

# Macroeconomics II

## Lecture 08

Endogenous growth models

AK model

## Lecture 8

# Chap 08 Endogenous models of growth. The AK model.

### •1 evolution of the neoclassical models after Solow

- endogenous models; learning by doing: Arrow (1962); technological change: Romer (1986, 1987, 1990), Lucas (1988); creative destruction: Aghion and Howitt (1992);
- from *proximate* to *fundamental* causes of growth and the role of institutions: North (1990), Acemoglu, Robinson and Johnson (2005).

### •2 endogenous models of economic growth: what are we talking about?

- the effect of “learning by doing” in the neoclassical models of economic growth;
- the production function in the AK model and no *steady state* in the AK model
- a comparison (?) with the Harrod-Domar (H-D) model: be careful!

## Reading

Jones, C., Vollrath, D. (2013), *Introduction to Economic Growth*, Norton, capítulo 9, pp. 215-227.

# endogenous models of economic growth: main concepts

**Solow** model: exogenous model of economic growth

the concept of steady state;

there are endogenous mechanisms that drive the economy towards the steady state:

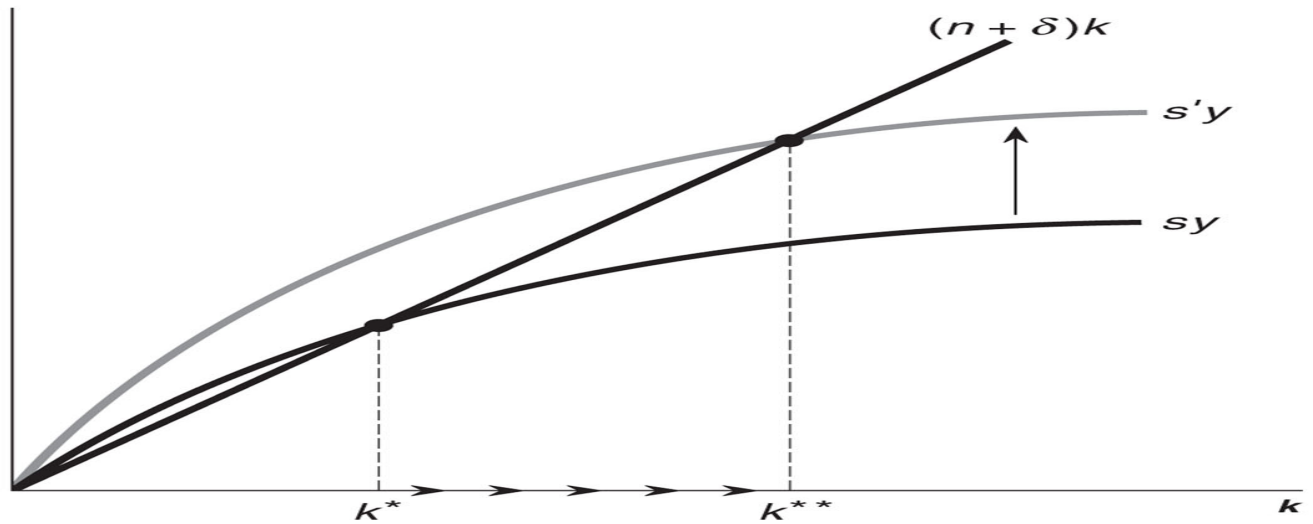
- substitutable production factors;
- capital accumulation is characterized by diminishing returns (declining marginal product of each unit of capital).

in steady state, the economy grows due to exogenous factors (technological progress at the rate “g”; population at the rate “n”), which are not “explained” by the model;

