ISEG - Lisbon School of Economics and Management
2017/2018

**Statistics I**
**Problem set 1 - Probability**
(version: 13/02/2018)

1. A box contains 5 balls of which 2 are black. The black balls are numbered 1 and 2, the others 3 to 5. Two balls are randomly taken out one after the other without replacement. The numbers on the two balls are observed.

   (a) Enumerate all the elements of the sample space associated to this random experiment.

   (b) Define in the sample space the following events:
   - $A_1$−The first ball observed is black.
   - $A_2$−The second ball observed is black.
   - $A_3$−The two balls taken out are black.
   - $A_4$−At least one of the two balls are black.
   - $A_5$−Exactly one of the two balls are black.
   - $A_6$−The sum of the numbers in the two balls is greater than seven.

   (c) Make a chart of the sample space and the events defined above.

2. Two light bulbs will be kept on until both fail. The life time of both bulbs are registered. None of the light bulbs have a life time greater than 1600 hours. Make a chart of the sample space and the following events:

   (a) $A$ – None of the light bulbs has a life time longer than 1000 hours.

   (b) $B$ – Just one of the bulbs has a life time greater than 1000 hours.

   (c) $C$ – The life time of one of the bulbs doubles the life time of the other.

   (d) $D$ – The sum of the life times of the two bulbs is greater than 2000 hours.

3. Define and classify the sample spaces associated with the following random experiments:

   (a) Observation of the number of spots when a six face die is thrown.

   (b) A coin is tossed and the face-up is observed.

   (c) A die is cast followed by the toss of a coin.

   (d) A coin is tossed until a head comes.

4. In a College, 70% of the students have a desktop computer at home, 40% have a laptop computer and 30% have both. If a student is randomly chosen, evaluate the probability of the student:

   (a) Has at least one of the two types of computers.

(b) Has no computer.

(c) Has one of the two types of computers.

5. The weather forecast says that it will rain next Saturday with probability 0.25 and that it will rain next Sunday with probability 0.25. Can one say that according to the weather forecast the probability of rain next weekend is 0.5?

6. An electronic system is composed of two sub-systems, A and B. From previous rehearsals it is known that : the probability of A failure is 0.2, the probability that only B fails is 0.15 and the probability that A and B fail simultaneously is 0.15. Evaluate the probability that:

   (a) B fails.

   (b) Only A fails.

   (c) Fail at least one of them, A or B.

   (d) Neither A nor B fails.

   (e) A and B don't fail simultaneously.

7. The workers of a firm regularly use public transports (bus, metro, train) to go from home to work. It is known that:

   - 54% use exclusively one of these public transports (bus-22%, metro-25%, train-7%);
   - 44% use at least two of the three public transports.
   - Additionally, 18%use bus and metro; 17% use bus and train; 19% use metro and train.

   (a) Having in mind that there are other means of transport beyond the above mentioned, calculate the percentage of workers that don't use any of the above mentioned means of transport.

   (b) Calculate the percentage of workers that use all the above mentioned means of transport.

8. If a person visits his dentist, suppose that the probability that he will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24, the probability that he will have a tooth extracted is 0.21, the probability that he will have his teeth cleaned and a cavity filled is 0.08, the probability that he will have his teeth cleaned and a tooth extracted is 0.11, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist will have at least one of these things done to him?

9. A seventeenth century French gambler, the Chevalier du Mere, had run out of takers for his bet that, when a fair cubical dice was thrown four times, at least one 6 would be scored.
   (a) Explain why one should not take this bet.
   (b) He therefore changed the game. He bet that he could get a “double 6” in 24 throws of a pair of fair dice. Explain why one should take this bet.
10. What is the probability of at least one boy in a family of 4 children? What is the probability that all the children are of the same sex? (Assume boys and girls and equally likely).
11. Take the succession of natural numbers, $1, 2, ..., n$, and chose two randomly without replacement. Evaluate the probability of one of them be lower than $k$ and the other greater than $k$, where $k = 1, ..., n-1$.
12. Find $P\left[A^c \cup (A \cap B)\right]$, if $P\left[A\right] = 0.2$, $P\left[B\right] = 0.6$, $P\left[A \cup B\right] = 0.7$.
13. Consider a computer system that generates randomly a key-word for a new user composed of 5 letters (eventually repeated) of an alphabet of 26 letters (no distinction is made between capital and lower case letters). Calculate the probability that there is no repeated letters in the key-word.
14. Consider a box with 20 balls – 7 black, 5 white and the others yellow.
    (a) Two ball are extracted without replacement. What is the probability that at least one is black?
    (b) If 6 balls are extracted with replacement, what is the probability that there are two of each colour?
    (c) A random experiment consist of throwing a regular die and then take out of the box ,without replacement, a number of balls equal to the number of spots on the die. Calculate the probability that all extracted balls are black.
15. A factory uses three machines to produce the same product. Machines A, B and C produce respectively 40%, 35% and 25% of total production. The percentage of defective parts produced by each machine are respectively 4%, 2% and 1%. If a piece is randomly selected from the total production.
    (a) What is the probability that it is not defective?
    (b) Knowing that it is defective what is the probability that it has been produced by machine A?
    (c) If two pieces are successively removed with replacement from the total production, what is the probability that one of them be defective and the other not?

16. Let A and B be independent events of the same sample space. If $P(A) = 1/3$ and $P(B) = 3/4$,
    (a) Calculate $P(A \cup B)$ and $P(B|A \cup B)$.
    (b) Show that the complement of $A$ is independent of the complement of $B$.

17. Bean seeds from supplier A have an 85% germination rate and those from supplier B have a 75% germination rate. A seed-packaging company purchases 40% of its bean seeds from supplier A and 60% from supplier B and mixes these seeds together.
    (a) Find the probability $P(G)$ that a seed selected at random from the mixed seeds will germinate.
    (b) Given that a seed germinates, find the probability that the seed was purchased from supplier A.

18. (*The Monty Hall Game Show Problem*) Suppose you are on a game show, and you are given the choice of three doors: Behind one door there is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what is behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No.2?" Answer this question considering the following facts and assumptions:
    - The host always opens a door that was not picked by the contestant
    - The host must always open a door to reveal a goat and never the car.
    - If the player initially chooses the door behind which there is the car, assume that the host selects a door to open, from those available, with equal probability.

19. A woman and a man (who are unrelated) each has two children. We know that at least one of the woman's children is a boy and that the man's oldest child is a boy. Given this information: What is the probability that the woman has two boys? What is the probability that the man has two boys?

20. (*The birthday problem*) Suppose five people are in a room. What is the probability that there is at least one shared birthday among these five people? (Assume that all years have 365 days and that all 365 possible birthdays are equally likely). Repeat the same exercise assuming that there are 23 people in the room.