LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

January 19, 2018

2 hours

Group 1

Consider a consumer whose utility function is $u(x_1, x_2) = x_1 + 2x_2$, where x_1 represents the quantity of good 1 and x_2 represents the quantity of good 2.

a) (0.5 marks) Formulate the consumer choice problem.
b) (2.5 marks) Find this consumer's demand for goods 1 and 2.
c) (1 mark) Determine the indirect utility function.
d) (1 mark) Determine the expenditure function.

Group 2

Consider an expected-utility maximiser with utility from wealth $u(w)$ where w is wealth. The agent has initial wealth w_0 and incurs a loss L with probability π; wealth does not change with probability $1 - \pi$. The agent can buy actuarially fair insurance, that is by paying an insurance premium πK, the agent will receive from the insurance company a compensation K if the loss occurs. Show that a risk-averse agent will buy full insurance, and that a risk-loving agent will buy no insurance. Give an intuition for these results.

Group 3

1. (3 marks) In a perfectly competitive market, a firm has a Leontief production function of the form $y = \text{Min}\{\alpha x_1, \beta x_2\}$, with $\alpha > 0$ and $\beta > 0$.

a) Carefully sketch the isoquant map for this technology.
b) Calculate the cost function and conditional input demands.

2. (2 marks) Show that a two-input production function may exhibit decreasing marginal returns and increasing returns to scale.

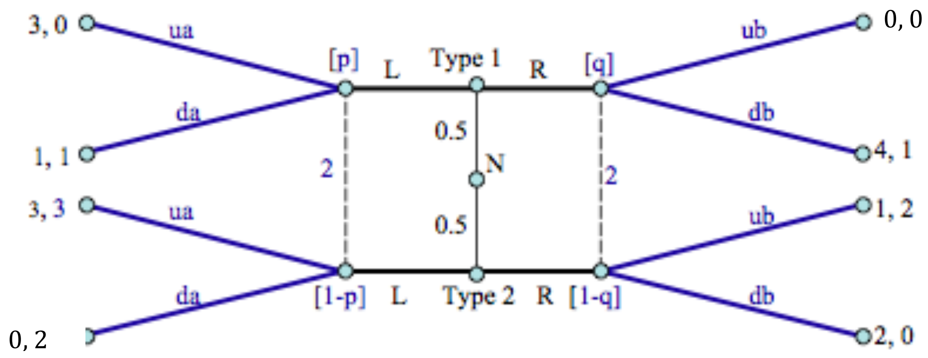
Group 4

1. (2 marks) Assume all firms in a competitive market share the same constant returns to scale technology. Explain why, in this case, the long-run equilibrium number of firms is indeterminate.

2. (3 marks) In a Cournot duopoly ($J = 2$), let each duopolist have constant average and marginal costs, but suppose that marginal costs are such that $0 \leq c1 < c2$. Show that firm 1 will have greater profits and produce a greater share of market output than firm 2 in the Nash equilibrium.

Group 5

Compute the weak perfect Bayesian Nash equilibria of the following game.



Group 6

Consider the following N player game: "Choose a number between 1 and 100. The winner is the person whose number is closest to 1/3 times the average of all chosen numbers and wins €100, whereas the others get nothing. The €100 prize is split evenly if there are ties."

a) (1 mark) Are there any strictly dominated strategies in this game? Explain.

b) (2 marks) Explain why choosing the number 33 weakly dominates choosing a higher number.

c) (2 marks) What is the unique strategy that survives iterative elimination of dominated strategies? Explain.

SOLUTIONS

Group 1

a) Choose x_1 and x_2 to Max $x_1 + 2x_2$ s.t. $p_1x_1 + p_2x_2 \leq m$, $x_1, x_2 \geq 0$.

b) Solving the problem above using the Kuhn-Tucker conditions, one obtains:

$x(p_1, p_2, m) = (m/p_1, 0)$ if $p_1 < p_2/2$; $x(p_1, p_2, m) = (0, m/p_2)$ if $p_1 > p_2/2$; $x(p_1, p_2, m) = (k, (m - p_1k)/p_2)$ s.t. $0 \leq k \leq m/p_1$.

c) $v(p_1, p_2, m) = m/p_1$ if $p_1 \leq p_2/2$; $v(p_1, p_2, m) = 2m/p_2$ if $p_1 > p_2/2$.

d) $e(p_1, p_2, u) = up_1$ if $p_1 \leq p_2/2$; $e(p_1, p_2, u) = mp_2/2$ if $p_1 > p_2/2$.

