Interest Rate & Credit Risk Models

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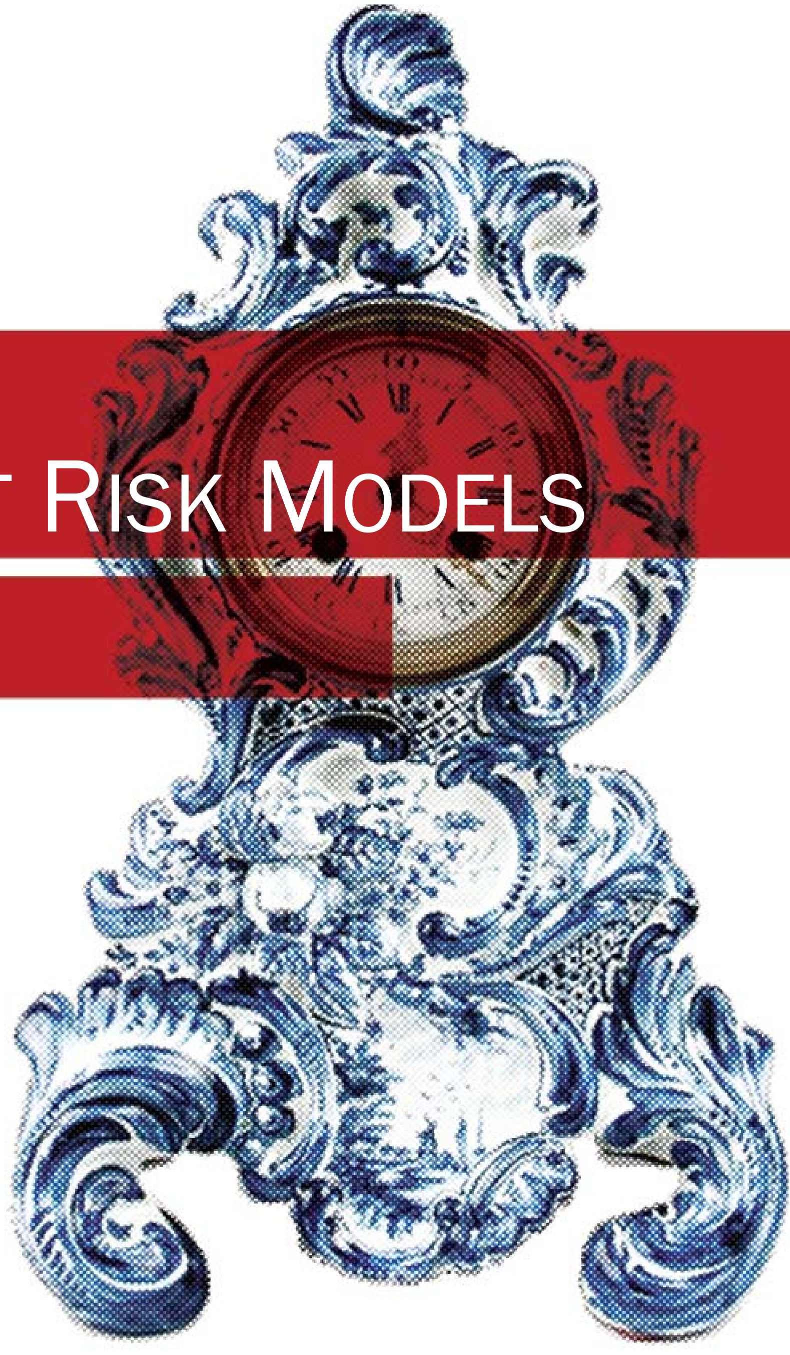

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OBJECTIVES

Main Goal

- Be able to evaluate and manage credit and interest rate risk in increasingly complex debt markets.

Interest Rate Theory

- Understand the distinction between different interest rates in the market: spot rates, forward rates, yield-to-maturity, etc.
- Identify under which conditions a stochastic interest rate model is needed and when a deterministic interest rate models is sufficient.
- Identify and apply the stochastic spot rate models, deriving the basic term structure properties from the spot rate.
- Handle forward rate models and understanding the fundamental difference the spot and the forward rate modeling approach.

OBJECTIVES

Credit Risk Theory

- Understand the approach of the structural models, in particular the Merton Model.
- Be familiar with main reduced form models.
- Understand the limitations of structural and reduced form models when used to model portfolio credit derivatives.
- Be able to incorporate correlated defaults.

PART I

FIXED INCOME MARKETS AND INTEREST RATE RISK

1.1 - DEFINITIONS

- Fixed-income market - the global financial market on which various medium to long term fixed interest rate instruments, such as bonds, swaps, FRAs, swaptions and caps are traded.

