CHAPTER 13
FORECASTING VOLATILITY I

In ARMA models considered in this course we have assumed that volatility is constant

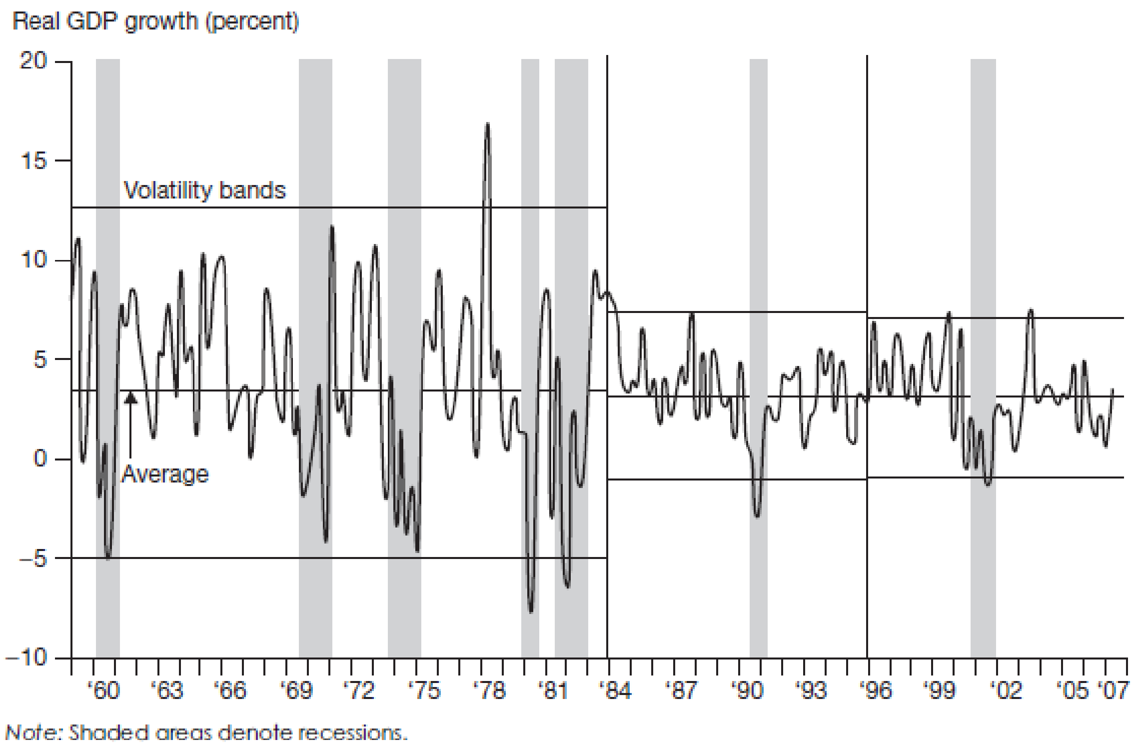


Figure 13.2 U.S. Real GDP with Volatility Bands

But there is evidence that for some financial series **volatility is time varying**