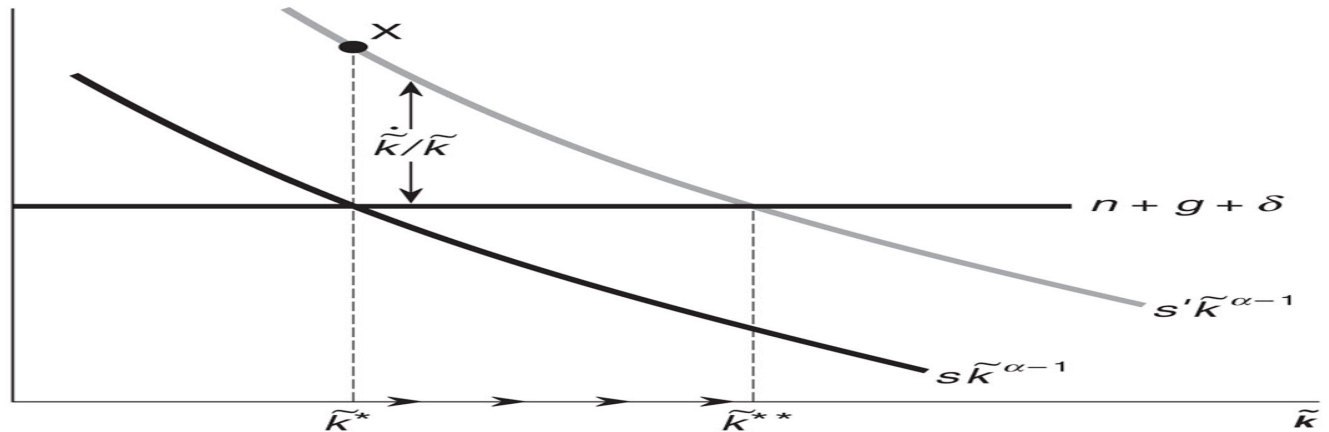
**in the Solow model, economic policy has only temporary effects on the rate of economic growth (the effect last during the transition towards the “new” steady state); it has no long-run effects on the rate of economic growth.**

Do remember (again!) the Solow model!

**FIGURE 2.4 AN INCREASE IN THE INVESTMENT RATE**



**FIGURE 2.11 AN INCREASE IN THE INVESTMENT RATE: TRANSITION DYNAMICS**



## growth theory since the 1950s

**neoclassical growth models** (Solow, etc), in the 1950s and 1960s:

savings (= investment) rate is exogenous (it is crucial to explain the steady state income level) (there is a social planner who decides?)

technological progress is exogenous (it is crucial to explain the long-run per capita growth rate) (does the technological progress come from heaven?)

they are called exogenous growth models

**unsatisfactory!** -> two theoretical reactions:

- growth models with endogenous savings (mid 1960s)  
Cass (1965) and Koopmans (1965), that go back to Ramsey (1928), Young (1928), Knight (1944)
- growth models with endogenous technological progress (mid 1980s)  
Arrow (1962), Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991)  
-> endogenous growth models

## growth theory in the 1970s

in between the two theoretical contributions (1970s): a great vacuum of growth economic theory of about 15 years!

main emphasis on short-term analysis in the 1970s (real business cycles, rational expectations, general equilibrium models, etc);

very much technical approaches and very little empirical applications on economic growth;

development economists emerged with great emphasis on the study of growth in less developed countries;

**growth** economics vs. **development** economics

## The “new” models of endogenous growth

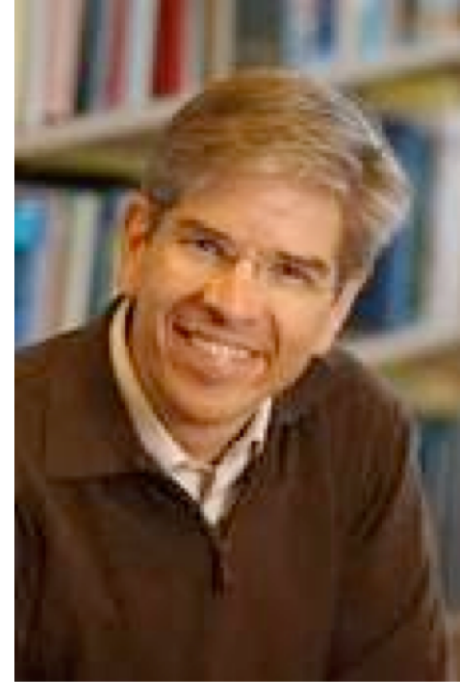
The models of exogenous growth are not adequate to explain economic growth:

the long-run growth rate of GDP is explained by the technological progress (exogenous, which means it is not explained by the working of the economic system)

In the **1970s and up to the mid-1980s**, macroeconomics was focused mainly in short term issues

By **mid-1980s** some economists made theoretical work devoted to fill gaps in the explanatory models of growth; examples:

- Romer (1986, 1987) deals with technological progress as an endogenous variable (the process of generation of new ideas, research and innovation, R&D);
- population growth as an endogenous variable (dependent on GDP per capita), and not as an exogenous variable



## endogenous growth models: the role of technological progress

difficulty of incorporation of technological progress in neoclassical models because competitive assumptions cannot be made regarding the creation of new ideas (imperfect competition/ideas as quasi-public goods)

main contribution of Paul Romer:

- Romer, P. (1986). “Increasing returns and long-run growth”. *Journal of Political Economy*, 90: 6 (Dec.), 1257-1278
- Romer, P. (1987). “Growth based on increasing returns due to specialization”. *American Economic Review*, 77:2 (May), 56-62
- Romer, P. (1990). “Endogenous technological change”. *Journal of Political Economy*. 98:5 (October), Part II, S71-S102

author of the incorporation of R&D theories and imperfect competition into the growth models.



