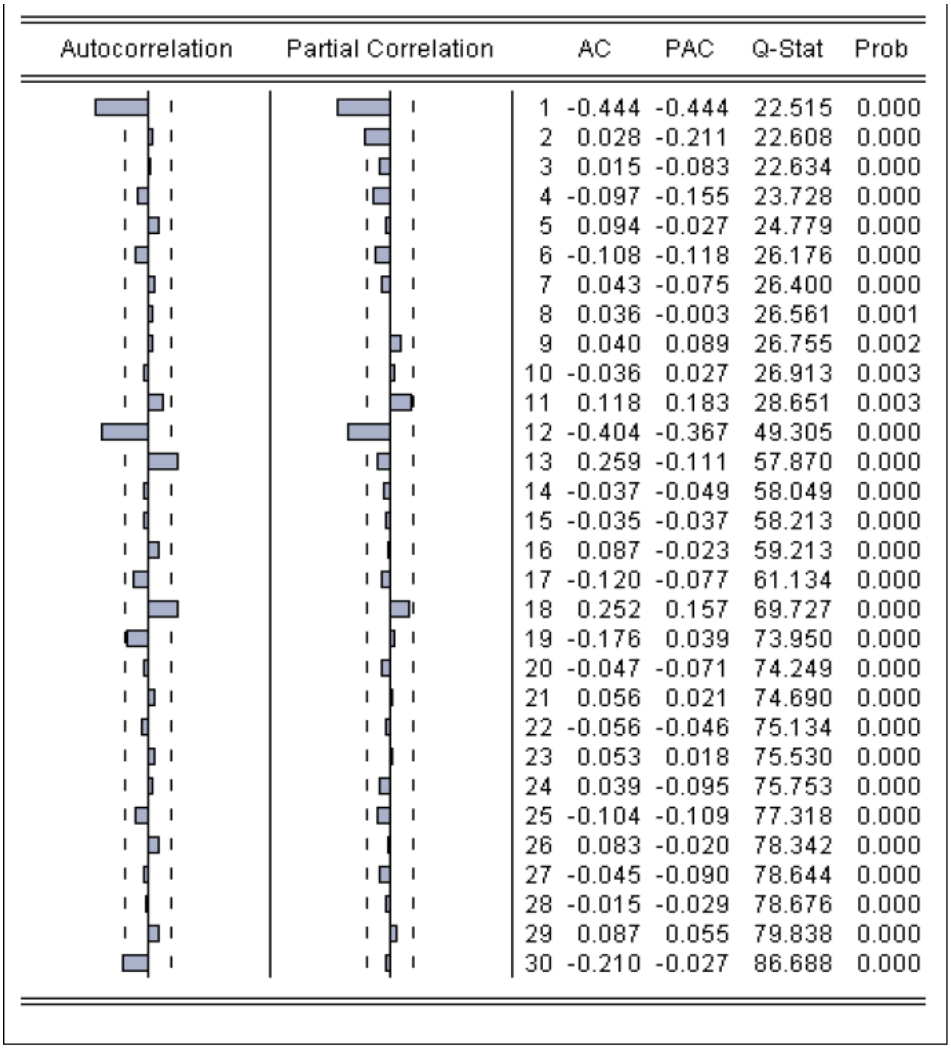
Week6:

11. Write the equation that defines a process $ARMA(0,1)(0,1)_{12}$ with parameters $\theta_{12} = 0.8$ and $\theta_1 = 0.6$ and find the ACF of the process.
12. Write the equation that defines a process $ARMA(1,0)(1,0)_4$ with parameters $\phi_4 = 0.8$ and $\phi_1 = 0.6$ and find the PACF of the process.
13. Suppose that you want to analyze a given time series data with the correlogram of Figure 1 . According to this information, what is the best model for this time series? Justify your answer.



