

**ANSWER EACH GROUP ON A SEPARATE SHEET OF PAPER.
ALWAYS USE 3 DECIMAL PLACES IN YOUR CALCULATIONS.**

GROUP I

1. Consider the information below on the distribution of firms by employment size classes.

Table: Distribution of firms by employment size

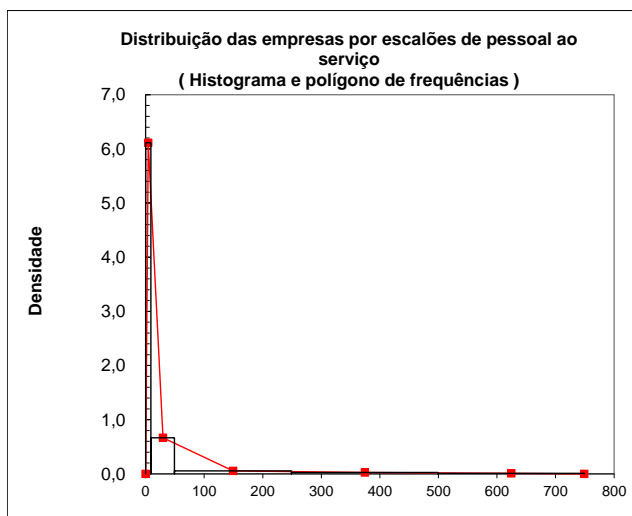
Employment size classes	% firms
0-9	55
10-49	26
50-249	11
250-499	6
≥ 500	2

Source: Survey

- (1.50 val) a) Depict graphically the histogram and the frequency polygon.

Employment size classes	MP _j	% firms (f _j)	Cum f _j	a _j	h _j
0-9	4,5	55	55	9	6.111
10-49	29.5	26	81	39	0.667
50-249	149.5	11	92	199	0.055
250-499	374.5	6	98	249	0.024
≥ 500	624.5	2	100	249	0.008

Histogram and frequency polygon



(1.25 val) b) Compute the mean and median values of the distribution.

Mean

$$\bar{X}^* = \frac{F_1 C_1 + \dots + F_m C_m}{n} = \sum_{j=1}^m f_j C_j = 61.55$$

Median:

$$Me = l_i(Me) + \frac{0,5 - cum f(Me-1)}{f(Me)} a(Me) = 0 + (0.5 - 0) / 0.55 * 9 = 8.18$$

(1.25 val) c) Compute the standard deviation and the coefficient of variation of this distribution.

Employment size classes	fj(MPj-xbar)^2
0-9	1,790.086
10-49	267.073
50-249	850.872
250-499	5,876.262
>=500	63,380,254
Standard deviation	122.974
CV	1.998

(1.00 val) d) Based on the results obtained previously analyse and classify the asymmetry profile of the distribution.

The mean is strongly greater than the median, which suggests there is strong positive asymmetry and the distribution is skewed to the left. The shape of the histogram also indicates the distribution is skewed to the left. The majority of firms are very small size but a few firms are very large.

2. The distribution of the monthly wages (in euros) of the workers of a given firm is given below:

Table: Distribution of monthly wages

Monthly wages	Number of workers
500 - 1000	75
1000 - 3000	58
3000 - 5000	22
5000 - 8000	20
8000 - 10000	15
10000 - 20000	10

Source: HR department

(2.00 val) a) Analyse and discuss the degree of inequality of the distribution of wages in this firm.

Calculation of Gini Index:

Wages	fj	cum fj	Yj	yj	cum yj	cum fj- cum yj
500 – 1000	37.5	37.5	56250	8.33	8.33	29.17
1000 - 3000	29	66.5	116000	17.18	25.51	40.99
3000 - 5000	11	77.5	88000	13.03	38.54	38.96
5000 - 8000	10	87.5	130000	19.25	57.79	29.71
8000 - 10000	7.5	95	135000	19.99	77.79	17.21
10000 - 20000	5	100	150000	22.21	100.00	
Gini					0.43	

The GI is high and thus there is considerable inequality in the distribution of wages.

GROUP II

Consider the following information on the evolution of imports in Legoland:

Year	2005	2006	2007	2008	2009	2010	2011	2012
Rate of change (%)	0.8	1.4	2.4	0.0	-2.9	1.9	-1.6	-3.2

(1.0 val) a) Compute the chain index of imports for each year from 2005 to 2012.

	2005	2006	2007	2008	2009	2010	2011	2012
Chain index	100.8	101.4	102.4	100	97.1	101.9	98.4	96.8

(1.0 val) b) Given that the value of imports in 2004 was 160,000 M€, compute the value of imports in 2008.

$$\text{Imports 2008} = 160\,000 * 1.008 * 1.014 * 1.024 * 1 = 167\,462.830 \text{ M€}$$

(1.25 val) c) Compute the average annual growth rate of imports between 2005 and 2012.

$$r_{2012,2005} = (1.014 * 1.024 * 1 * 0.971 * 1.019 * 0.984 * 0.968)^{(1/7)} - 1 = -0.309\% \text{ per year}$$

(1.25 val) d) Compute the 2008 fixed base index of imports for each year from 2005 to 2012.

