Problems - Part 2

1. An investor model has decided to model the rate of wage inflation in year t with the AR(2) model

$$I_t = 0.02 + 0.3 \left(I_{t-1} - 0.02\right) + 0.09(I_{t-2} - 0.02) + 0.005Z_t,$$

where $Z_t \sim N\left(0,1\right).$ The current value of wage inflation is $I_t = 0.022$.

(a) State the key statistical properties and discuss the economic plausibility of this model.

(b) By considering $\begin{bmatrix} I_t \\ I_{t-1} \end{bmatrix}$, show that the model can be written in vector form to produce a Markov model.

(c) During the last year, the rate of wage inflation was such that $I_t = 1.05I_{t-1}.$Calculate the probability that it will increase by at least 5% again in the coming year. Comment on your answer.

2. A government in a developed country is convinced that in order to win a national election, all it needs to do is ensure that the annual rate of inflation is between 1% and 3% at the time of the national election. The central bank has informed that the government that the annual force of inflation I_t at each month t can be modelled by:

$$I_t = 0.95I_{t-1} + 0.001Z_t.$$

The current annual rate of inflation is 2.9%.

(a) Find the distribution of I_{12}.

(b) Assuming that the government and the central bank are correct, calculate the probability that the government will win the next election in one year's time.

(c) Explain, with reasons, whether inflation in this model is mean-reverting and give an example of another financial quantity that would normally be considered to be mean-reverting.