Degree:	2nd	Field of Study:	Actuarial Science

Code:	MODFIN	Course name:	Models in Finance	Credits ECTS:	8

Scientific field:	Mathematical Analysis and Mathematical Finance	Department:	Mathematics

Curricular year:	2nd	Semester	1st	X	Type	Compulsory	X
			2nd			Optional	

Lecturer:	João Miguel Espiguinha Guerra

Contact hours	Total workload
39	212

Aims and scope

– The aim of this course is to develop the necessary skills in order to understand and apply the mathematical methods, of analytical, stochastic and numerical type, that play an important role in financial stochastic models either in discrete or continuous time. In particular, we are interested in models for the valuation of derivative securities. These skills are also important in order to communicate with other financial professionals and to critically evaluate modern financial theories.

Summary

- Brownian motion
- The Itô integral and Itô's Formula
- Stochastic Differential Equations
- Girsanov's Theorem
- Stochastic models of security prices
- Introduction to the valuation of derivative securities
- The Binomial model
- The Black-Scholes model
- Models for the term structure of interest rates
- Credit risk models

Teaching and assessment methodologies

In classes, we shall discuss the syllabus topics in sequential order. However, we will point the multiple connections and relations between the topics and models. In classes, the syllabus topics will be presented and we shall stimulate critical discussion about the different models and their underlying financial theories. The students should read parts of the books of the main bibliography and selected papers will be recommended for a deeper study of a particular area.
Assessment: The final grade is awarded on the basis of a written exam .

Main bibliography

- Institute and Faculty of Actuaries, *Subject CT8 Financial Economics Core Technical Core Reading for the 2017 exams*, Institute and Faculty of Actuaries, 2016.
- Guerra, J. (2013), *Stochastic Calculus for Models in Finance*, Lecture Notes, ISEG, 2013.
- Hull, J. (2008) *Options, futures and other derivatives*, 7th ed., Prentice Hall.
- Mikosch, T. (1998), *Elementary Stochastic Calculus with Finance in view*, World Scientific.
- Oksendal, B. (2003), *Stochastic Differential Equations: An Introduction with Applications*, 6th edition, Springer.
- Björk, Tomas (2004), *Arbitrage Theory in Continuous Time*, second edition, Oxford University Press.

PROGRAMME

1. STOCHASTIC CALCULUS
    1.1. The Brownian motion
        1.1.1. Definition
        1.1.2. Main properties of the Brownian motion
        1.1.3. The geometric Brownian motion
        1.1.4. Martingales in discrete and in continuous time
    1.2. The Itô integral
        1.2.1. The Itô integral for deterministic functions
        1.2.2. The Itô integral for simple processes
        1.2.3. Main properties of the Itô integral
        1.2.4. The Itô integral for adapted and square-integrable processes
    1.3. Itô's Formula
        1.3.1. The one dimensional Itô formula or Itô lemma. Examples of application.
        1.3.2. The multidimensional Itô formula
        1.3.3. The martingale representation theorem
    1.4. Stochastic Differential Equations
        1.4.1. Itô processes and diffusions
        1.4.2. The existence and uniqueness theorem
        1.4.3. The geometric Brownian motion and mean reverting processes
        1.4.4. The Ornstein-Uhlenbeck process
    1.5. The Girsanov Theorem
        1.5.1. Change of probability measures
        1.5.2. The Girsanov Theorem

2. STOCHASTIC MODELS OF SECURITY PRICES
    2.1. The properties of the lognormal distribution and the lognormal model
    2.2. Empirical tests of the lognormal model
    2.3. Time series "cross-sectional" and longitudinal properties

3. VALUATION OF DERIVATIVE SECURITIES
    3.1. Introduction to the valuation of derivative securities
        3.1.1. Arbitrage and complete markets
        3.1.2. Factors that affect option prices
        3.1.3. Forward and futures contracts
        3.1.4. European and American options
        3.1.5. Bounds for options prices
        3.1.6. The Put-Call parity and arbitrage opportunities
    3.2. The Binomial model
        3.2.1 The Binomial model with one time step
        3.2.2 The Binomial model with two time steps
        3.2.3 The Binomial model with n time steps

- 3.2.4 The risk-neutral pricing measure for a binomial lattice and the risk-neutral pricing approach
- 3.2.5 The binomial trees recombination
- 3.2.6 How to calculate the value of European and American options using the binomial model
- 3.2.7 How to calibrate a binomial model
- 3.2.8 The state-price deflator approach to pricing

3.3. The Black-Scholes model

- 3.3.1. Application of the Girsanov theorem and of the martingale representation theorem in continuous time
- 3.3.2. The Black-Scholes model assumptions
- 3.3.3. The Black-Scholes PDE and the Black-Scholes formula
- 3.3.4. How to calculate the value of European options using the Black-Scholes option-pricing model
- 3.3.5. The Black-Scholes model for assets with dividends
- 3.3.6. The martingale method. The risk-neutral pricing and the equivalent martingale measure
- 3.3.7. How to control the risk using the Delta hedging
- 3.3.8. The Deflator approach and its equivalence to the risk-neutral pricing approach
- 3.3.9. The Greeks and their interpretation: Delta, Gamma, Vega, Rho, Lambda and Theta
- 3.3.10. Examples of some exotic options and their cash flow characteristics.

4. TERM STRUCTURE AND CREDIT RISK MODELS

4.1. Models for the term structure of interest rate

- 4.1.1. Introduction
- 4.1.2. Desirable characteristics of term-structure models
- 4.1.3. The risk-neutral approach to the pricing of zero-coupon bonds and interest-rate derivatives
- 4.1.4. The state-price deflators approach to the pricing of zero-coupon bonds and interest-rate derivatives
- 4.1.5. Features of the Vasicek bond price model
- 4.1.6. Features of the Cox-Ingersoll-Ross (CIR) bond price model
- 4.1.7. The Hull-White model
- 4.1.8. Limitations of one-factor models

4.2. Credit risk models

- 4.2.1 Introduction
- 4.2.2 Credit event and recovery rate
- 4.2.3 Structural models. Reduced form models and intensity based models
- 4.2.4 The Merton model
- 4.2.5 Two-state models for credit ratings with a constant transition intensity
- 4.2.6 The Jarrow-Lando-Turnbull model
- 4.2.7 Two-state models for credit ratings with stochastic transition intensity