By **volatility** we mean variability: σ^2, σ, MAD or other

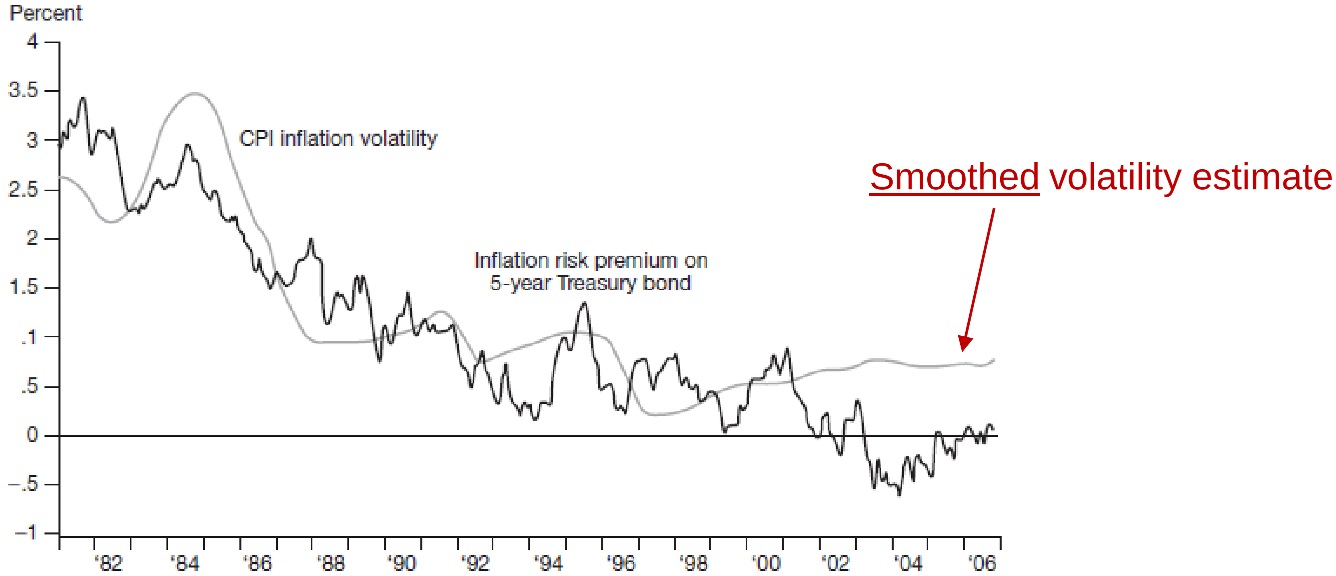


Figure 13.3 Volatility of Consumer Price Index Inflation

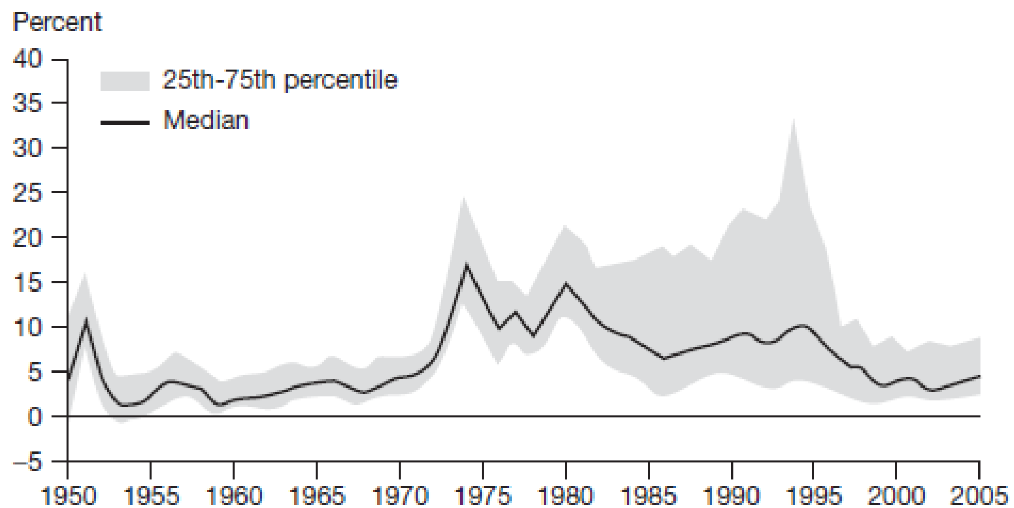


Figure 13.4 Global Inflation Rate

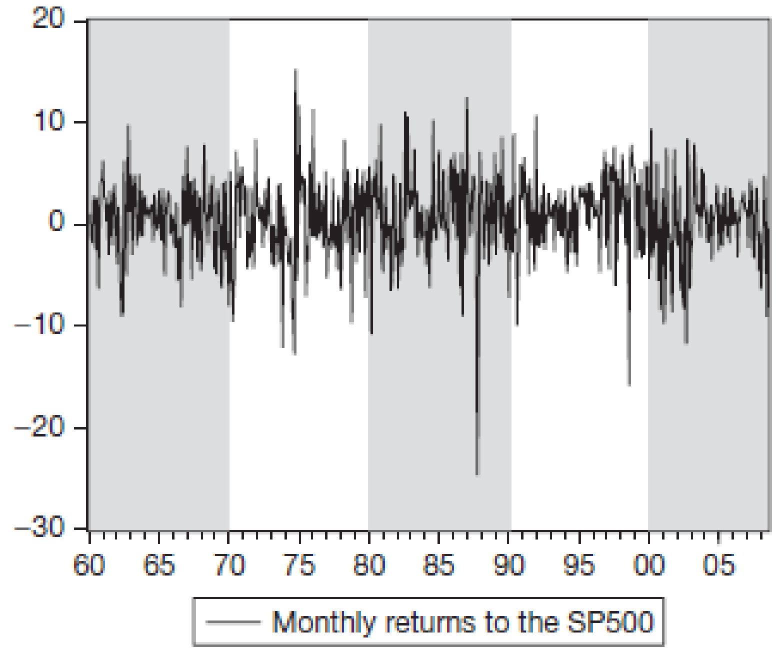


Figure 13.5 Monthly Returns of the SP500 Index

	1960–1969	1970–1979	1980–1989	1990–1999	2000–2008 (July)
Sample standard deviation	3.45%	4.59%	***4.79%***	3.88%	4.05%
Maximum **Minimum**	9.67% –8.99%	***15.10%*** –12.71%	12.38% ***–24.54%***	10.58% –15.76%	9.23% –11.65%