- Several fixed-income markets operate => many concepts of interest rates have been developed.

- Interest Rate Risk - changes in the net present value (the price) of a stream of future cash flows resulting from changes in interest rates.

- Management of interest rate risk - pricing and hedging of interest rate products and balance sheets.

INTEREST RATE RISK IN BANKS BALANCE SHEETS

- In banks, the goal is to measure the sensitivity of the balance sheet and the P&L to interest rate shifts.

- 2 types of interest rate risk:

  - Risk of Net Interest Income fluctuation

  - Risk of optionality embedded in assets and liabilities, e.g. prepayment of loans and early redemption of deposits, impacting on cash-flows.

- Target variables:

  - Net Interest Income: captures cash-flows in a given period (e.g. 1 year)

  - NPV of assets minus liabilities: allows to capture all balance sheet cash-flows, usually through duration.

- Earnings-at-risk (EaR):

  - Impact on earnings (NI or EV) from several very unfavorable scenarios for interest rates.

INTEREST RATE RISK IN BANKS BALANCE SHEETS

- Sources of interest rate risk:
  - liquidity flows:
    - Direct – new loans, issued debt or deposits received
    - Indirect - prepayments, early redemptions
  - repricing of existing assets and liabilities
- Measurement:
  - interest rate or repricing gaps - corresponding to the differences between the assets and the liabilities to be repriced in different time buckets (usually up to 1 year), excluding non-interest rate bearing balance sheet items (e.g. fixed assets and capital, even though capital may be considered as a fixed rate liability). As in liquidity risk, these gaps may be static or dynamic.
  - difference between average repricing term of assets and liabilities (with fixed rates, corresponds to the differences between residual maturities).

INTEREST RATE RISK IN BANKS BALANCE SHEETS

- EBA guidelines:

- FIs must measure their exposure to IRR in the banking book, in terms of both potential changes to economic value (EV), and changes to expected NII or earnings, considering:
  - different scenarios for potential changes in the level and shape of the yield curve, and to changes in the relationship between different market rates (i.e. basis risk);
  - assumptions made on non-interest bearing assets and liabilities of the banking book (including capital and reserves);
  - assumptions made on customer behaviour for 'non-maturity deposits' (i.e. the maturity assumed for liabilities with short contractual maturity but long behavioural maturity);
  - behavioural and automatic optionality embedded in assets or liabilities, considering:
    (a) impacts on current and future loan prepayment speeds from the underlying economic environment, interest rates and competitor activity;
    (b) the speed/elasticity of adjustment of product rates to changes in market interest rates; and
    (c) the migration of balances between product types, due to changes in their features.

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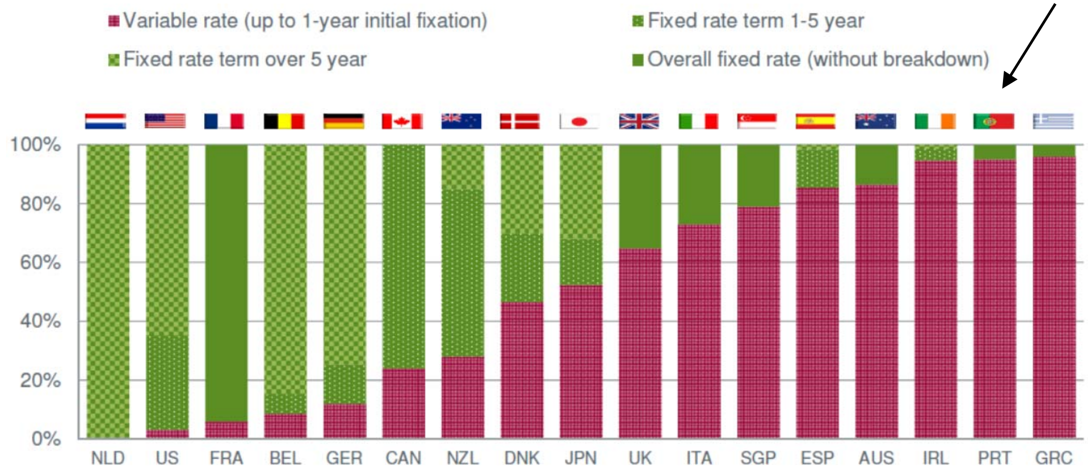
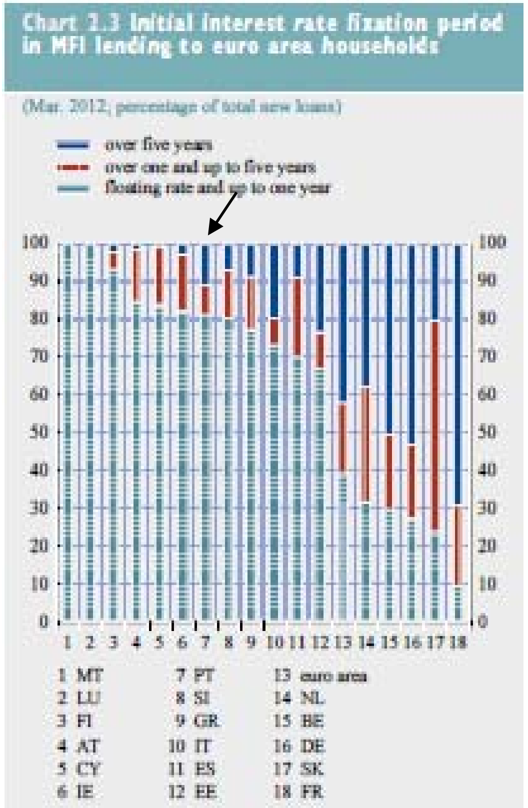
INTEREST RATE RISK IN BANKS BALANCE SHEETS



