



LISBON
SCHOOL OF
ECONOMICS &
MANAGEMENT
UNIVERSIDADE DE LISBOA

Macroeconomics II

Lecture 09

Romer model
Extensions



Theoretical Lecture 09

Chap 08 The Romer model

- the model of endogenous growth by Romer: main assumptions;
- production function;
- production of new ideas and productivity of research (externality due to duplication; the spillover effect of research);
- economic growth in the *steady state*;
- long-run effect of research policy.

Reading

Jones, C., Vollrath, D. (2013), *Introduction to Economic Growth*, Norton, capítulo 5, pp. 97-119.

Classical

Romer, P. (1990), “Endogeneous Technological Change”, *Journal of Political Economy*, 98, October 1990: S71-S102

The Romer model

To endogenize the technological progress: **research as an economic activity** (R&D)

production function:

$$(1) Y = K^\alpha (A \cdot L_Y)^{1-\alpha}$$

A(t) – level of technology in the economy, which is measured by the stock of ideas accumulated until the present; the model intends to explain the growth of A(t).

for a given level of technology A, the production function has constant returns to scale in K e L_Y

A is an input for production (*stock* of ideas: the use of *patents*)

production function has increasing returns to scale in K, L_Y and A

the generation of the inputs: physical capital (K), labour (L) and ideas (A)

physical capital

$$(2) \quad dK/dt = s_k Y - \delta K$$

labour

$$(3) \quad (dL/dt)/L = n \quad n \text{ is exogenous}$$

$$(4) \quad L = L_Y + L_A \quad \text{labour in the production of final goods } (L_Y) \text{ and in research } (L_A)$$

$$(5) \quad L_A/L = s_R \quad s_R \text{ constant; exogenous? endogenous?}$$

ideas

A – the growth of A is **endogenous**

$A(t)$, the growth of the stock of ideas

$$dA/dt = \theta^* \cdot L_A$$

dA/dt the evolution of the discovery of new ideas

θ^* productivity of research (the rate of producing new ideas by the researchers)

L_A number of researchers

$\theta^* = \theta(A)$, the productivity of researchers is a function of the **stock of ideas**

increasing function? a large set of accumulated ideas (large stock) facilitates the discovery of new ideas; positive spillover (much of what has been discovered so far facilitates the generation of new ideas);

decreasing function? a large set of accumulated ideas (large stock) becomes more difficult to discover “new” ideas (hardly you can discover new things since so much is already known ...)

$\theta^* = \theta \cdot A^\Phi$ rate at which new ideas are produced, with $\Phi > 0$
(increasing) ou $\Phi < 0$ (decreasing)

$\theta^* = \theta(L_A)$, the productivity of the researchers is a function of the **number of researchers**

a large number of researchers facilitates the creation of research networks and then strengthens the ability of each research and his/her team to make more and better research ($\lambda > 1$ below)

externality associated to **duplication**: some ideas may be not new ideas, since they may have been already discovered/or being discovered simultaneously by other researchers ($\lambda < 1$ below)

(6) $dA/dt = \theta \cdot L_A^\lambda \cdot A^\Phi$ using L_A^λ because the productivity of research depends on the number of researchers looking for new ideas

$\lambda < 1$ (**duplication** or repetition effect), or $\lambda > 1$

$\Phi < 0$ (decreasing with A), or $\Phi > 0$ (increasing with A, **spillover** effect)

The Romer model

$$(1) Y = K^\alpha (A \cdot L_Y)^{1-\alpha}$$

$$(2) dK/dt = s_k Y - \delta K$$

$$(3) (dL/dt)/L = n$$

$$(4) L = L_Y + L_A$$

$$(5) L_A/L = s_R$$

$$(6) dA/dt = \theta L_A^\lambda \cdot A^\Phi$$

growth of the economy in *steady state*

Assuming \underline{s}_R constant, the growth of GDP per capita in *steady state* is explained by the technological progress (as in Solow model):

$$g_y = g_k = g_A$$

What is (and what explains) the rate of technological progress, g_A ?

Rem: this growth rate is endogenous in the model!

$$dA/dt = \theta L_A^\lambda \cdot A^\Phi$$

$$(dA/dt)/A = \theta \cdot (L_A^\lambda \cdot A^\Phi)/A = \theta \cdot L_A^\lambda / A^{1-\Phi}$$

$$\text{or} \quad g_A = \theta \cdot L_A^\lambda / A^{1-\Phi}$$

the rate of technological progress in *steady state*

$$g_A = \theta \cdot L_A^\lambda / A^{1-\Phi}$$

in *steady state* g_A is constant: growth rate of the numerator = growth rate of denominator

$$\lambda \cdot (dL_A/dt)/L_A = (1 - \Phi) \cdot (dA/dt)/A$$

in *steady state* $(dL_A/dt)/L_A = n$ (growth rate of population)

$$\lambda \cdot n = (1 - \Phi) \cdot g_A$$

Technological progress

$$g_A = \lambda \cdot n / (1 - \Phi)$$

g_A is explained by the parameters of the production function of ideas (λ e Φ) and the growth rate of population (n)

$$g_A = \lambda \cdot n / (1 - \Phi)$$

$$(6) \quad dA/dt = \theta L_A^\lambda \cdot A^\Phi$$

interpretation

let the special case $\lambda = 1$ and $\Phi = 0$ (for an easier explanation)

then: $dA/dt = \theta \cdot L_A$, from equation (6)

If L_A is constant, in each period $\theta \cdot L_A$ new ideas are created. This means that the growth rate of A , g_A , is decreasing (the stock rises by equal amounts, so that the growth rate decreases). The possibility to get a non-decreasing growth rate of g_A is to prevent the decrease of the number of researchers and, instead, to rise. This requires the population to rise. This explains n in the equation above.

conclusion

If the population does not increase, **economic growth will not happen**, even keeping research activity and technological progress in the economy.