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.444	-0.444	22.515	0.000
		2	0.028	-0.211	22.608	0.000
		3	0.015	-0.083	22.634	0.000
		4	-0.097	-0.155	23.728	0.000
		5	0.094	-0.027	24.779	0.000
		6	-0.108	-0.118	26.176	0.000
		7	0.043	-0.075	26.400	0.000
		8	0.036	-0.003	26.561	0.001
		9	0.040	0.089	26.755	0.002
		10	-0.036	0.027	26.913	0.003
		11	0.118	0.183	28.651	0.003
		12	-0.404	-0.367	49.305	0.000
		13	0.259	-0.111	57.870	0.000
		14	-0.037	-0.049	58.049	0.000
		15	-0.035	-0.037	58.213	0.000
		16	0.087	-0.023	59.213	0.000
		17	-0.120	-0.077	61.134	0.000
		18	0.252	0.157	69.727	0.000
		19	-0.176	0.039	73.950	0.000
		20	-0.047	-0.071	74.249	0.000
		21	0.056	0.021	74.690	0.000
		22	-0.056	-0.046	75.134	0.000
		23	0.053	0.018	75.530	0.000
		24	0.039	-0.095	75.753	0.000
		25	-0.104	-0.109	77.318	0.000
		26	0.083	-0.020	78.342	0.000
		27	-0.045	-0.090	78.644	0.000
		28	-0.015	-0.029	78.676	0.000
		29	0.087	0.055	79.838	0.000
		30	-0.210	-0.027	86.688	0.000

Week 7:

14. Are the following processes stationary/causal? Are the following processes invertible? Justify your answers. Consider that $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$.
    a. $y_t = \varepsilon_t + 0.8\,\varepsilon_{t-1} - \varepsilon_{t-2}$
    b. $y_t = 0.6\,y_{t-1} + 0.4y_{t-2} + \varepsilon_t$
    c. $y_t = (1 - 0.7L + 0.3L^2)\,\varepsilon_t$

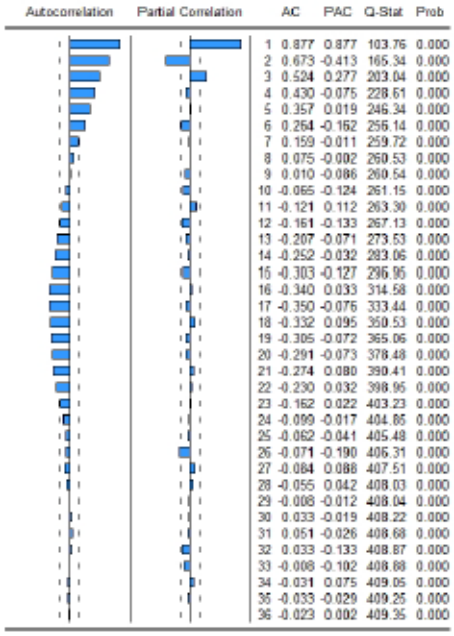
15. Consider the process ARMA(1,1) with $\phi = 0.8$ and $\theta = 0.5$ and with mean equal to 10.
    a. Formulate the equation that defines the process

Week 8:

16. For the following processes identify the orders of the autoregressive and moving average part and write the ARMA representation without the lag operator:
    i. $Y_t = (1 - 0.5L)\varepsilon_t$
    ii. $(1 + 0.8L)Y_t = (1 - 1.2L)\varepsilon_t$
    iii. $(1 - 0.7L + 0.4L^2)Y_t = (1 - 1.2L)\varepsilon_t$
    iv. $(1 + 0.8L)Y_t = (1 - 0.7L + 0.4L^2 + L^3)\varepsilon_t$
17. Consider the following models where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$:
    i. $Y_t = Y_{t-1} + \varepsilon_t - 1.5\varepsilon_{t-1}$
    ii. $Y_t = 0.8Y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$
    iii. $Y_t = 1.1Y_{t-1} + 0.8Y_{t-1} + \varepsilon_t - 1.7\varepsilon_{t-1} + 0.72\varepsilon_{t-2}$
    iv. $Y_t = 0.6Y_{t-1} + \varepsilon_t - 1.2\varepsilon_{t-1} + 0.2\varepsilon_{t-2}$

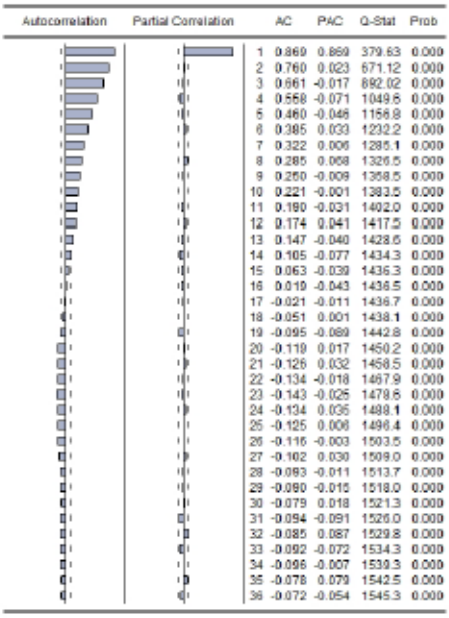
    a) Verify if Y_t is stationary and invertible.
    b) Characterize the behavior of the ACF and PACF.