Source: Fitch (2016), 2016 Fitch Credit Outlook Conference, Lisbon, 28 Jan.



Source: European Central Bank (2012), "Financial Stability Review", June.

INTEREST RATE RISK IN BANKS BALANCE SHEETS

- Hedging of gaps is done through the spot market, forward/futures, options or swaps, as well as by changing the pricing structure of balance sheet.
- Portuguese banks usually have positive interest rate gaps, as credit rates are mostly indexed to money market rates (e.g. Euribor), while among liabilities only bonds issued are usually indexed, given that term deposits are typically short term liabilities (though may be renewed) and their interest rates are typically fixed by the bank.
- Therefore, short term interest rate decreases are, *ceteris paribus*, unfavorable to banks (as long as repricing gaps are shorter for assets than for liabilities).
- However, we must also bear in mind that higher interest rates may reduce credit risk.

INTEREST RATE RISK IN BANKS BALANCE SHEETS

- **Interest rate risk** management in FIs creates a demand for mathematical models:

  - Calculation of durations/modified durations for bonds
  - Pricing of exotic options
  - Prepayment models for loans and deposits in banks
  - Behavior models for key balance sheet items, namely as a function of interest rates
  - Estimation of the Term Structure of Interest Rates

1.2 - From Bonds to Interest Rates

- **Definition of** a **zero-coupon (or discount) bond** of maturity *T:*

  - financial security paying the holder the principal (aka face or nominal value) at a single pre-specified date *T* in the future, i.e., with no intermediary cash-flows (coupons).

  - Assuming that the principal is 1 unit of cash:

$$\begin{aligned} p(t,T) &= \text{price, at } t\text{, of a } T\text{-bond.} \\ p(T,T) &= 1. \end{aligned}$$

- The price of the bond will tend to 1 along time (pull-to-par) and at the maturity date it will be 1.

Being t (current time) < S < T

At time t:

- Sell one S-bond

Issue 1 S-bond, receiving *p(t,S)*

- Buy exactly $p(t,S)/p(t,T)$ $T-$bonds

Q(T)

- Net investment at t: 0\$.

The amount raised is equal to the amount invested:
Q(S)*P(S)=Q(T)*P(T)
1*p(t,S)=[p(t,S)/p(t,T)]*p(t,T)
p(t,S)=p(t,S)

At time S:

- Pay 1\$

(the principal of the bond issued, may be funded again by the bond markets)

At time T:

- Collect $p(t,S)/p(t,T) \cdot 1\$$

(assuming it is a zero coupon bond, it only pays the principal)

Net Outcome

- The contract is made at t.
- An investment of 1 at time S has yielded $p(t,S)/p(t,T)$ at time T.
- The equivalent constant rates, R, are given as the solutions to

Continuous rate:

Continuously compounded return

$$e^{R\cdot(T-S)}\cdot 1 = \frac{p(t,S)}{p(t,T)}$$

The investment of 1 at time S (the amount that was paid at S) will allow to obtain the return p(t,S)/p(t,T)

Simple rate:

$$[1 + R\cdot(T-S)]\cdot 1 = \frac{p(t,S)}{p(t,T)}$$

- **However, as we are at *t* and *S* is a future moment (*S*>*t*), *R* is not a spot, but a forward rate, obtained solving the continuous rate formula in the previous slide, in order to *R*:**