## endogenous growth models: the role of technological progress (cont.)

incorporation of Schumpeterian ideas of progress (as “creative destruction”)

Aghion, P., Howitt, P. (1992). “A model of growth through creative destruction”. *Econometrica*, 60:2 (March), 323-351

+ new lines of research:

- Diffusion of technology and its role in economic growth (a line of empirical research in progress);
- Notice the relevance of FDI and its growth in the last decades

## from proximate to fundamental causes of growth

North, D. (1990). *Institutions, Institutional Change and Economic Performance*. Cambridge: Cambridge University Press.

*“The factors we have listed (innovation, economies of scale, education, capital accumulation, etc.) are not causes of growth; **they are growth**”* (North and Thomas, 1973, p. 2, italics in original)

## the role of institutions to explain economic growth:

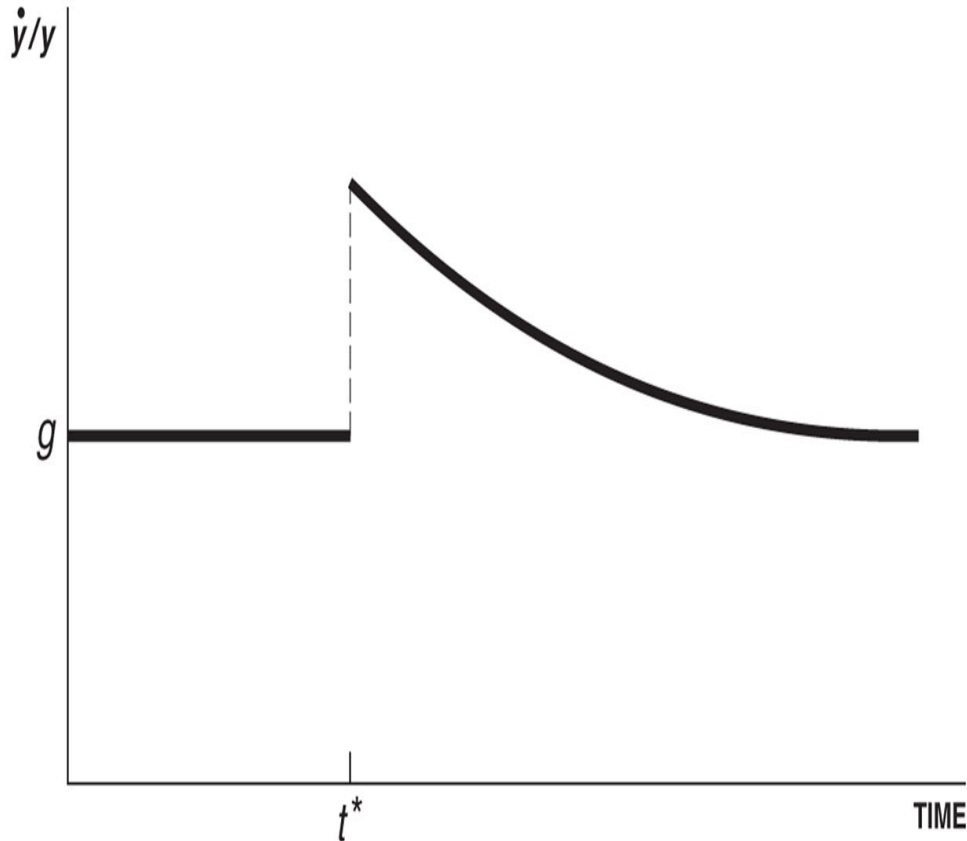
*“Institutions are the rules of the game in a society or, more formally, are the humanly devised constraints that shape human interaction (...) In consequence [institutions] structure incentives in human exchange, whether political, social, or economic”* (North 1990, p. 3)

## Why nations fail:

***“Nations fail today because their extractive economic institutions do not create the incentives needed for people to save, invest, and innovate”***

(Acemoglu, D., & Robinson, J. (2012). *Why nations fail: the origins of power, prosperity and poverty*. New York: Crown Business, p 372)

