LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Macroeconomics II

Lecture 07

Extensions of the Solow model
Evidence for the Solow model

1

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Theoretical Lecture 07

Chap 07 The Solow-Swan model of economic growth (2/2)

The extension of Solow model with human capital

- **human capital as a production factor**
- **human capital in the Solow model**
- **Solow model: a synthesis (comparing several versions)**

For reading

Jones & Vollrath (2013), *Introduction to Economic Growth*, Norton, capítulo 3, pp. 54– 63

Further readings

N. Gregory Mankiw; David Romer; David N. Weil (1992), A Contribution to the Empirics of Economic Growth, *The Quarterly Journal of Economics*, Vol. 107, No. 2. (May, 1992), pp. 407-437.

2

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Extensions of the **Solow model** including the accumulation of **human capital**:

According to **Lucas** (1988), "By an individual's 'human capital' I will mean, for the purposes of this section, simply his general skill level, so that a worker with human capital h(t) is the productive equivalent of two workers with ½ h(t) each, or a half-time worker with 2 h(t)" (p.17)

skill level increases by::
•formal schooling
•specific professional training
•*learning-by-doing* (profissional experience)

3

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

"human capital" as a production function

$Y = F(K, AL) = K^{\alpha} (AL)^{1-\alpha}$

$A(t) = A_0 e^{gt}$

L active population

$Y = F(K, AH) = K^{\alpha} (AH)^{1-\alpha}$

H skilled active population (skilled labour)

$H = e^{\Psi u}L$ (generation of skill labour by schooling)

u = 0 => H = L (all labour is unskilled)

d ln H/du = (dH/du)/H = ψ

(Ψ = 0.10 means that one additional year of schooling rises skill level by 10%)

u, fraction of time spent learning skills;

ψ, increase (%) of skills when u rises by 1

4

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

variables of the Solow model

Inserting the factor $h = e^{\Psi\alpha}$

y , k (**basic** Solow model)

y/A , k/A (Solow model **with technology**)

y/Ah **(output-technology ratio)** **, k/Ah** **(capital-technology ratio)** (Solow model **with technology** and **human capital**)

Cobb-Douglas production function

$y = k^{\alpha}$ (**basic** Solow model)

$y = k^{\alpha}A^{1-\alpha}$ (Solow model **with technology**)

$y = k^{\alpha}(Ah)^{1-\alpha}$ (Solow model **with technology** and **human capital**)

5

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Solow model: a synthesis of the versions

Solow growth model

A. production function

B. accumulation of physical capital

- with no technological progress
- with technological progress (labour augmenting)
- with technological progress (labour augmenting) and human capital

6

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Production function

Solow basic model

$Y = K^{\alpha}L^{1-\alpha}$

$L = L_0e^{nt}$

$y = Y/L; k=K/L$

$Y/L = K^{\alpha}.L^{1-\alpha}/L = K^{\alpha}L^{-\alpha}$

$y = k^{\alpha}$

7

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Production function Solow model with technology

$Y = K^{\alpha} (AL)^{1-\alpha}$

$y = Y/L; k=K/L$

$L = L_0e^{nt}$

$Y/L = K^{\alpha}. A^{1-\alpha}.L^{1-\alpha}/L = K^{\alpha}. L^{-\alpha}. A^{1-\alpha}$

$A = A_0e^{gt}$

$y = k^{\alpha}A^{1-\alpha}$

$y^{\#} = Y/AL; k^{\#} = K/AL$

$Y/AL = K^{\alpha} (AL)^{-\alpha}$

$y^{\#} = (K/AL)^{\alpha}$ and **$y^{\#} = k^{\#\alpha}$**

8

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Production function

Solow model with technology and human capital

$Y = K^{\alpha}(AH)^{1-\alpha}$

$Y = K^{\alpha}.(AhL)^{1-\alpha}$

$Y/L = K^{\alpha} L^{-\alpha}(Ah)^{1-\alpha}$

$y = k^{\alpha}(Ah)^{1-\alpha}$

$y^{\#} = Y/AhL$; $k^{\#} = K/AhL$

$Y/AhL = K^{\alpha}(AhL)^{-\alpha}$

$y^{\#} = (K/AhL)^{\alpha}$

$y^{\#} = k^{\#\alpha}$

$L = L_0 e^{nt}$

$A = A_0 e^{gt}$

$H = e^{\Psi u} L = h . L$

($h = e^{\Psi u}$, constant)