	2004	2005	2006	2007	2008	2009	2010	2011	2012
Fixed base index 2008=100	95.544	96.308	97.656	100.000	100.000	97.100	98.945	97.362	94,246

GROUP III

Consider the evolution of sales of a given firm in the table below. You also know that the value of sales in 2016, at current prices, was 3,350,024 €.

Table: Evolution of sales of a given firm

Year	2011	2012	2013	2014	2015	2016
Value of sales (Rate of change)	3.3	4.3	-2.1	0.4	2.5	2.8
Sales Price Index (2010=100)	101.2	103.8	104.5	104.9	105.8	106.9

Source: Commercial department

(0.75 val) a) Compute, for each year, the 2010 fixed base index of value of sales.

2010	2011	2012	2013	2014	2015	2016
100.000	103.300	107.742	105.479	105.901	108.549	111.588

(0.75 val) b) Compute, for each year, the value of sales at current prices.

2010	2011	2012	2013	2014	2015	2016
3,002,132.83	3,101,203.21	3,234,554.95	3,166,629.30	3,179,295.81	3,258,778.21	3,350,024.00

(1.50 val) c) Compute, for each year, the value of sales at constant prices of 2011.

2010	2011	2012	2013	2014	2015	2016
3,038,158.42	3,101,203.21	3,153,535.27	3,066,630.48	3,067,156.69	3,117,092.20	3,171,397.84

(1.50 val) d) Compute the value of sales in 2015 at 2010 prices and the value of sales in 2013 at 2016 prices.

Sales in 2015 at prices of 2010 = $3,258,778.21 / 1.058 = 3,080,131$

Sales in 2013 at prices of 2016 = $3,166,629.30 * (1.069 / 1.045) = 3,239,355.715$

(1.00 val) e) Assuming that in 2017 prices will increase by 1.4% and quantities sold will increase by 2.2%, compute the value of sales in that year at current prices and at prices of 2011.

Sales in 2017 at current prices = $3,350,024.00 * 1.014 * 1.022 = 3,471,656.671$

Sales in 2017 at prices of 2011 = $3,171,397.84 * 1.022 = 3,241,168.592$

GROUP IV

You were asked to analyse the correlation between credit card transactions (million euros) (Y) and GDP (million euros) (X) in a given country. In addition, you are given the following information relating to these two variables for the latest 12-year period:

Mean of X = 1,128.75	Mean of Y = 122.00
Variance of X = 6,525.52	Variance of Y = 51.50
Covariance between X and Y = 547.92	

(1.25 val) a) Using the information given compute and discuss the correlation coefficient between X and Y.

$$r_{yx} = S_{YX} / S_x * S_y$$

$$S_x = (6,525.52)^{0.5} = 80.781$$

$$S_y = (51.50)^{0.5} = 7.176$$

$$r_{yx} = 547.92 / (80.781 * 7.176) = 0.945$$

The coefficient of linear correlation is very strong and equal to 0.945 – this means there is strong linear association between these variables. The sign is positive because of the positive covariance.

(1.00 val) b) Compute the linear regression model for these two variables and discuss its relation with the coefficient of correlation calculated in part a).

$$b_1 = \frac{S_{YX}}{S_x^2} = 547.92 / 6,525.52 = 0.084$$

$$b_0 = \bar{Y} - b_1 \bar{X} = 122.00 - 0.084 * 1,128.75 = 27.185$$

$$Y = 27.185 + 0.084 * X$$

The slope of the regression line is b_1 and is positive. It indicates that a one unit increase in X is associated with an increase of b_1 in Y, other things remaining the same. It therefore also shows there is a positive linear association between X and Y. The intercept of the regression line is given by b_0 and indicates the value of Y when X is equal to zero.

(0.75 val) c) Explain the difference between the information provided by the values obtained for the coefficient of correlation and the covariance.

The coefficient of correlation takes values ranging between 1 and -1. The closer the value is to 1/-1 the stronger the degree of positive/negative linear association. The closer the value is to 0, the weaker the degree of linear correlation between any two variables. Therefore, the interpretation of the coefficient of correlation is very straightforward. Furthermore, because it is a unit free measure (because it normalizes the covariance between X and Y by the product of the standard deviation of X and the standard deviation of Y), it can be used to compare the degree of linear correlation in a direct way for any pair of variables. On the contrary, the covariance between X and Y is scale dependent and cannot be easily interpreted nor used to compare the degree of linear correlation between different pairs of variables.