Effects of Economic Policy

Economic policy may have effect on long-run economic growth?

example of an economic policy measure: incentives to research, creating/increasing research subsidies; **rise of s_R**

$$(dA/dt)/A = \theta \cdot L_A^\lambda / A^{1-\Phi}$$

let us assume that $\lambda = 1$ and $\Phi = 0$

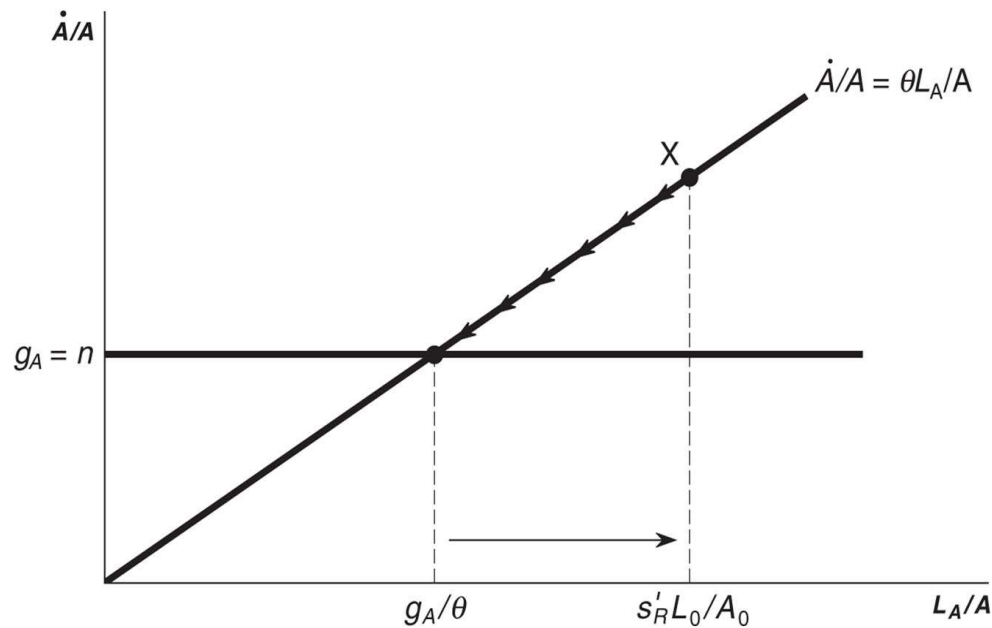
then:

$$(dA/dt)/A = \theta \cdot L_A/A = \theta \cdot s_R \cdot L/A$$

We know that in *steady state*, with $\lambda = 1$ and $\Phi = 0$, $g_A = n$

What is the effect of a policy of incentives to research by rising s_R ?

FIGURE 5.1 TECHNOLOGICAL PROGRESS: AN INCREASE IN THE R&D SHARE



$s'_R > s_R$ s_R increased (number of researchers in I&D)

$s'_R \cdot L_0 > s_R \cdot L_0$, the number of ideas increases;

technological progress > population growth (n)

=> L_A/A decreases

=> g_A decreases

the economy returns to the previous *steady state*

steady state: $\theta \cdot s_R \cdot L_0/A_0 = g_A \Rightarrow s_R \cdot L_0/A_0 = g_A/\theta$