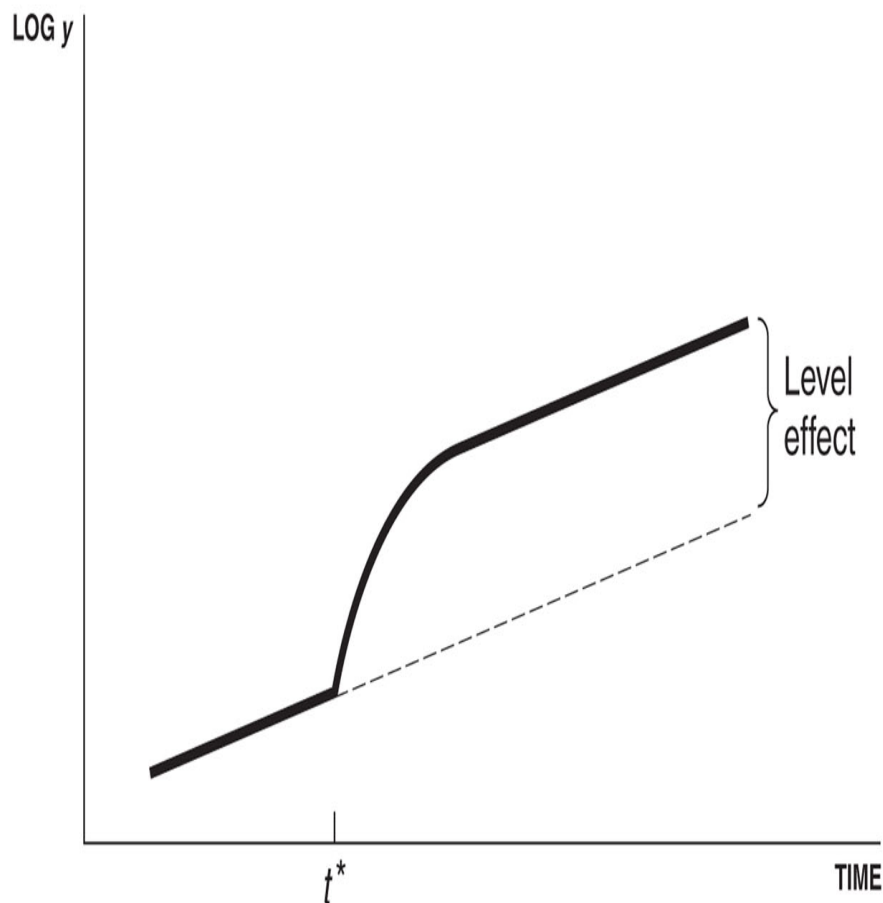
**FIGURE 2.12 THE EFFECT OF AN INCREASE IN INVESTMENT ON GROWTH**



**Solow model:**

the rise of the investment rate  
has no effect on the long-run  
growth rate of GDP *per capita*

FIGURE 2.13 THE EFFECT OF AN INCREASE IN INVESTMENT ON  $y$



**Solow model:**

the rise of the investment rate

**has positive effect** on the long-run

level of GDP *per capita*

## endogenous growth models: a preliminary theoretical question

Are there **endogenous** mechanisms that may “explain” a sustainable growth path of the economy?

recall the last version of the Solow model (with technological progress, imbedded in human capital)

$$Y = K^\alpha \cdot (AH)^{1-\alpha}$$

$$H = h \cdot L$$

skilled active population (by schooling and professional training)

$$h(u) = e^{\psi u}$$

u (fraction of time spent on learning skills);  $\psi$  (increase % of skills when u rises by 1)

$$A(t)$$

technological progress imbedded in human capital

$$Y = K^\alpha \cdot (A \cdot h(u) \cdot L)^{1-\alpha}$$

## an alternative model (of endogenous growth): the AK model



**Rebelo, Sergio** (1991), “Long-Run Policy Analysis and Long-Run Growth”,  
*The Journal of Political Economy*, Vol. 99, No. 3 (Jun., 1991), pp. 500-521

introducing “learning by doing” in the models of economic growth (following Arrow)

one way of introducing technological progress is to admit that there exists a **level of knowledge per unit of human capital** that is explained by the level of development of the economic activity, such that each generation of physical capital incorporates accumulated (improved) skill experience (and, then, higher productivity of labour):

$$A = c \cdot (K/h.L)$$

(c is a constant; proportion of physical capital per unit of human capital)

## the production function of the AK model

the production function  $Y = K^\alpha L^{1-\alpha}$  comes:

$$Y = K^\alpha (A \cdot h \cdot L)^{1-\alpha}$$

$$Y = K^\alpha (\underbrace{c \cdot (K/h \cdot L)}_{\text{level of knowledge per unit of human capital used in the economic activity}} \cdot \underbrace{h \cdot L}_{\text{schooling and professional training}})^{1-\alpha} =$$

$$= c^{1-\alpha} \cdot K =$$

$$= A \cdot K$$

level of knowledge  
per unit of human capital  
used in the economic  
activity

schooling and  
professional training

## AK model

(1)  $Y = A.K$                       **production function**

(2)  $dK/dt = s.Y - \delta.K =$               **physical capital accumulation**

From (1) and (2) comes

$$dK/dt = s.AK - \delta.K$$

$$(dK/dt)/K = s.A - \delta$$

The **growth rate** of the economy is an increasing function of the **rate of investment** ( $s$ )

from (1), the growth rate of output is equal to the growth rate of capital

it corresponds to  $\alpha = 1$

what does this mean?



