is allowed.

1. Represent all possible combinations of A and B in the space $(\sigma, \bar{R})$. Explain. .... [5p]
   Solution:
   Because the two risky assets have *perfect positive correlation* of returns, all combinations are on two straight lines. This results from the fact that the expected return of a portfolio is always linear in the weights and in this case so is the volatility. To see this note

$$\sigma_p^2 = x^2\sigma_A^2 + (1-x)^2\sigma^2 B + 2x(1-x)\underbrace{\rho_{AB}}_{1}\sigma_A\sigma_B = [x\sigma_A + (1-x)\sigma_B]^2$$

$$\sigma_p = |x\sigma_A + (1-x)\sigma_B| \ ,$$

   where x is the proportion of the investment made in asset A.

   ...Sketch missing ...

2. Determine the minimum risk portfolio and interpret its expected return. .... [10p]
   Solution:
   Because $\rho_{AB} = 1$ and shortselling is allowed, we know it is possible to fully eliminate risk, i.e. there is a combination of A and B with $\sigma_p = 0$. So by setting $\sigma_p^2 = 0$ we get

$$x\sigma_A + (1-x)\sigma_B = 0 \Leftrightarrow x = -\frac{\sigma_B}{\sigma_A - \sigma_B} = -\frac{30\%}{15\% - 30\%} = 2$$

   and we conclude the riskless combination requires and investment of $x = 200\%$ in A and short-selling of B, $(1-x) = -100\%$. Its expected return is

$$R_f = 2\bar{R}_A - \bar{R}_B = 2 \times 15\% - 25\% = 5\%.$$

3. Write down the equation for the efficient frontier. .... [5p]
   Solution:
   The efficient frontier (EF) is the straight line passing trough the point $(0\%, 5\%)$, A and B. Since A and B have the same Sharpe ratio we compute that of B to get the slope of the EF

$$EF: \quad \bar{R}_p = R_f + \frac{\bar{R}_B - R_f}{\sigma_B}\sigma_p \quad \Leftrightarrow \quad \bar{R}_p = 0.05 + \frac{25\% - 5\%}{30\%}\sigma_p \quad \Leftrightarrow \quad \bar{R}_p = 0.05 + \frac{2}{3}\sigma_p$$

4. The indifference curves of Mr. Gamble's risk tolerance function are given by

$$\bar{R}_p = -\sigma_p^2 - 2\sigma_p + K \ ,$$

   for some constant K.

   (a) What can you conclude about Mr. Gamble risk profile? Explain and sketch his indifference curves in space $(\sigma, \bar{R})$ .... [7.5p]

Solution:
From the indifference curves of Mr.Gamble we get

$$\left(\frac{\partial \bar{R}_p}{\partial \sigma_p}\right)_{IC} = -2\sigma_p - 2 < 0 \ .$$

We can thus conclude that this investor is a risk lover, since he has indifference curves that are decreasing in $(\sigma, \bar{R})$ space. So, he is willing do accept lower expected return investments if they bear more risk.

(b) If he would see investments A and B as *alternatives* which would he prefer? By how much would $\bar{R}_B$ need to change to make him indifferent between A and B? .... [7.5p]
Solution:
He will prefer B to A, since B has both a higher expected return and volatility.
For Mr. Gamble to be indifferent B would need to lie on the same indifference curve as A
For A

$$K_A = \bar{R}_A + \sigma_A^2 + 2\sigma_A = 0.15 + 0.15^2 + 2 \times 0.15 = 0.4725$$

If we impose the expected utility level K_A on B and use the IC we get

$$\bar{R}_B = -\sigma_B^2 - 2\sigma_B + K_A \Leftrightarrow \bar{R}_B = -0.3^2 - 2 \times 0.3 + 0.4725 = -3.75\% \ ,$$

and conclude the expected return on B would need to be negative and of -3.75% for Mr. Gamble to be indifferent between both assets.

(c) Consider now combinations of A and B. What investment would you recommend if there were no restrictions? Explain. .... [5p]
Solution:
Since he is a risk lover we can focus on efficient portfolios, because for any fixed risk level, those are the ones that maximize expected return and a risk lover likes both risk and expected return. His optimum can be understood as, first maximize risk and then for the maximal risk maximize expected return. Or, maximize risk along the efficient frontier (EF). Since our EF is a straight line, without restriction he can always increase his risk level by increasing how much he shortsells of A to invest in B. The optimum is to have $x_A = -\infty$ and $x_B = 1 + \infty$.

(d) What if he cannot shortsell? Explain. .... [5p]
Solution:
If he cannot shortsell then the efficient frontier is the segment of line between A and B.

All feasible investments – without shorlltselling – have a risk σ_p lower than the risk of B, since the line stops at B, i.e. $\sigma_p \leq 30\%$.

Since our investor is risk lover, the best for him is to invest everything in B and get the maximum possible risk $\sigma_B = 30\%$. So the optimum would be $x_A = 0\%$ and $x_B = 100\%$.