Group 2

The agent solves the following problem: choose $0 \leq K \leq L$ to Max $\pi u(w_0 - L + K - \pi K) + (1-\pi)u(w_0 - \pi K)$. The derivative of the objective function is:

$\pi(1-\pi)[u'(w_0 - L + K - \pi K) - u'(w_0 - \pi K)]$

If the agent is risk-averse ($u''<0$), since $w_0 - L + K - \pi K < w_0 - \pi K$ for any $K < L$, we have $u'(w_0 - L + K - \pi K) - u'(w_0 - \pi K) > 0$ for any $K < L$. Then, $K = L$ (full insurance) is the solution.

If the agent is risk-loving ($u''>0$), since we have $w_0 - L + K - \pi K < w_0 - \pi K$ for any $K < L$, we have $u'(w_0 - L + K - \pi K) - u'(w_0 - \pi K) \leq 0$ for any $L \geq K \geq 0$. Then, $K = 0$ (no insurance) is the solution.

Group 3

a) The isoquants have straight angles (over the line $x_2 = \alpha/\beta x_1$).

b) Since the two inputs are perfect complements, we must have $\alpha x_1^* = \beta x_2^*$. Then, $\text{Min}\{\alpha x_1, \beta x_2\} = \alpha x_1 = \beta x_2$ and $x_1(w_1, w_2, y) = y/\alpha$ and $x_2(w_1, w_2, y) = y/\beta$.
Then, $c(w_1, w_2, y) = w_1 y/\alpha + w_2 y/\beta$.

2. For example, the production function $f(k,l) = k^{0,5}l^{0,8}$ has decreasing marginal returns for both inputs, but increasing returns to scale since $f(tk,tl) = t^{1,3}k^{0,5}l^{0,8} > tf(k,l)$.

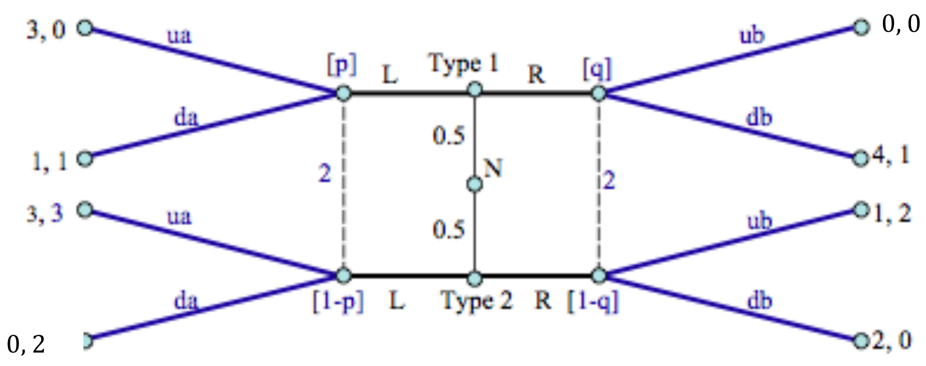
Group 4

1. With constant returns to scale, the only possibility for there to be a long run equilibrium is for prices to be such that maximum profit is 0 for any level of production. And, in these conditions, one may have a large number of firms producing a small number of units or a small number of firms producing a lot. All of this is compatible with equilibrium.

2. Solve the Cournot problem: Firm i chooses qi to Max P(q1+q2)qi – ciqi. Considering the first-order conditions for both firms, one can conclude that qi* decreasing in ci.

Group 5

Compute the weak perfect Bayesian Nash equilibria of the following game.



Equilibria are: [LR, daub, p=1, q=0], [RL, uadb, p=0, q=1], [LL, uaub, p=0,5, q≤2/3].

Group 6

Consider the following N player game: "Choose a number between 1 and 100. The winner is the person whose number is closest to 1/3 times the average of all chosen numbers and wins €100, whereas the others get nothing. The €100 prize is split evenly if there are ties."

a) Yes, there are strictly dominated strategies. For example, 100 is strictly dominated by 99.

b) Since the maximum number that can be chosen is 100, the maximum average is 33,33 and the closest strategy to this number is 33. It follows that for any choice of numbers of the other players, 33 always gives a payoff that is higher or equal than any number higher than 33.

c) 1. (Check the book!).