1st Part: 35 Marks. All answers shall be given in the space available. All True/False questions have equal marking. During the test no comments or questions should be asked. Write your name and number on every sheet on the place available. No mobile phones, or any device with bluetooth or wifi, are allowed at any time.

Name: $\qquad$ Number: $\qquad$
In the following group of questions, every right answer has $\mathbf{2 . 5}$ marks each, wrong answers have - $\mathbf{- 2 . 5}$ each (2.5 penalty mark). [Each group of questions will have a mark between $\mathbf{0}$ (minimum) and 10 (maximum)]

Write True (T) or False (F), with an X in the appropriate entry.

1. Consider deferred annuities, perpetuities, variable payment annuities and debts:

| $\ddot{a}_{\overline{10} \mid 5 \%}>\ddot{a}_{\overline{10} \mid 1 \%}$. | T |  |
| :--- | :--- | :--- |
| The formula $s_{\bar{n} \mid i}=\left((1+i)^{n}-1\right) / i$ results from the sum of the payments of an annuity with <br> variable payments in decreasing geometric progression. |  |  |
| Keeping $i$ and $n$ unchanged, the future value of an annuity, $s_{\bar{n} \mid i}$, does not change if the annuity <br> is deferred. |  |  |
| In a "constant amortization loan" both interest and principal payments are decreasing as <br> payments are made. |  |  |

2. Consider bonds, leasing and shares:

| A share grants its owner the benefit from future profits of the company the share refers to. |  |  |
| :--- | :--- | :--- |
| A Leasing is a rental contract used by companies and individuals for buying fixed assets. |  |  |
| Shares are equity securities. |  |  |
| A Zero Coupon Bond does not pay interest. |  |  |

In the next group of questions, tick $\sqrt{ }$ or write $X$ in the box next to the answer you consider to be correct (only one is). In each group, a correct answer has 5 marks and a wrong answer gets $\mathbf{- 1 . 2 5}$ marks (penalty 1.25).
3. Mr. Brad faces an annuity whose value at some moment in time is $500 s_{5 \mid 8 \%}(1.08)^{-5}$. He is considering the calculations: (i) $500 \ddot{a}_{\overline{5} \mid 8 \%}(1.08)^{-1}$; (ii) $500 a_{5 \mid 8 \%}$; (iii) $250\left({ }_{3 \mid} a_{\overline{2} \mid 8 \%}\right)+500 a_{\overline{3} \mid 8 \%}+250 s_{\overline{2} \mid 8 \%}(1.08)^{-5}$. The following calculations are equivalent to $500 s_{5 \mid 8 \%}(1.08)^{-5}$, choose the best / more complete option:
a) (i) ;
b) (i) and (ii)
c) (i), (ii) and (iii)
d) None of others
4. A laptop computer has a selling price of $€ 1,957.60$ (after a certain discount). Mr. Brad has been given the option to pay the computer in six monthly instalments of $€ 332$ each, being the first one within a year. Considering a annual interest rate of $6 \%$ compounded monthly, advise Mr. Ed the best option:
a) Pay $€ 1,957.60$ immediately
b) Pay the annuity
c) Indifferent
; d) Not enough information
5. Consider the following information about a constant amortization loan:

| Period | Debt at <br> beginning of <br> the period | Interest | Payment | Amortization | Accumulative <br> Amortization | Debt at end <br> of the <br> period |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | $121.878,92 €$ | $731,27 €$ | $3.871,08 €$ | $3.139,81$ | $6.260,89 €$ | $118.739,12 €$ |

The accumulative amortization at end of period 12 is :
a) $€ 10,131.97$
b) None of the others
c) $€ 12,576.42$
d) $€ 9,419.54$

## Quantitative Finance Formulas

Interest accumulation: $\mathrm{Fv}=\mathrm{Pv}+\mathrm{I}$

Simple interest: $\mathrm{Fv}=\mathrm{Pv}(1+\mathrm{i} \cdot \mathrm{t})$
Compound interest: $\mathrm{Fv}=\mathrm{Pv}(1+i)^{\mathrm{t}}$
Simple discount: $\quad D=F v \cdot d \cdot t$

I=Interest; P=Principal; i=interest rate
$\mathrm{t}=$ number of periods
Effective rates conversion:

$$
i_{L}=\left(1+i_{S}\right)^{L / S}-1 ; i_{S}=\left(1+i_{L}\right)^{s / L}-1
$$