18. In the following figure you may find the ACF and PACF of four time series.



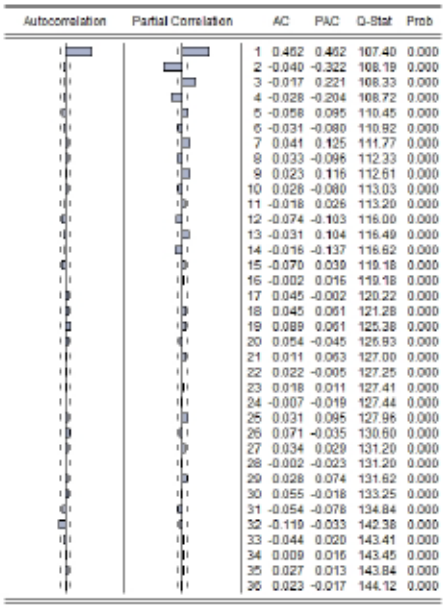
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.877	0.877	103.76	0.000
		2	0.673	-0.413	165.34	0.000
		3	0.524	0.277	203.04	0.000
		4	0.430	-0.075	228.61	0.000
		5	0.357	0.019	246.34	0.000
		6	0.264	-0.162	256.14	0.000
		7	0.159	-0.011	259.72	0.000
		8	0.075	-0.002	260.53	0.000
		9	0.010	-0.086	260.54	0.000
		10	-0.065	-0.124	261.15	0.000
		11	-0.121	0.112	263.30	0.000
		12	-0.161	-0.133	267.13	0.000
		13	-0.207	-0.071	273.53	0.000
		14	-0.252	-0.032	283.06	0.000
		15	-0.303	-0.127	296.95	0.000
		16	-0.340	0.033	314.58	0.000
		17	-0.350	-0.076	333.44	0.000
		18	-0.332	0.095	350.53	0.000
		19	-0.305	-0.072	365.06	0.000
		20	-0.291	-0.073	378.48	0.000
		21	-0.274	0.080	390.41	0.000
		22	-0.230	0.032	398.95	0.000
		23	-0.162	0.022	403.23	0.000
		24	-0.099	-0.017	404.85	0.000
		25	-0.062	-0.041	405.48	0.000
		26	-0.071	-0.190	406.31	0.000
		27	-0.084	0.088	407.51	0.000
		28	-0.055	0.042	408.03	0.000
		29	-0.008	-0.012	408.04	0.000
		30	0.033	-0.019	408.22	0.000
		31	0.051	-0.026	408.68	0.000
		32	0.033	-0.133	408.87	0.000
		33	-0.008	-0.102	408.88	0.000
		34	-0.031	0.075	409.05	0.000
		35	-0.033	-0.029	409.25	0.000
		36	-0.023	0.002	409.35	0.000