FIGURE 5.2 \dot{A}/A OVER TIME

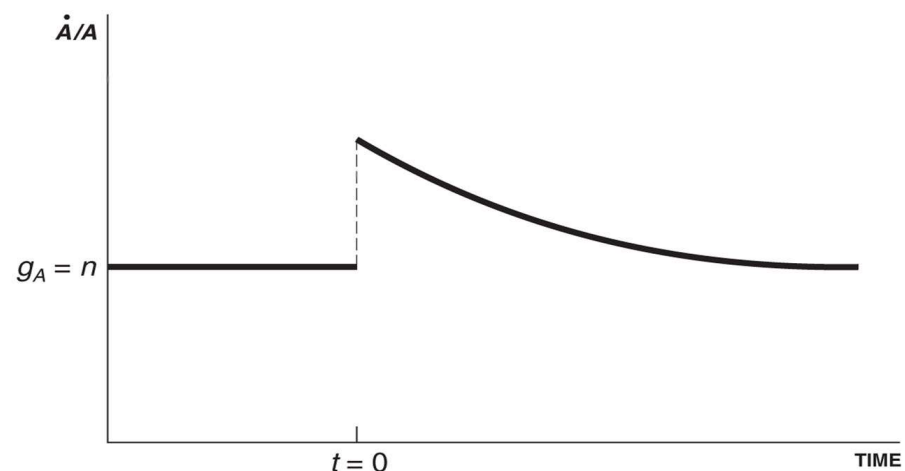
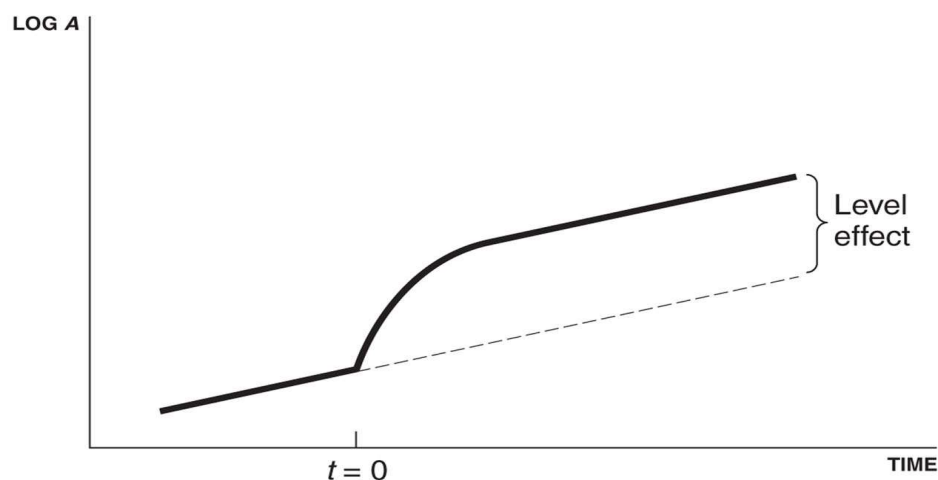


FIGURE 5.3 THE LEVEL OF TECHNOLOGY OVER TIME



A permanent increase of s_R increases the technological progress (and economic growth) only in a temporary way, not permanently in the long-run.

But it increases permanently (in the long-run) the level of technology.

technological progress and growth

(to remind)

growth accounting: growth of factors and of TFP (TFP, total factor productivity)

growth of TFP through technological progress

technological progress in the models of exogenous growth

growth in steady state: rate g

rate g is exogenous

technological progress in the models of endogenous growth

rate g is endogenous

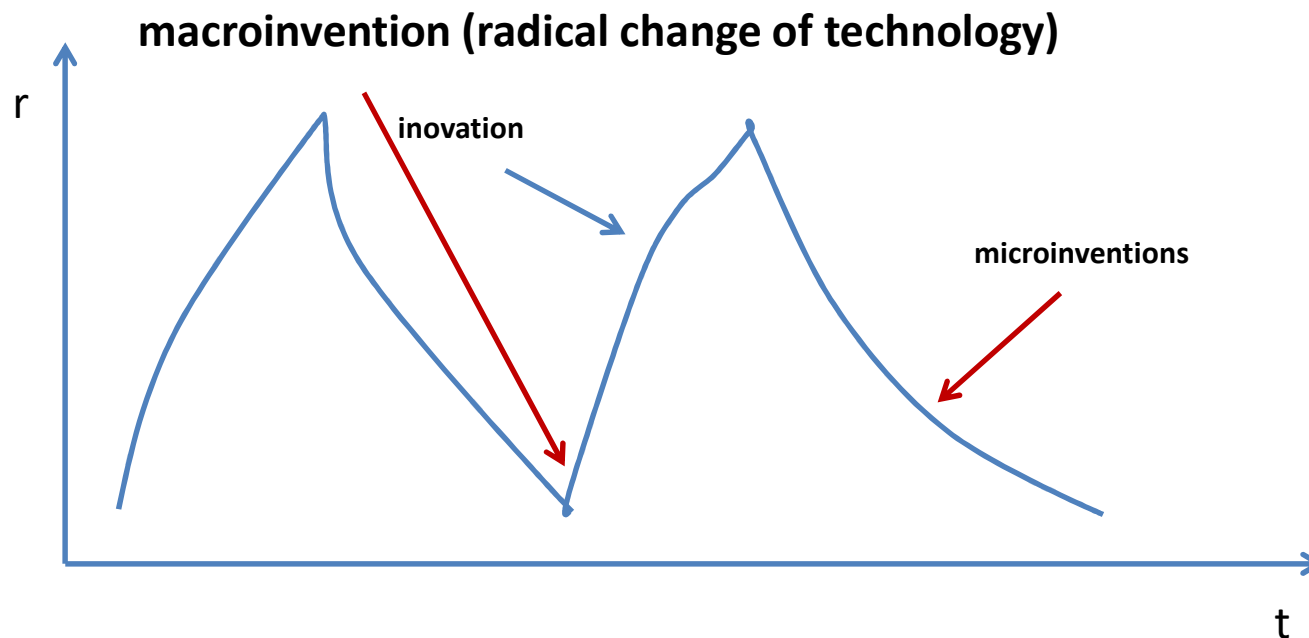
technological progress is “explained” by the working of the economy (it is an output of a sector of economic activity)

what is technological progress ?



