

# Stochastic Calculus – Masters in Mathematical Finance

## Program 2017/2018

### 1-Introduction

- 1.1. What is stochastic calculus?
- 1.2. A very brief history of stochastic calculus

### 2 – A review of basic concepts in probability and stochastic processes

- 2.1. Stochastic processes
- 2.2. Conditional expectation
- 2.3. Martingales in discrete time
- 2.4. Martingale transform (or discrete stochastic integral) in discrete time
- 2.5. Martingales in continuous time

### 3 – Brownian motion

- 3.1. Brownian motion: Definition, continuous and non-differentiable sample paths
- 3.2. Main properties of Brownian motion.
- 3.3. Brownian motion with drift, Geometric Brownian motion and the “Brownian bridge”.
- 3.4. Quadratic variation of Brownian motion

### 4 – The stochastic integral

- 4.1. Motivation
- 4.2. Stochastic integral for simple processes
- 4.3. The stochastic integral for adapted processes.
- 4.5. Main properties of the stochastic integral as a process: martingale property, continuity of sample paths and the quadratic variation.
- 4.6. Extensions of the stochastic integral

### 5 – The Itô formula

- 5.1. One dimensional Itô formula
- 5.2. Multidimensional Itô formula
- 5.3. Itô integral representation Theorem
- 5.4. Martingale representation Theorem

### 6 – Stochastic Differential Equations

- 6.1. Motivation and examples
- 6.2. Some examples: stochastic differential equation for the geometric Brownian motion and the Langevin equation for the Ornstein-Uhlenbeck process.
- 6.3. Existence and uniqueness of solutions Theorem for SDE's
- 6.4. Examples: Ornstein-Uhlenbeck process with mean reversion and a financial application: the Vasicek model for interest rates
- 6.5. Linear stochastic differential equations
- 6.6. Strong and weak solutions for stochastic differential equations
- 6.7. Numerical approximations: the Euler and the Milstein schemes

- 6.8. Markov property of the solutions of SDE's
- 6.9. Stratonovich stochastic integral and Stratonovich SDE's

#### 7 – Stochastic differential equations and partial differential equations

- 7.1. Infinitesimal generator for a diffusion
- 7.2. Stochastic representation for solutions of parabolic PDE's: Feynman-Kac formulas
- 7.3. The heat equation and the Brownian motion
- 7.4. The Kolmogorov backward equation

#### 8 – Girsanov Theorem

- 8.1. Changes of probability measures
- 8.2. Girsanov Theorem: elementary version
- 8.3. Girsanov Theorem: a more general version

#### 9 – Application to financial markets and derivatives pricing

- 9.1. The Black-Scholes model
- 9.2. No arbitrage and the Black-Scholes equation
- 9.3. The equivalent martingale measure and risk neutral valuation
- 9.4. The Black-Scholes formula

### **Recommended Bibliography**

- \_J. Guerra, Cálculo Estocástico, Texto de Apoio – notas das aulas teóricas, 2012.
- \_B. Oksendal, Stochastic Differential Equations, 6th. Edition, Springer, 2003
- \_T. Mikosch, Elementary Stochastic Calculus with Finance in view, World Scientific, 1998.
- \_D. Nualart, Stochastic Calculus (Lecture notes, Kansas University):  
<http://www.math.ku.edu/~nualart/StochasticCalculus.pdf>

### **Optional Bibliography**

- \_Tomas Björk, Arbitrage Theory in Continuous Time, Oxford University Press, 3<sup>rd</sup> Editions, 2009.
- \_P. E. Kloeden and E. Platen, Numerical Solution of Stochastic Differential Equations, Springer, 1992.
- \_I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calculus, 2nd edition, Springer, 1991.
- \_F. Klebaner, Introduction to Stochastic Calculus with Applications, 3rd edition, Imperial College Press, 2012.
- \_Steven Shreve, Stochastic Calculus for Finance II: Continuous-Time Models, Springer, 2004.

## **Assessment**

Final Exam