(a) Time series x_t



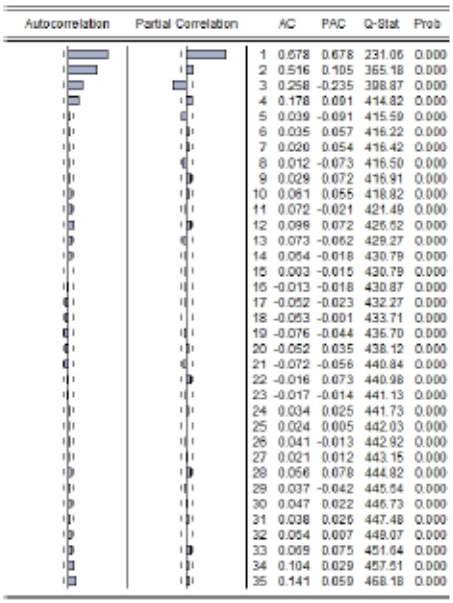
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.869	0.869	379.63	0.000
		2	0.760	0.023	671.12	0.000
		3	0.661	-0.017	892.02	0.000
		4	0.558	-0.071	1049.6	0.000
		5	0.460	-0.046	1156.8	0.000
		6	0.385	0.033	1232.2	0.000
		7	0.322	0.006	1285.1	0.000
		8	0.285	0.058	1326.5	0.000
		9	0.250	-0.009	1358.5	0.000
		10	0.221	-0.001	1383.5	0.000
		11	0.190	-0.031	1402.0	0.000
		12	0.174	0.041	1417.5	0.000
		13	0.147	-0.040	1428.6	0.000
		14	0.105	-0.077	1434.3	0.000
		15	0.063	-0.039	1436.3	0.000
		16	0.010	-0.043	1436.5	0.000
		17	-0.021	-0.011	1436.7	0.000
		18	-0.051	0.001	1438.1	0.000
		19	-0.095	-0.089	1442.8	0.000
		20	-0.119	0.017	1450.2	0.000
		21	-0.126	0.032	1458.5	0.000
		22	-0.134	-0.018	1467.9	0.000
		23	-0.143	-0.025	1478.6	0.000
		24	-0.134	0.035	1488.1	0.000
		25	-0.125	0.006	1496.4	0.000
		26	-0.116	-0.003	1503.5	0.000
		27	-0.102	0.030	1509.0	0.000
		28	-0.093	-0.011	1513.7	0.000
		29	-0.090	-0.015	1518.0	0.000
		30	-0.079	0.018	1521.3	0.000
		31	-0.094	-0.091	1526.0	0.000
		32	-0.085	0.087	1529.8	0.000
		33	-0.092	-0.072	1534.3	0.000
		34	-0.096	-0.007	1539.3	0.000
		35	-0.078	0.079	1542.5	0.000
		36	-0.072	-0.054	1545.3	0.000

(b) Time series y_t



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.462	0.462	107.40	0.000
		2	-0.040	-0.322	108.19	0.000
		3	-0.017	0.221	108.33	0.000
		4	-0.028	-0.204	108.72	0.000
		5	-0.058	0.095	110.45	0.000
		6	-0.031	-0.080	110.82	0.000
		7	0.041	0.125	111.77	0.000
		8	0.033	-0.096	112.33	0.000
		9	0.023	0.116	112.61	0.000
		10	0.026	-0.080	113.03	0.000
		11	-0.018	0.026	113.20	0.000
		12	-0.074	-0.103	116.00	0.000
		13	-0.031	0.104	116.49	0.000
		14	-0.016	-0.137	116.62	0.000
		15	-0.070	0.039	119.18	0.000
		16	-0.002	0.015	119.18	0.000
		17	0.045	-0.002	120.22	0.000
		18	0.045	0.061	121.28	0.000
		19	0.089	0.001	125.38	0.000
		20	0.054	-0.045	126.93	0.000
		21	0.011	0.063	127.00	0.000
		22	0.022	-0.005	127.25	0.000
		23	0.018	0.011	127.41	0.000
		24	-0.007	-0.019	127.44	0.000
		25	0.031	0.095	127.96	0.000
		26	0.071	-0.035	130.60	0.000
		27	0.034	0.029	131.20	0.000
		28	-0.002	-0.023	131.20	0.000
		29	0.028	0.074	131.62	0.000
		30	0.055	-0.018	133.25	0.000
		31	-0.054	-0.078	134.84	0.000
		32	-0.119	-0.033	142.38	0.000
		33	-0.044	0.020	143.41	0.000
		34	0.009	0.016	143.45	0.000
		35	0.027	0.013	143.84	0.000
		36	0.023	-0.017	144.12	0.000