Facts and concepts, technological progress: invention and innovation

- **invention**: discovery of new ideas
- **innovation**: implementation of the new ideas in the economic activity





data research & development (R&D)

Expenditure on R&D as % of GDP in 2001 (OECD):

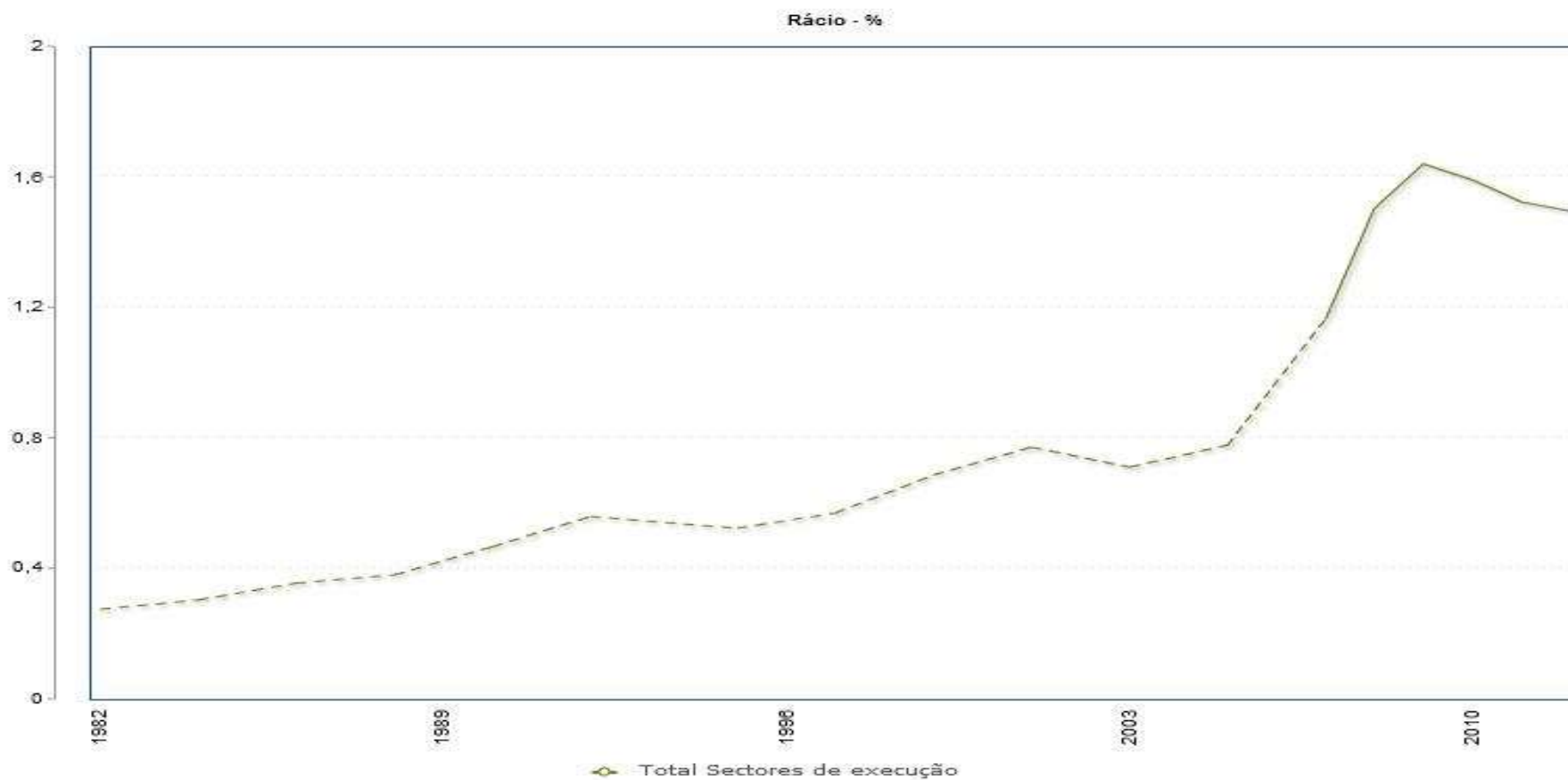
Portugal	0,83
Spain	0,96
Germany	2,49
USA	2,82
Japan	3,09

other indicators:

nr. of scientists

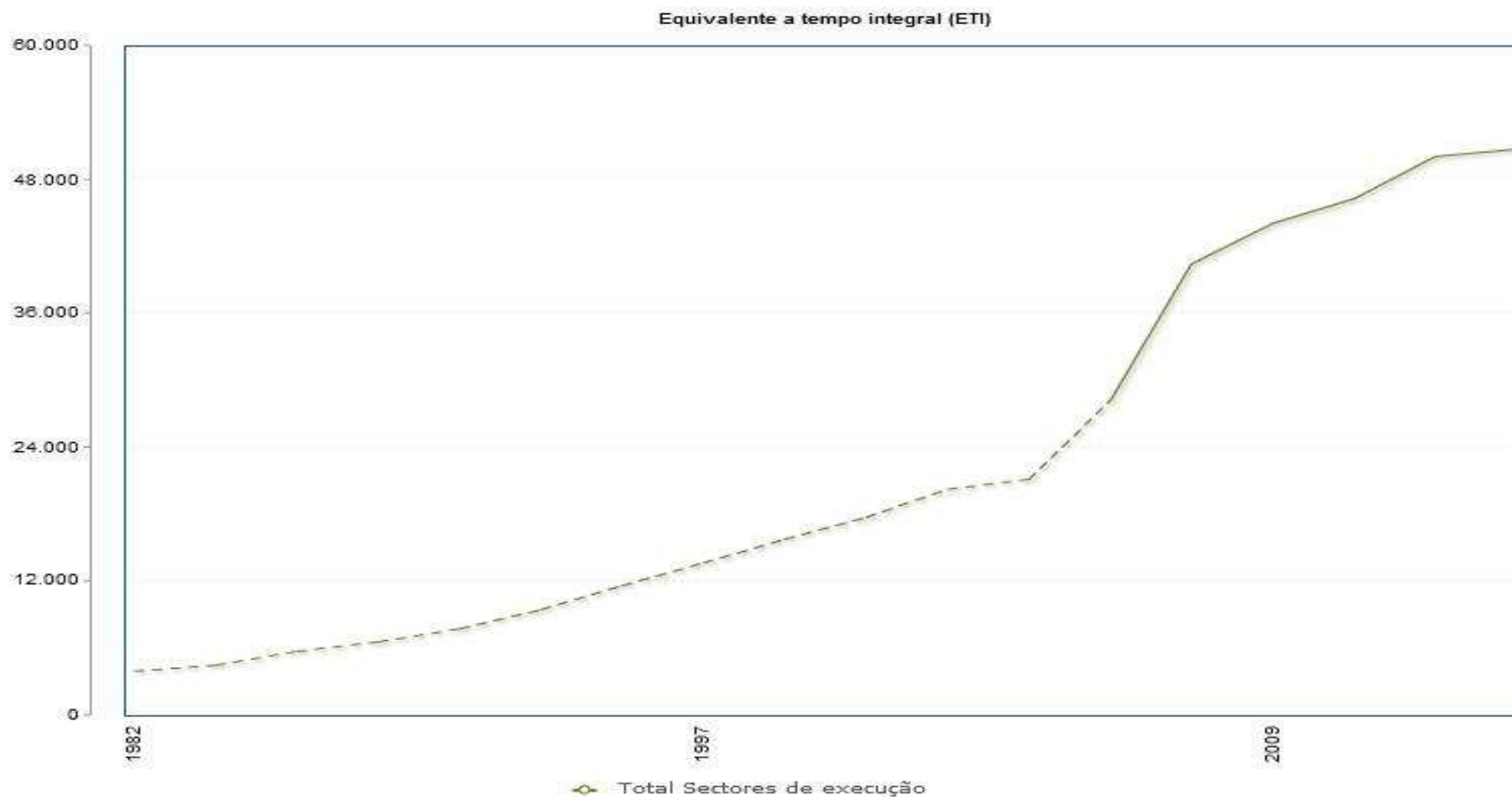
nr. of registered patents

PORTUGAL: Expenditure on R&D as % of GDP



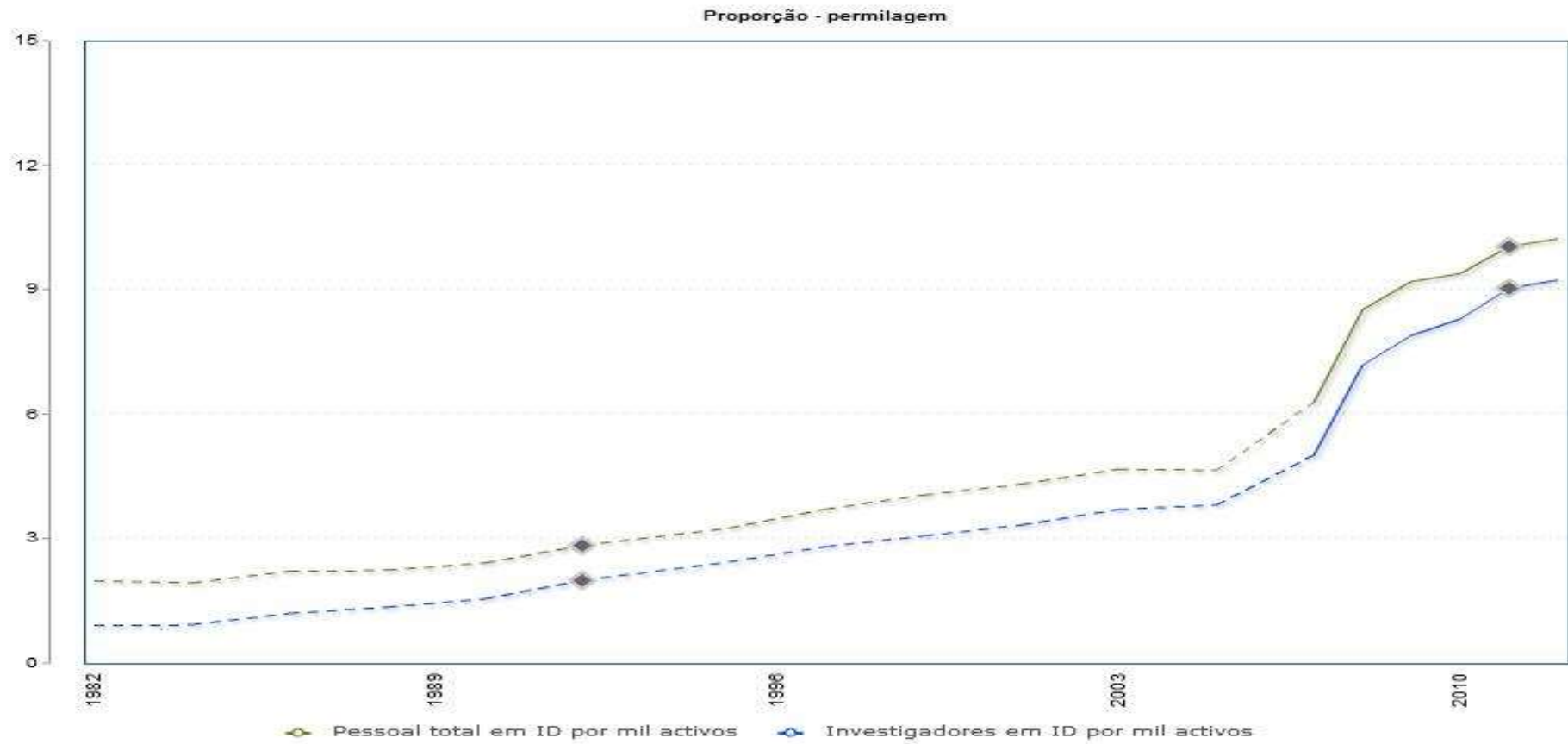
Fontes/Entidades: INE-BP, DGEEC/MEC, PORDATA

