LISBON SCHOOL OF ECONOMICS & MANAGEMENT UNIVERSIDADE DE LISBOA

Quantitative Finance – 1st year/Eco-Finance-Management, 1st sem., Exam 2 31.01.2018, 18:00

1st Part (70 marks) Total Time: 2h

1st Part: 70 Marks. All answers shall be given in the space available. All True/False questions have equal marking. During the test no comments or questions should be asked. Write your name and number on every sheet on the place available. No mobile phones, or any device with bluetooth or wifi, are allowed at any time.

Name:____**Number:**_________

In the following group of questions, every right answer has 2.5 marks each, wrong answers have –2.5 each (2.5 penalty mark). *Each group of questions will have a mark between* ***0*** *(minimum) and* ***10*** *(maximum).*

Write True (**T**) or False (**F**), *with an "**X**" in the appropriate entry.*

1. Consider Simple and Compound Interest calculation:

	T	F
For compound interest and a positive interest rate, the effective monthly interest rate is proportional to the annual nominal rate with quarterly accumulation.		
Consider a positive annual interest rate, and the statement: "Similarly to a stopped clock and the right hour… *there are only two moments in time that the accumulation value of a given principal in compound interest and simple interest are the same*".		
Simple discount corresponds to the present value of a unit principal discounted one year.		
The interest rate $i_A^{(m)}$ divided by m corresponds to the effective accumulation rate of some principal.		

2. Consider discount and ordinary annuities:

	T	F
An ordinary annuity is a sequence of equal payments dispersed, or received, at any time intervals.		
The notation $a_{\overline{10}|8\%}$ stands for an ordinary unit payment annuity with 10 equal payments where a constant interest rate of 8% is applicable.		
The term of an annuity is the same as the corresponding life.		
A Pension payment is an example of a contingent annuity.		

3. Consider any kind of annuities and debts:

	T	F
The *annual percentage* rate (APR) is the rate at which the cash value of a loan equals the present value of the payments.		
A "Down Payment" is any periodic payment of a loan.		
$\ddot{a}_{\overline{10}|5\%} = (1.05).a_{\overline{10}|5\%}$.		
In a Constant-Principal Loan the principal amortizations are constant throughout the life of the loan.		

4. Consider bonds, leasing and shares:

	T	F
A bond is a share of a loan.		
The Maturity Value of a bond is its Redemption Value.		
A Leasing contract grants the use of a specific fixed asset for a specific time without the need for the asset's property.		
The market value of a stock is assured by the corresponding face or issuance value.		

In the next group of questions, tick √ or write X in the box next to the answer you consider to be correct (only one is). In each group, a correct answer has 5 marks and a wrong answer gets -1.25 marks (penalty 1.25).

5. Consider simple interest. Mr. Ben calculates the annual nominal rate, compounded twice a year, equivalent to a quarterly effective interest rate of 1%. The right answer is:

a) 2.01% ☐; **b)** 4.00% ☐; **c)** 4.02% ☐; **d)** 2.00% ☐.

6. Consider compound interest. Mr. Ben invested €10,000.00 at rate $i_A^{(12)} = 15\%$. What is the application term necessary in order to increase the principal invested by 50% (approximately)?

a) 4.032 years ☐; **b)** 5 years ☐; **c)** 2.720 years ☐; **d)** Any of the previous ☐.

7. In compound interest, Mr. Ben did a four year application of €10,000 at a nominal (annual) rate of 2% compounded semi-annually. Write the interest earned during the second year, approximately.

a) €101.00 ☐; **b)** None of the others ☐; **c)** €205.04 ☐; **d)** €103.03 ☐.

8. Consider simple interest and a semi-annual interest rate of 2.5%. Calculate the necessary investment amount that Mr. Ben needs to do in order to accumulate a total of €1,050.00 within 24 months:

a) €1,001.14 ☐; **b)** €1,000.00 ☐; **c)** €954.54 ☐; **d)** Any of the others ☐;

9. Consider the following information, with normal years (two, *k* and *k+1*), about the amortization schedule of an existing loan, from Mr. Ben's company:

Year	**Debt at beginning of the year**	**Interest**	**Payment**	**Principal Paid**	**Accumulative Amortization**	**Debt at end of the period**
k	€99,932.90	€666.22	€733.77		€134.65	
k+1		€665.77	€733.77			

Argue on the Amortization Method used, from those studied:

a) Amortizations are decreasing ☐; **b)** Amortizations are constant ☐; **c)** Not enough information ☐; **d)** Amortizations are increasing ☐;

10. A certain perpetual bond is issued 10% over the par, sold at a value of €110. The coupon rate is 5.0% and bondholders require a return of 4.0%. What is the value of the bond?

a) €200 ☐; **b)** €100 ☐; **c)** **€125** ☐; **d)** **€137.50** ☐.

Name:____Student No.:__________

2^{nd} Part (130 marks)

In this group write your calculations in the space below the question and write the final answer in the box provided. Do not forget to present all formulae and intermediate calculations needed.

1. (45 marks)

Mr. Ben is buying a new car and requires a loan of €24,000 to pay for it. A car dealer offers two alternatives for the loan:

- **i.** Monthly payments for three years, starting one month after purchase, with an annual interest of 12% compounded montly; or
- **ii.** Monthly payments for four years, also starting one month after purchase, with annual interest 15% compounded montly.

