Solutions to Problems - Part 1

1.

(a)

$$dX_t = \mu dt + \sigma dB_t$$

(b)

$$dS_t = \left(\mu + \frac{1}{2}\sigma^2\right) S_t dt + \sigma S_t dB_t$$

(c)

$$P\left[S_t < S_{t-1}\right] = ... = P\left[B_t - B_{t-1} < \frac{-\mu}{\sigma}\right] = ... = \Phi\left(-0.24\right) = 0.405$$

2.

(a) In the long term, $\mathbb{E}\left[\sigma_t\right] = 0.2$ and $Var\left[\sigma_t\right] = 0.002976$. The long term distribution is $N\left[0.2; 0.002976\right]$

(c) $\mu = 0.2$ and $\frac{\beta^2}{2\alpha} = Var\left[\sigma_t\right] = 0.002976$.

(d) The parameter values can be such that, in the long term, we have that correlation between times $t-1$ and t is the same for both models, i.e.,

$$corr\left[\sigma_{t-1}, \sigma_t\right]$$

is the same for both models.

(e)

$$dV_t = \left[-2\alpha V_t + 2\alpha\mu\sqrt{V_t} + \beta^2\right] dt + 2\beta\sqrt{V_t} dB_t.$$