9

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

capital accumulation (Solow basic model)

$dK/dt = s.Y - \delta K$

$(dK/dt)/K = s.Y/K - \delta = (s.Y/L)/(K/L) - \delta = s.y/k - \delta$

$k = K/L$

$(dk/dt)/k = (dK/dt)/K - (dL/dt)/L$

$(dk/dt)/k = s.y/k - \delta - n = s.y/k - (\delta + n)$

$(dk/dt)/k = s.k^{\alpha-1} - (\delta + n)$

$dk/dt = s.y - (\delta + n).k$

FIGURE 2.8 TRANSITION DYNAMICS

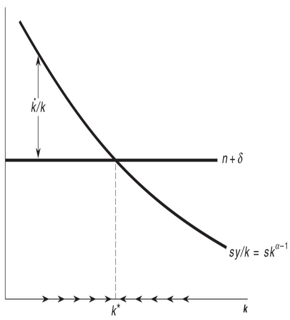
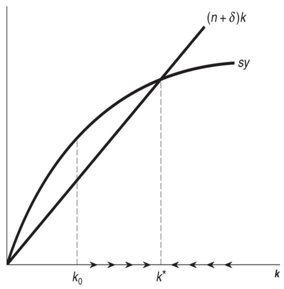


FIGURE 2.2 THE BASIC SOLOW DIAGRAM



LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

capital accumulation (Solow basic model with technology)

dK/dt = s.Y – δK

(dK/dt)/K = s.Y/K – δ = (s.Y/L)/(K/L) – δ = s.y/k – δ

$k^{\#}$ = K/AL

$(dk^{\#}/dt)/k^{\#}$ = (dK/dt)/K – (dA/dt)/A - (dL/dt)/L

$(dk^{\#}/dt)/k^{\#} = s.y^{\#}/k^{\#} – \delta – g – n = s.y/k – (\delta + g + n)$

$(dk^{\#}/dt)/k^{\#} = s.k^{\#\alpha-1} – (\delta + g + n)$

$dk^{\#}/dt = s.y^{\#} – (\delta + g + n).k^{\#}$

FIGURE 2.11 AN INCREASE IN THE INVESTMENT RATE: TRANSITION DYNAMICS

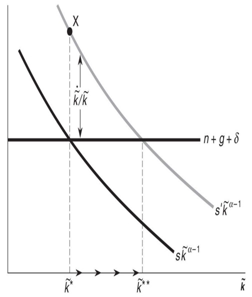
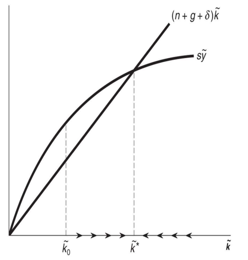


FIGURE 2.9 THE SOLOW DIAGRAM WITH TECHNOLOGICAL PROGRESS



11

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

steady-state in the most complete case (with technology and human capital)

h is a constant,

$y^{\#} = k^{\#\alpha}$	**production function**
$dk^{\#}/dt = s_k y^{\#} - (n + g + \delta)\, k^{\#}$	**physical capital accumulation**

in the ***steady-state***

$dk^{\#}/dt = 0 \;\Rightarrow\; s_k y^{\#} = (n + g + \delta)\, k^{\#} \;\Rightarrow\; s_k k^{\#\alpha} = (n + g + \delta)\, k$

$k^{\#1-\alpha} = (s_k/(n + g + \delta))$

$k^{\#*} = (s_k/(n + g + \delta))^{1/(1-\alpha)}$

$y^{\#*} = (s_k/(n + g + \delta))^{\alpha/(1-\alpha)}$

being y# = Y/hAL = y/hA, then:

$y^*(t) = h.A(t).\,(s_k/(n + g + \delta))^{\alpha/(1-\alpha)}$

12

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

$$y^*(t) = h.A(t). (s_k/(n + g + \delta))^{\alpha/(1-\alpha)}$$

Interpretation:

The **richest** countries are those that have higher rates of **investment in physical capital**, devote a higher fraction of **time on learning skills** and higher **levels of technology**.