PORTUGAL: Nr de researchers (Full Time Equivalent) in R&D



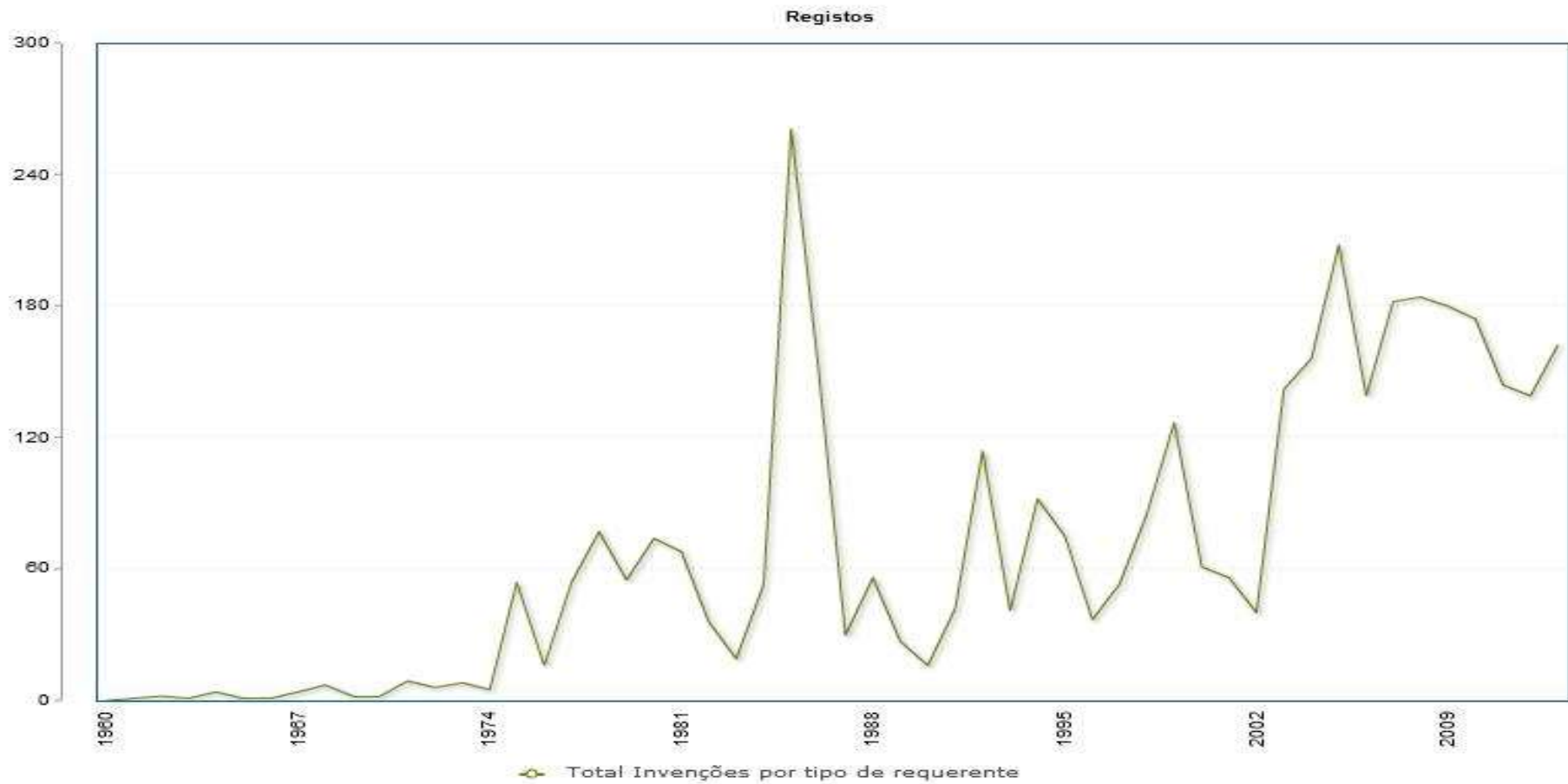


Nr of workers in R&D as % of active population





Nr of inventions/patents issued in Portugal



output of I&D: ideas (or “*knowledge*”)

ideas (or “*knowledge*”) as an economic good:

não-rivalry (knowledge is shared by several people)