Relation between nominal and effective rates:
$\mathrm{i}_{\mathrm{A}}(\mathrm{m})=\mathrm{m}\left[\left(1+\mathrm{i}_{\mathrm{A}}\right)^{1 / m}-1\right]$
Continuous compounding:
Nominal rate: $\delta=\ln \left(1+\mathrm{i}_{\mathrm{A}}\right)$
Future Value: $\mathrm{S}=\mathrm{Pe}^{\delta \mathrm{t}}$
Present Value: $\mathrm{P}=\mathrm{Se}^{-\delta t}$
Present value of a n payment annuity immediate of
1 per period: $a_{\bar{n} \mid i}=\frac{1-(1+i)^{-n}}{i}$
Accumulated value of a $n$ payment annuity immediate of 1 per period:
$s_{\bar{n} \mid i}=\frac{(1+i)^{n}-1}{i}=a_{\bar{n} \mid i}(1+i)^{n}$
Present value of annuity due:

$$
\ddot{a}_{\bar{n} \mid i}=1+a_{\overline{n-1} \mid i}=a_{\bar{n} \mid i}(1+i)
$$

Accumulated value of annuity due:

$$
\ddot{s}_{\bar{n} \mid i}=s_{\bar{n} \mid i}(1+i)
$$

Present value of deferred annuity:

$$
{ }_{k \mid} a_{\bar{n} \mid i}=a_{\bar{n} \mid i}(1+i)^{-k}
$$

Accumulated value of deferred annuity:

$$
k \mid S_{\bar{n} \mid i}=S_{\bar{n} \mid i}
$$

Forborne annuities
$F V=R . S_{n \mid i}(1+i)^{p}$
p- number of intervals between the last payment and FV.
Present value of perpetuity immediate: $a_{\bar{\infty} \mid i}=\frac{1}{i}$
Increasing arithmetic progression:

$$
\begin{aligned}
& (C-h) a_{\bar{n} \mid i}+h(I a)_{\bar{n} \mid i} ; \quad(I a)_{\bar{n} \mid i}= \\
& \frac{\ddot{a}_{\bar{n} \mid i} \mathrm{n}(1+i)^{-n}}{i}
\end{aligned}
$$

Decreasing arithmetic progression:

$$
(D-h) a_{\bar{n} \mid i}+h(D a)_{\bar{n} \mid i} ; \quad(D a)_{\bar{n} \mid i}=\frac{n-a_{\bar{n} \mid i}}{i}
$$

Geometric progression: $\quad C \frac{1-r^{n}(1+i)^{-n}}{1+i-r}$
$\mathrm{M}^{\text {thly }}$ payable annuity:

$$
a_{\bar{n} \mid i}^{(m)}=a_{\bar{n} \mid i} \frac{i}{i^{(m)}} ; \quad s_{\bar{n} \mid i}^{(m)}=s_{\bar{n} \mid i} \frac{i}{i^{(m)}}
$$

Leasing:
Lease payment=PMT + I
$\mathrm{Pv}=\mathrm{PMT} a_{\bar{n} \mid i}, \mathrm{I}=\mathrm{RV} \cdot i$
Leasing (for an annuity immediate):
$V c=E+R a_{\bar{n} \mid i}+R V(1+i)^{-n}$, where
Vc: value of the contract; E: entry value
RV = residual value; $P M T=$ periodic payment
Linear Interpolation:
$R n=R 1+[(R 2-R 1) /(t 2-t 1)] .(t n-t 1)$
Rn - unknown rate
R1 and R2 - two known

In this group write your calculations in the space below the question and write the final answer in the box provided. Do not forget to present all formulae and intermediate calculations needed.

Name: $\qquad$ Number: $\qquad$

## 1. (45 marks)

Mr. Brad's sharehold company, Omega, issued a bond loan with the following terms:

- Date of issue: 01/01/yy.
- Nominal Value: €10.00.
- N. ${ }^{\circ}$ of bonds issued: 20,000.
- Issue value at par;
- Loan term: 3 years.
- Semi-annual coupon rate: 3.0\%.
- Payment of semi-annual interest. The first payment will occur one semester after issuance.
- Mode of Redemption (above the par): Repayments semi-annually of equal number of bonds, starting one year after the issuance date;
Redemption premium: $€ 0.50$ per bond during the first two repayments and $€ 1.00$ per bond after that.
a) Compute the total value of the bond loan and the redemption premium to be paid.


## R:

b) Fill out the bond amortization table (Euros).

| Semester | Debt at beginning <br> of the period | Interest | $N^{\circ}$ de bonds repaid | Amortization | Premium | Total Payment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

## 2. (20 marks)

Mr. Brad financed a car aquisition with a leasing contract that has the following conditions:

- Term of leasing contract: 5 years;
- Quarterly effective rate of $3.0 \%$, the first payment to be paid 3 months after the date of the contract;
- Down payment of $7 \%$ of the car value;
- Residual value with last payment and equal to $10 \%$ of the car value, assume $€ 2,000$ for the residual value;
- Constant payments every quarter (at the end of each period).

Compute the payments that are associated with the leasing contract.

