LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

Masters in Economics and Masters in Monetary and Financial Economics

Midterm Test – Solution Topics

Maximum duration: 1h30 — 10th of November of 2017

Question 1

(3 marks) A coach is to select one athlete to compete in biathlon. When choosing among any set of athletes, he always prefers the fastest runner and the sharpest shooter. Is his preference relation complete? Is it transitive? Explain.

A: The preference relation is not complete (if athlete A is faster but worse shooter B, A and B cannot be compared). It is transitive.

Question 2

Suppose preferences are represented by the Cobb-Douglas utility function, $u(x_1, x_2) = x_1^a x_2^b$, a, b > 0. Let p_1 be the price of good 1, let p_2 be the price of good 2, and let income be equal to y.

a. (0.5 marks) Compute this consumer's marginal rate of substitution (MRS). A: $MRS = ax_2/bx_1$.
b. (0.5 marks) Show that this consumer's preferences can also be represented by the function $u'(x_1, x_2) = a\ln x_1 + b \ln x_2$, a, b > 0. A: Both utility functions have the same MRS. In fact, $u'(x_1, x_2) = \ln[u (x_1, x_2)]$, which implies both utility functions represent the same preferences.
c. (3 marks) Derive the Hicksian (or compensated) demands for goods 1 and 2. A: Solve the expenditure minimization problem Min $p_1x_1+p_2x_2$ s.t $x_1^a x_2^b = u$ to obtain $x_1(p_1, p_2, u) = u^{1/(a+b)}(ap_2/bp_1)^{b/(a+b)}$ and $x_2(p_1, p_2, u) = u^{1/(a+b)}(bp_1/ap_2)^{a/(a+b)}$
d. (1 mark) Determine the expenditure function. A: Substituting the Hicksian demand curves in the expenditure function, we have $e (p_1, p_2, u) = u^{1/(a+b)} p_1^{a/(a+b)} p_2^{b/(a+b)} [(a/b)^{b/(a+b)} + (b/a)^{a/(a+b)}]$.
e. (1 mark) Determine the indirect utility function. A: Inverting the expenditure function, we obtain

$v (p_1, p_2, m) = m^{(a+b)} p_1^{-a} p_2^{-b} [(a/b)^{b/(a+b)} + (b/a)^{a/(a+b)}]^{-(a+b)}$.

Question 3

Suppose you have to pay 2 euros for a ticket to enter a competition. The prize is 19 euros and the probability that you win is 1/3. You have an expected utility function with u(x) = log(x) and your current wealth is 10 euros.

a. (3 marks) What is the certainty equivalent of this competition? A: The expected utility of the lottery is u(g) = 1/3ln(27) + 2/3ln(8) = ln(12). Since the certainty equivalent (CE) is such that u(CE) = u(g), the CE of this lottery is 12.
b. (1 mark) What is the risk premium? A:The expected value of the lottery is 1/3*27 + 2/3*8 = 43/3 = 14,33. Therefore, the risk premium is P= E(g) – CE = 2,33.
c. (2 marks) Should you enter the competition? A: Yes! The certainty equivalent of the lottery is 12, greater than the current wealth.

Question 4a

Suppose preferences are represented by the utility function $u(x_1, x_2) = \min\{2x_1, x_2\}$. Let p_1 be the price of good 1, let p_2 be the price of good 2, and let income be equal to y.

a. (1 mark) Compute this consumer's marginal rate of substitution (MRS). A: $MRS(x_1, x_2) = \infty$ when $x_2 > 2x_1$, $MRS(x_1, x_2) = 0$ when $x_2 > 2x_1$, and it is not defined when $x_2 = 2x_1$.

b. (3 marks) Derive the Marshalian demands for goods 1 and 2. A: Goods 1 and 2 are perfect complements. At the solution, one must have $2x^*_1 = x^*_2$ and $p_1 x^*_1 + p_2 x^*_2 = m$, so that $x_1(p_1, p_2, m) = m/(p_1+2p_2)$ and $x_2(p_1, p_2, m) = 2m/(p_1+2p_2)$.

c. (1 mark) Determine the indirect utility function. A: Substituting the Marshallian demand curves into the utility function, one obtains $v(p_1, p_2, m) = 2m/(p_1+2p_2)$.

Question 4b

Consider a firm whose technology is $f(x_1, x_2) = \min\{x_1, 3x_2\}$. Let w_1 be the price of input 1 and let w_2 be the price of input 2.

a. (3 marks) Determine the conditional input demands. A: When solving the cost minimization problem Min $w_1x_1+w_2x_2$ s.t $\min\{x_1, 3x_2\} = y$, one must have $x^*_1 = 3x^*_2$ and $\min\{x^*_1, 3x^*_2\} = y$, so that $x_1(w_1, w_2, y) = y$ and $x_2(w_1, w_2, y) = y/3$.

b. (1 mark) Compute the cost function. A: $c(w_1, w_2, y) = y(w_1 + w_2/3)$.

a. (1 mark) Check whether the cost function is homogeneous in the prices of inputs. A: $c(tw_1, tw_2, y) = y(tw_1 + tw_2/3) = ty(w_1 + w_2/3) = tc(w_1, w_2, y)$. The function is homogeneous of degree 1 in prices.