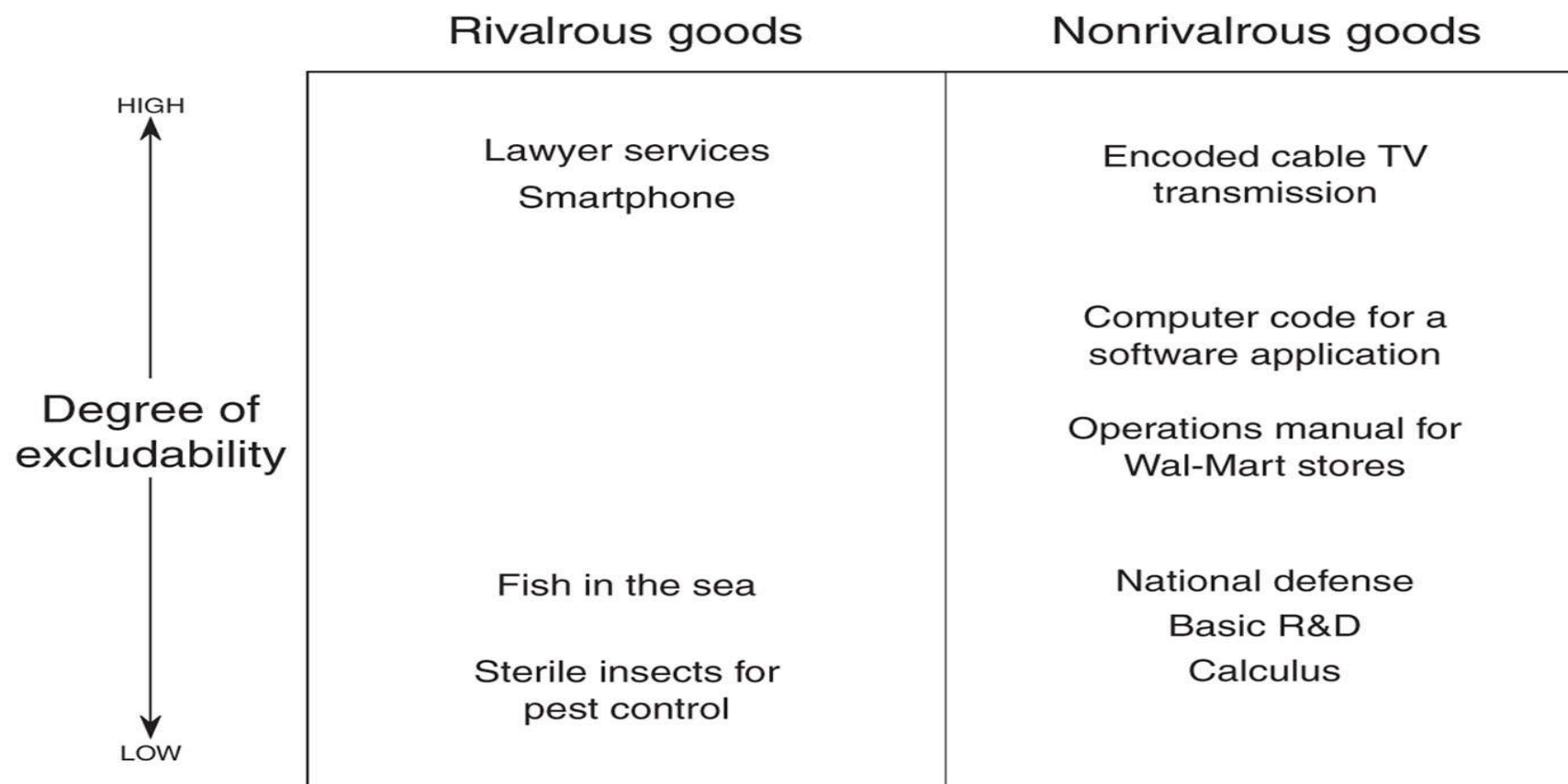
excludability (not possible to exclude the access to knowledge; but may have restrictions to the access: secret, patent, copyright)

consequences?

non-rivalry => **high fixed cost of production** and **low marginal cost** for its production (almost null) (ex: the production of a software)