(c) Time series w_t



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.678	0.678	231.06	0.000
		2	0.516	0.105	365.18	0.000
		3	0.258	-0.235	398.87	0.000
		4	0.178	0.091	414.82	0.000
		5	0.039	-0.091	415.59	0.000
		6	0.035	0.057	416.22	0.000
		7	0.020	0.054	416.42	0.000
		8	0.012	-0.073	416.50	0.000
		9	0.029	0.072	416.91	0.000
		10	0.061	0.055	418.82	0.000
		11	0.072	-0.021	421.49	0.000
		12	0.099	0.072	426.52	0.000
		13	0.073	-0.062	429.27	0.000
		14	0.054	-0.018	430.79	0.000
		15	0.003	-0.015	430.79	0.000
		16	-0.013	-0.018	430.87	0.000
		17	-0.052	-0.023	432.27	0.000
		18	-0.053	-0.001	433.71	0.000
		19	-0.076	-0.044	436.70	0.000
		20	-0.052	0.035	438.12	0.000
		21	-0.072	-0.056	440.84	0.000
		22	-0.016	0.073	440.98	0.000
		23	-0.017	-0.014	441.13	0.000
		24	0.034	0.025	441.73	0.000
		25	0.024	0.005	442.03	0.000
		26	0.041	-0.013	442.92	0.000
		27	0.021	0.012	443.15	0.000
		28	0.056	0.078	444.92	0.000
		29	0.037	-0.042	445.54	0.000
		30	0.047	0.022	446.73	0.000
		31	0.038	0.026	447.48	0.000
		32	0.054	0.007	449.07	0.000
		33	0.069	0.075	451.64	0.000
		34	0.104	0.029	457.51	0.000
		35	0.141	0.059	468.18	0.000

(d) Time series z_t

According to the previous figures identify an appropriate ARMA model for each series. Justify.

19. Consider the following estimation outputs for a fitted model on a prince index (IPI).

    b. Write the estimated model in equation form.

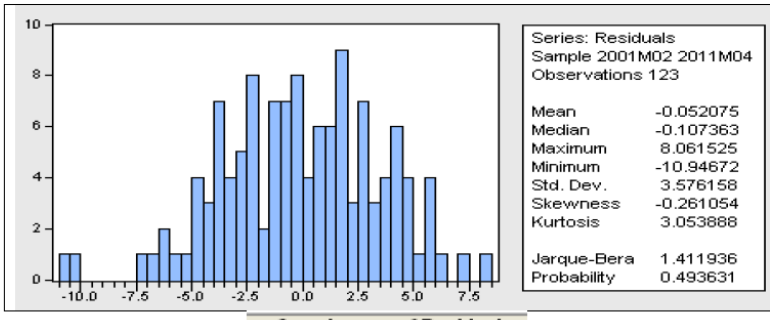
    c. Comment on the residuals distribution.

    d. Is the proposed model acceptable?

Dependent Variable: D(IPI,1,12)
Sample (adjusted): 2001M02 2011M04
Included observations: 123 after adjustments
MA Backcast: 2000M01 2001M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.740267	0.060517	-12.23246	0.0000
SMA(12)	-0.887638	0.023475	-37.81162	0.0000

R-squared	0.557799	Mean dependent var	-0.071479
Adjusted R-squared	0.554144	S.D. dependent var	5.378399
S.E. of regression	3.591289	Akaike info criterion	5.411026
Sum squared resid	1560.580	Schwarz criterion	5.456753
Log likelihood	-330.7781	Hannan-Quinn criter.	5.429600
Durbin-Watson stat	2.249783		



Correlogram of Residuals

Sample: 2001M02 2011M04
Included observations: 123
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.128	-0.128	2.0665	
		2	0.012	-0.005	2.0842	
		3	0.232	0.237	8.9770	0.003
		4	-0.162	-0.111	12.376	0.002
		5	0.013	-0.029	12.399	0.006
		6	0.156	0.119	15.601	0.004
		7	-0.139	-0.055	18.168	0.003
		8	0.032	-0.015	18.308	0.006
		9	0.194	0.162	23.366	0.001
		10	-0.147	-0.048	26.323	0.001
		11	0.084	0.023	27.287	0.001
		12	0.221	0.187	34.072	0.000
		13	-0.153	-0.030	37.332	0.000
		14	0.024	-0.083	37.411	0.000
		15	-0.051	-0.152	37.788	0.000
		16	-0.073	0.044	38.544	0.000
		17	-0.001	-0.051	38.544	0.001
		18	-0.033	-0.079	38.706	0.001
		19	-0.139	-0.100	41.557	0.001
		20	0.103	0.077	43.153	0.001
		21	0.033	0.040	43.312	0.001
		22	-0.193	-0.160	48.972	0.000
		23	0.218	0.158	56.288	0.000
		24	-0.204	-0.188	62.736	0.000
		25	-0.091	-0.069	64.034	0.000
		26	0.087	0.023	65.235	0.000
		27	-0.231	-0.078	73.750	0.000
		28	0.057	0.049	74.276	0.000
		29	0.004	-0.099	74.278	0.000
		30	-0.134	0.017	77.241	0.000