Range	18.66	27.81	***36.92***	26.34	20.88
Interquartile interval	(–1.62, 2.69)	(***–2.21***, 3.56)	(–1.42, ***3.96***)	(–1.00, 3.80)	(–2.03, 2.10)
Interquartile range	4.31	***5.77***	5.38	4.80	4.13

Table 13.1 Summary of the Dispersion in the Monthly Returns to the SP500

13.3 Is There Time Dependence in Volatility?

Figure 13.6 Time Series of SP500 Index, Yen/Dollar Exchange Rate, and 10-year Treasure Note Yield

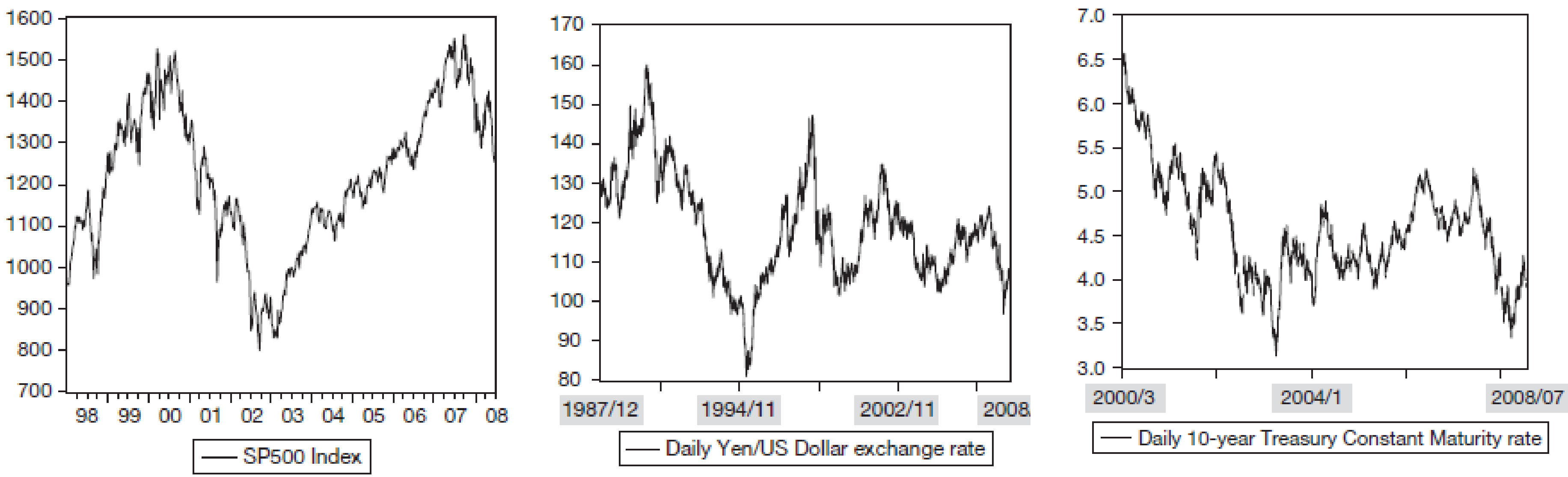
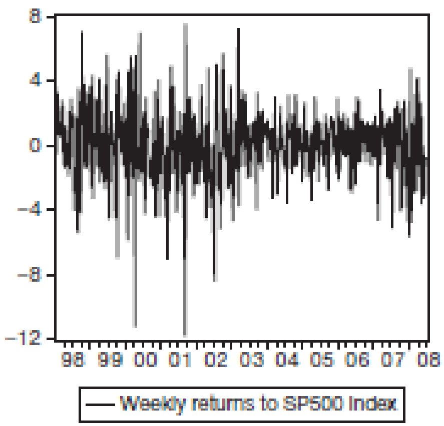