In the *steady state*, GDP per capita grows at the same growth rate as that of the **technology**.

13

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Solow model: several versions

basic Solow model

$Y = F(K,L)$ $y^* = (s/(n + g))^{\alpha/(1-\alpha}$ sstate: $r(k) = 0; r(Y) = n$
$r(y) = 0$

with $k = K/L$, $r(L) = n$

Solow model with technology

$Y = F(K,A.L)$ $y^*(t) = A(t). (s_k/(n + g + \delta))^{\alpha/(1-\alpha}$ sstate: $r(k^\# = 0; r(k) = g; r(Y) = n+g$
$r(y) = g$

with $k^\# = K/A.L = k/A$, $r(A) = g$; $r(L) = n$

Solow model with technology and human capital

$Y = F(K,A.H)$ $\mathbf{y^*(t) = h.A(t). (s_k/(n + g + \delta))^{\alpha/(1-\alpha}}$ sstate: $r(k^\# = 0; r(k) = g; r(Y) = n+g$
$r(y) = g$

with $k^\# = K/A.H = K/A.h.L$, $r(A) = g$, $r(L) = n$; $r(h) = 0$ (h constant)

14

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

the two major questions in macroeconomics of growth

a) why are some countries richer?

from $y^*(t) = h.A(t). (s_k/(n + g + \delta))^{\alpha/(1-\alpha}$

countries are **richer** if:

- •have higher investment rates in physical capital (s_k)
- •spend a large fraction of time accumulating skills (h)
- •have low population growth rates (n)
- •have higher levels of technology (A)

why there are such differences? Solow model does not explain!
see Jones (2013), pp. 58-63 for empirical analysis.

b) why are some countries growing faster?

in the steady state, r(y) = g

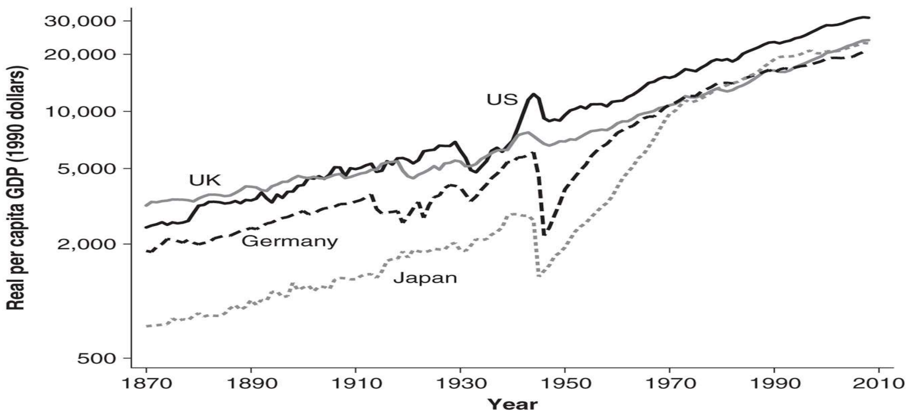
what explains g? g is "exogenous" in the Solow model!

15

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Convergence and differences in growth rates
the phenomenon of catching up

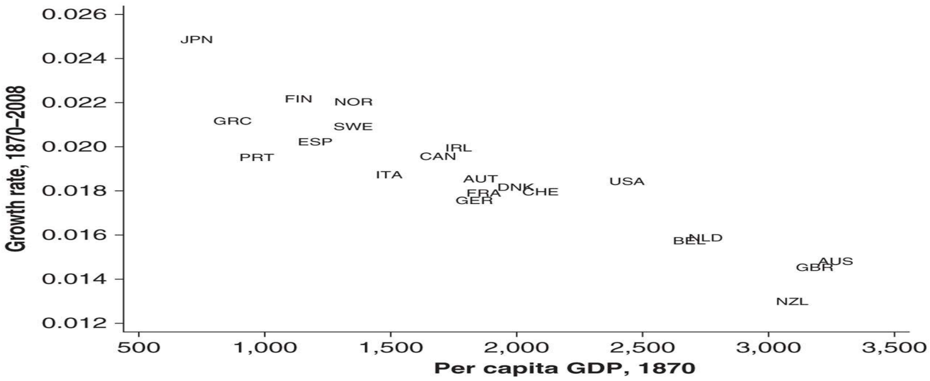
FIGURE 3.3 **PER CAPITA GDP, 1870–2008**



LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

convergence: those economies that were **richer** in the initial year of the period grew more slowly; those that were **poorer** grew faster.

FIGURE 3.4 **GROWTH RATE VERSUS INITIAL PER CAPITA GDP, 1870–2008**

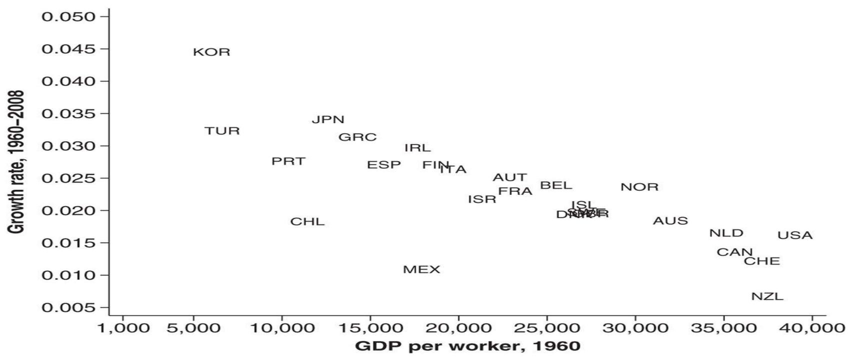


LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Does the empirical data support convergence?

looking at the OECD countries **Yes**!

FIGURE 3.5 **CONVERGENCE IN THE OECD, 1960–2008**



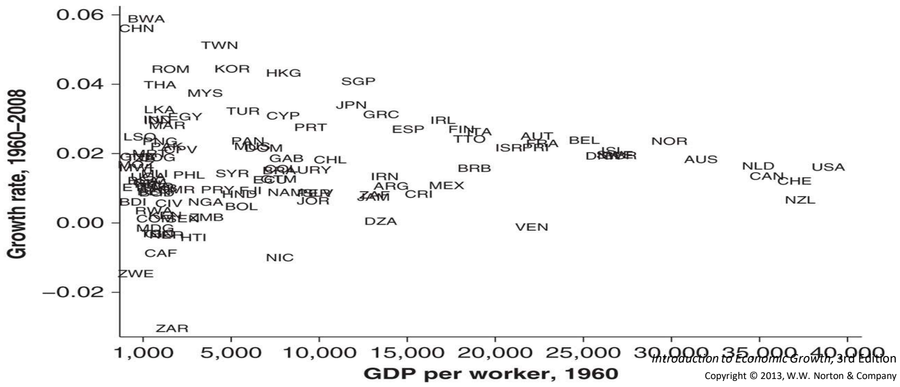
Introduction to Economic Growth, 3rd Edition

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Does the empirical data support convergence?

looking at the World: **No!**

FIGURE 3.6 THE LACK OF CONVERGENCE FOR THE WORLD, 1960–2008



LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

How does the Solow model explain convergence … and the lack of it?

To remind the key differential equation of the Solow model

<u>basic Solow model</u>
$(dk/dt)/k = s.y/k - \delta - n = s.y/k - (\delta + n)$

<u>Solow model with technology and human capital</u>
$(dk^{\#}/dt)/k^{\#} = s_k y^{\#}/k^{\#} - (n + g + \delta)$