1. The **forward rate for the period** $[S, T]$, **contracted at** t is defined by

$$R(t; S, T) = -\frac{\log p(t,T) - \log p(t,S)}{T - S}.$$

2. The **spot rate**, $R(S,T)$, for the period $[S,T]$ is defined by

$$R(S,T) = R(S; S, T).$$

3. **The instantaneous forward rate is the forward for a very short time to maturity, obtained by calculating the limit of the continuous rate formula in the previous slide, when S is very close to T:**

$$f(t,T) = -\frac{\partial \log p(t,T)}{\partial T} = \lim_{S \to T} R(t;S,T).$$

The forward rate equation in the previous slide corresponds to the symmetric of the first derivative, when S is very close to T

4. **The instantaneous short rate at t corresponds to the instantaneous forward when T is very close to t, i.e. when the maturity of the instantaneous forward is close to zero.**

$$r(t) = f(t,t).$$

Continuous and Simple Interest Rates

1. The **simple forward rate L(t;S,T) for the period** $[S,T]$, **contracted at** t is defined by

Solving the last equation of slide 14 in order to R

Simple rate:

$$[1 + R \cdot (T - S)] \cdot 1 = \frac{p(t,S)}{p(t,T)}$$

$$L(t;S,T) = \frac{1}{T-S} \cdot \frac{p(t,S) - p(t,T)}{p(t,T)}$$

2. The **simple spot rate,** $L(S,T)$, for the period $[S,T]$ is defined by

Given that p(S,S) =1

$$L(S,T) = \frac{1}{T-S} \cdot \frac{1 - p(S,T)}{p(S,T)}$$

CONTINUOUS AND SIMPLE INTEREST RATES

The **simple spot rate**, $L(T, T+\delta)$, for the period $[T, T+\delta]$ is given by

$$L = \frac{1}{\delta} \cdot \frac{1-p}{p}$$

As T-S in the last equation now becomes δ and *p(S,T)* becomes *p*

COUPON-BEARING BONDS

- Coupon rate is the stated interest rate on a security
  - It is called the coupon rate because in the past bondholders kept coupons that had to be presented at the payment agents for interest payments
  - It is referred to as an annual percentage of face value
  - It is often paid twice a year
- The **current yield** gives you a first idea of the return on a bond

$$y_c = \frac{c}{P}$$

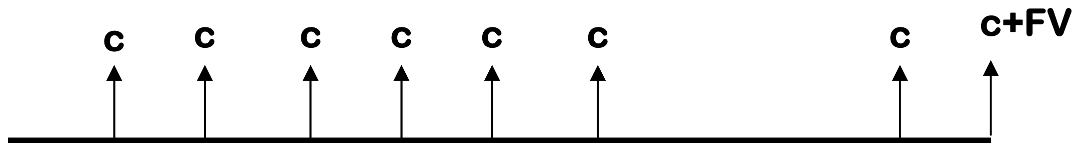
- Example
  - A $1,000 bond has a coupon rate of 7 percent
  - If you buy the bond for $900, your actual current yield is

$$y_c = \frac{70}{900} = 7.78\%$$

1.3 - YIELD-TO-MATURITY

- The **yield-to-maturity** (**YTM**) is the interest rate that makes the present value of the bond's payments equal to its price.

- YTM is the IRR of the cash-flows delivered by the bonds:
  - There is no closed formula to compute the YTM, given that it usually involves equations of order higher than 2.
  - However, YTM may be easily computed by iterative (trial-and-error) methodologies.
  - One of the conceptual problems of YTM is that each cash-flow is discounted using the same rate, implicitly assuming that the yield curve is flat.
  - Conversely, we may be discounting cash-flows of different bonds occurring at the same dates with different discount rates (yields).
  - The yield is equal to the coupon rate whenever the bond price corresponds to the redemption value.



- It is the solution to:
  - Simple YTM (annual)

$$P^c = \frac{FV}{(1+y)^T} + \sum_{n=1}^{T} \frac{c}{(1+y)^n}$$

(other)

$$P^c = \frac{FV}{\left(1+\frac{y}{m}\right)^{mT}} + \sum_{n=1}^{mT} \frac{c}{\left(1+\frac{y}{m}\right)^n}$$

  - Continuous Time YTM

$$P^c = FVe^{-yT} + \sum_{n=1}^{mT} ce^{-y\left(\frac{n}{m}\right)}$$

- Therefore, the YTM depends on:
  - The bond’s maturity
  - The bond’s coupon rate
- Actually, the yield curve has a less pronounced shape and curvature when the coupons are higher, as the higher weight of intermediate cash-flows reduces the effect of higher maturities in the discount process.