Denote by R_1 and R_2 the monthly payments for options **(i)** and **(ii)**, respectively.

a) Calculate R_1.

R:

b) Calculate R_2.

R:

c) Help Mr. Ben to decide. Explain brief but clearly your option.

d) Compute the first three lines of the amortization table corresponding to **Option (i)**:

Period	Debt at start of the period	Interest	Payment	Amortization	Accumulative Amortization	Debt at end of the period
1	24,000.00					
2						
3						

2. (45 marks)

Mr. Ben is going to get a brand new, luxurious, SUV (**S**port **U**tility **V**ehicle) by a leasing contract. It has a contract value of €80,000 and the following conditions: (**i**) Term: 4 years; (**ii**) A down-payment of 15% of contract value, done upfront; (**iii**) Bi-annual constant rent payments, with first being paid one year after contract set in force; (**iv**) Effective annual rate: 10.25%; (**v**) Residual value: 10% of contract value, optional, paid with the last rental payment.

a) Calculate the Present Value of the Residual Value.

R:

b) Calculate the amount of each rental payment

R:

c) Calculate the value of the SUV immediately after the 3rd rental payment.

R:

Name:____**Student No.:**__________

3. (40 marks)

Mr. Ben's corporation issued a bond loan under the following terms:

• Date of issue: 01/03/*n*.

• Nominal Value: €10.00.

• N.º of bonds issued: 200,000.

• Issue value at par;

• Loan term: 5 years.

• Annual coupon rate: 6%.

• Payment of annual interest: The first payment will occur one year after issuance.

• Mode of Redemption: Equal annual reimbursements, with a premium of €0.20 per bond in the first two years and €0.50 for next years.

• Number of reimbursements: 4. Date of the first repayment: 2 years after issuance.

a) Compute the total value of the bond loan.

R:

b) Fill out the bond amortization table:

Year	Debt at beginning of the period	Interest	No. of bonds repaid	Amortization	Premium	Total Payment	Debt at end of the period
1							
2							
3							
4							
5							

Quantitative Finance Formulas

Interest accumulation: Fv = Pv + I

Simple interest: Fv = Pv (1 + i·t)

Compound interest: Fv = Pv $(1 + i)^t$

Simple discount: D = Fv·d·t

I=Interest; P=Principal; i=interest rate

t=number of periods

Effective rates conversion:

$i_L = (1 + i_S)^{L/S} - 1$; $i_s = (1 + i_L)^{s/L} - 1$

Relation between nominal and effective rates:
$i_A(m) = m[(1 + i_A)^{1/m} - 1]$

Continuous compounding:

Nominal rate: $\delta = \ln(1 + i_A)$

Future Value: $S=Pe^{\delta t}$

Present Value: $P=Se^{-\delta t}$

Present value of a n payment annuity immediate of 1 per period: $a_{\overline{n}|i} = \frac{1-(1+i)^{-n}}{i}$

Accumulated value of a n payment annuity immediate of 1 per period:

$$s_{\overline{n}|i} = \frac{(1+i)^n-1}{i} = a_{\overline{n}|i}(1+i)^n$$

Present value of annuity due:

$$\ddot{a}_{\overline{n}|i} = 1 + a_{\overline{n-1}|i} = a_{\overline{n}|i}(1+i)$$

Accumulated value of annuity due:

$$\ddot{s}_{\overline{n}|i} = s_{\overline{n}|i}(1+i)$$

Present value of deferred annuity:

$$_{k|}a_{\overline{n}|i} = a_{\overline{n}|i}(1+i)^{-k}$$

Accumulated value of deferred annuity:
$$_{k|}s_{\overline{n}|i} = s_{\overline{n}|i}$$

Forborne annuities

$FV=R.S_{n|i}(1+i)^p$

p- number of intervals between the last payment and FV.

Present value of perpetuity immediate: $a_{\overline{\infty}|i} = \frac{1}{i}$

Increasing arithmetic progression:

$(C-h)a_{\overline{n}|i} + h(Ia)_{\overline{n}|i}$; $(Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - \mathrm{n}(1+i)^{-n}}{i}$

Decreasing arithmetic progression:

$(D-h)a_{\overline{n}|i} + h(Da)_{\overline{n}|i}$; $(Da)_{\overline{n}|i} = \frac{n - a_{\overline{n}|i}}{i}$

Geometric progression: $C\,\frac{1-r^n(1+i)^{-n}}{1+i-r}$

M^{thly} payable annuity:

$a^{(m)}_{\overline{n}|i} = a_{\overline{n}|i}\frac{i}{i(m)}$; $s^{(m)}_{\overline{n}|i} = s_{\overline{n}|i}\frac{i}{i(m)}$

Leasing:

Lease payment=PMT + I

Pv=PMT $a_{\overline{n}|i}$, I=RV · i

Leasing (for an annuity immediate):

$Vc = E + Ra_{\overline{n}|i} + RV(1+i)^{-n}$, where

Vc: value of the contract; E: entry value

RV = residual value; PMT = periodic payment

Linear Interpolation:

Rn=R1+[(R2-R1)/(t2-t1)].(tn-t1)

Rn - unknown rate
R1 and R2 - two known