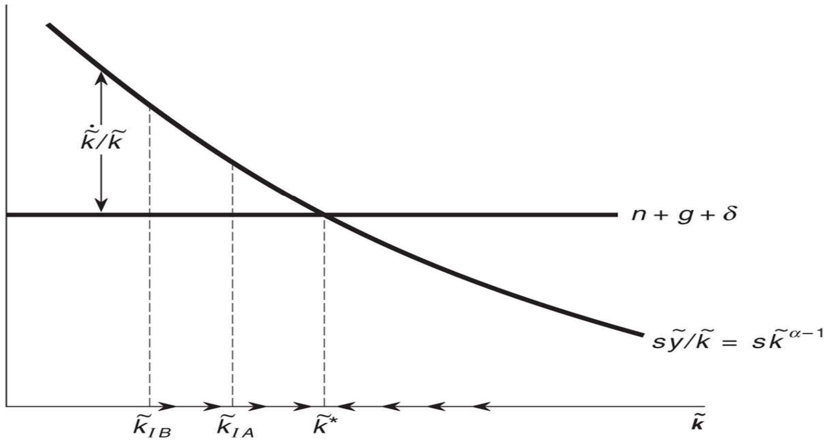
since $y^{\#} = k^{\#\alpha}$, it comes:

$$\mathbf{(dk^{\#}/dt)/k^{\#} = s_k .(k^{\#})^{\alpha-1} - (n + g + \delta)}$$

see next figure

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

FIGURE 3.7 TRANSITION DYNAMICS IN THE NEOCLASSICAL MODEL



Introduction to Economic Growth, 3rd Edition

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Interpretation

If two economies have the same levels of technology, the same rates of investment, the same rates of population growth and the same depreciation rate (that is, the same *steady state*), the convergence hypothesis should hold: poor countries should grow faster on average than rich countries.

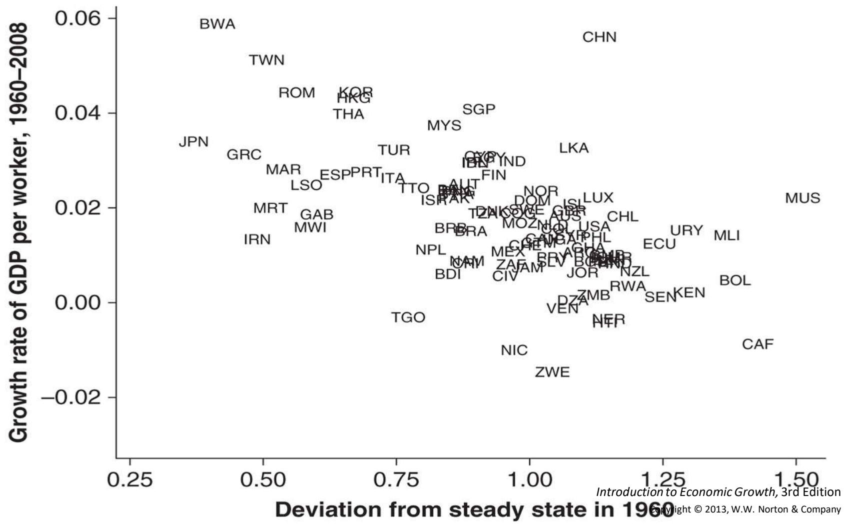
compare country A e country B.

But the countries do not have the same *steady states*.
Therefore, the countries are not expected to grow toward the same *steady state*.
This implies that countries that are "poor" relative to their "own" *steady states* do tend to grow more rapidly.

See next figure

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

FIGURE 3.8 "CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–2008



Introduction to Economic Growth, 3rd Edition

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Principle of transition dynamics:

The further an economy is "below" its steady state, the faster the economy should grow.

The further an economy is "above" its steady state, the slower the economy should grow.

24

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Robert Solow: a (self)criticism … or a challenge to the economic theory?

"I paid too little attention to the problems of effective demand".

"a theory of equilibrium growth (…) needs a theory of deviations from the equilibrium growth path"

"the problem of combining long-run and short-run macroeconomics has still not been solved" (written in 1987)

"growth theory was invented to provide a systematic way to talk about and to compare equilibrium paths for the economy. In that task it succeeded reasonably well. **In doing so, however, it failed to come to grips adequately with an equally important and interesting problem: the right way to deal with deviations from equilibrium growth**.

"So a simultaneous analysis of trend and fluctuations really does involve an integration of long-run and short-run, or **equilibrium and disequilibrium**"

25