## remind (again!) the Solow model

the *steady state* condition

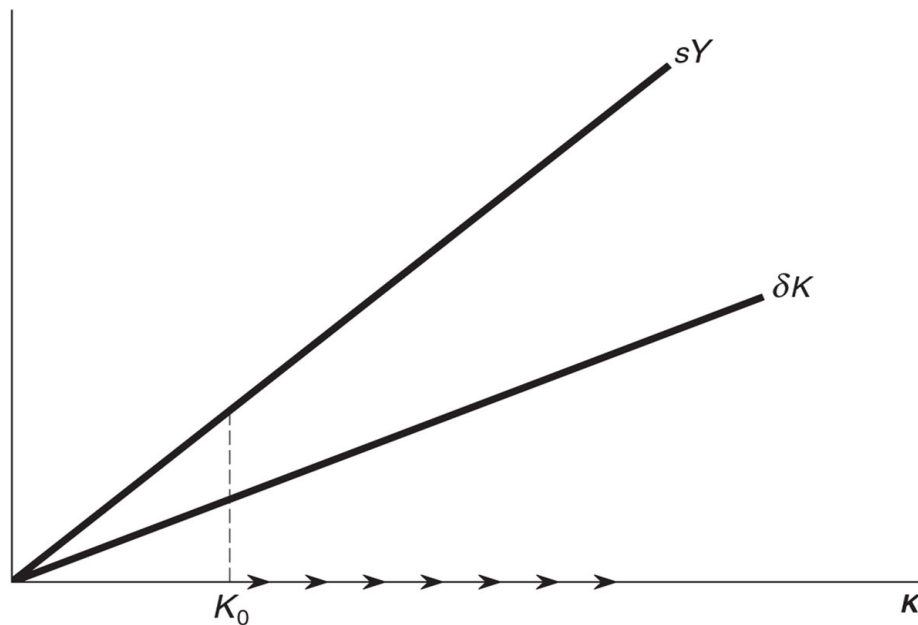
$$s.y = (n+g+\delta).k$$

implies that the value of  $y$  in steady state depends on the concavity of  $s.y$ , that is, it depends on  $\alpha$  (diminishing returns of capital accumulation  $\Rightarrow \alpha < 1$ ).

For a larger value of  $\alpha$  (where  $0 < \alpha < 1$ ) the longer is the transition period for the new steady state (i.e. the farther away the steady-state value of  $k^*$  is from  $k_0$ ). For  $\alpha = 1$ , this time is infinity!

- look at the effect of rising  $\alpha$  on the shift of the  $s.y$  curve (draw these curves);
- make an economic interpretation of a larger value for  $\alpha$  (rate of declining marginal productivity of physical capital);
- make an economic interpretation of the effect of a higher value of  $\alpha$  on the transition towards the new steady state;

FIGURE 9.1 THE SOLOW DIAGRAM FOR THE AK MODEL



AK generates  
endogenously  
economic growth

in the **AK model**  $\alpha = 1$ ; what does it mean?  
*constant returns* to the accumulation of capital;

## AK model: important interpretation

for  $\alpha < 1$ , in the transition process, due to diminishing returns of capital accumulation, each new unit of capital that is added to the economy is slightly (large  $\alpha$ )/highly (small  $\alpha$ ) less productive than the previous unit. Therefore, investment will fall to the level of depreciation, and the accumulation of capital will end; we reach a new ***steady-state***;

for  $\alpha = 1$ , in the transition process, due to constant returns to capital accumulation, each new unit of capital that is added to the economy is as productive as the previous one.

Therefore, investment will remain, and the accumulation of capital will never end; we will never reach a new ***steady-state***.

## is AK model comparable to HD model?

No!

$Y = AK$       AK model

$K = vY$       HD model

Or  $Y = (1/v)K$

but there are **big differences**

in HD model the production factors are complementary (K/L is a constant)/  
in the AK model the production factors are substitutable (K/L is not a constant);

in the HD model, v is the capital-output ratio (strictly related to the the physical capital)/  
in the AK model, A has a different meaning (see above).