- **increasing** returns to scale
- **imperfect** competition

FIGURE 4.1 ECONOMIC ATTRIBUTES OF SELECTED GOODS



microeconomic analysis

FIGURE 4.2 FIXED COSTS AND INCREASING RETURNS

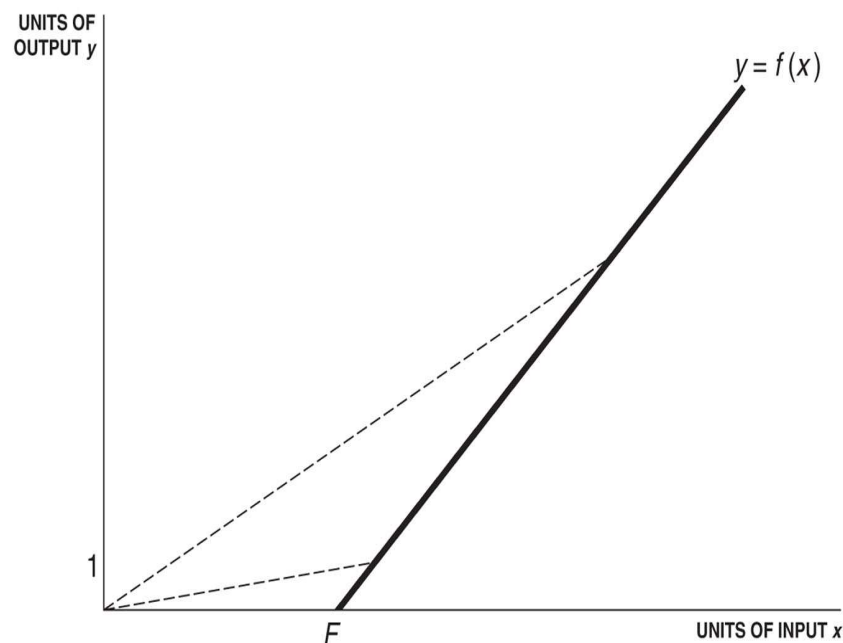
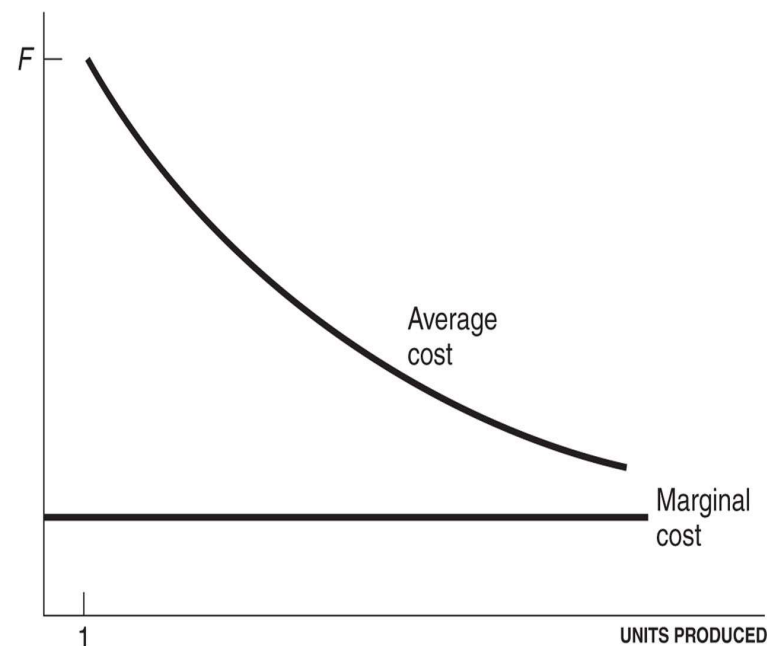


FIGURE 4.3 FIXED COSTS AND INCREASING RETURNS



average cost > marginal cost and then there is not a competitive equilibrium with $p =$ marginal cost (because in that case profits < 0); to have producers, it requires that $p >$ marginal cost (it is **not perfect competition**)

the **production of new ideas** requires the possibility of generating surplus, what is **not compatible** with perfect competition

The Romer model (summary)

to endogenize the technological progress:

- production function (it has ideas/“*knowledge*” as a production factor)
- equations to describe the creation of inputs (including the ideas/“*knowledge*”)

$$(1) \quad Y = K^\alpha (A \cdot L_Y)^{1-\alpha}$$

$$(2) \quad dK/dt = s_k Y - \delta K$$

$$(3) \quad (dL/dt)/L = n$$

$$(4) \quad L = L_Y + L_A$$

$$(5) \quad L_A/L = s_R$$

$$(6) \quad dA/dt = \theta^* \cdot L_A$$

$$(7) \quad \theta^* = \theta \cdot A^\Phi$$

$$dA/dt = \theta L_A^\lambda \cdot A^\Phi$$

p.f. constant returns to scale in K and L_Y ;
increasing returns in K , L_Y and A .

A(t) *stock* of knowledge
(nr of ideas invented until moment t)

θ^* productivity of research (nr. of new ideas
produced per researcher)

$0 < \lambda < 1$ externality associated with duplication

$\Phi > 0$ positive knowledge spillover in research

some of the ideas created by a
researcher may be not new ($\lambda < 1$);
but may also exist a network
effect ($\lambda > 1$)

much of what has been discovered
so far facilitates the generation of
new ideas (positive spillover); a large
set of accumulated ideas becomes more
difficult to discover "new" ideas (negative
spillover)