Table 13.2 Unit Root Testing: Value of the Dickey-Fuller Test

SP500 Index	Yen/Dollar exchange rate	10-year Treasury Note
–2.07 p-value = 0.25	–2.69 p-value = 0.08	–2.09 p-value = 0.24

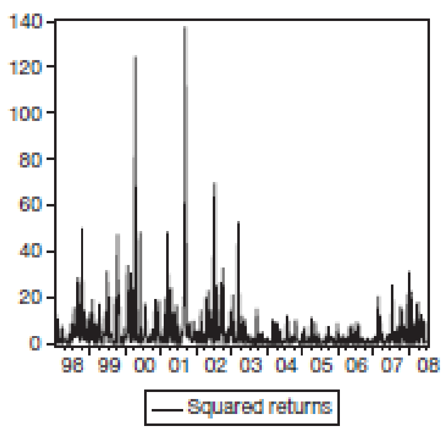
The time series are nonstationary -> we proceed to take the first difference of each series (in logs) to get the returns r_t

Figure 13.7 Transformations of Weekly Returns to the SP500 Index and Their Autocorrelations



Sample: 1/12/1998 7/07/2008
Included observations: 547

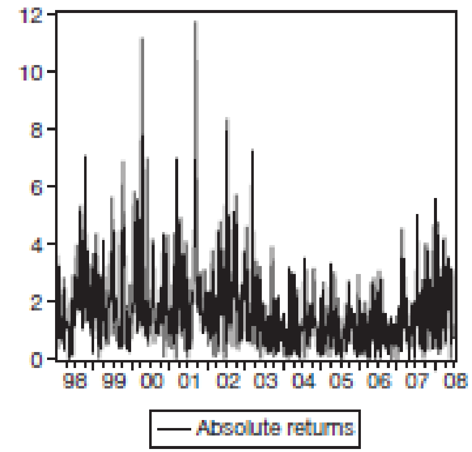
Autocorrection	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.077	-0.077	3.2986	0.069
		2	0.040	0.035	4.1955	0.123
		3	-0.024	-0.019	4.5169	0.211
		4	-0.039	-0.044	5.3473	0.253
		5	0.001	-0.003	5.3481	0.375
		6	0.048	0.051	6.6214	0.357
		7	-0.112	-0.107	13.568	0.059
		8	-0.009	-0.031	13.610	0.093
		9	-0.047	-0.039	14.823	0.096
		10	0.087	0.083	19.066	0.039
		11	0.058	0.065	20.964	0.034
		12	0.009	0.006	21.011	0.050



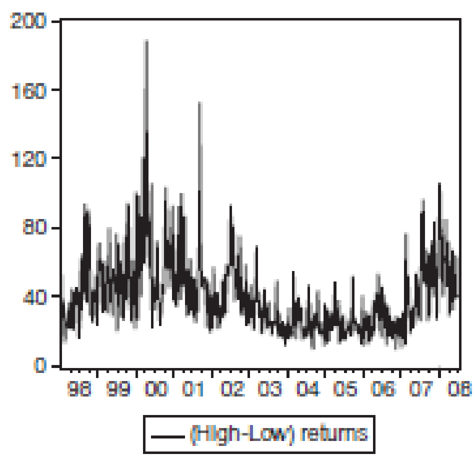
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.236	0.236	30.605	0.000
		2	0.058	0.002	32.441	0.000
		3	0.083	0.073	36.282	0.000
		4	0.065	0.030	38.590	0.000
		5	0.041	0.017	39.525	0.000
		6	0.062	0.045	41.644	0.000
		7	0.072	0.044	44.538	0.000
		8	0.002	-0.032	44.541	0.000
		9	0.031	0.030	45.083	0.000
		10	0.057	0.035	46.911	0.000
		11	0.068	0.045	49.486	0.000
		12	0.054	0.023	51.110	0.000

The series of returns does not display autocorrelation.
But the squared returns do.

The transformations r_t^2, $|r_t|$ and $r_t^{HI} - r_t^{LO}$ have positive autocorrelations.



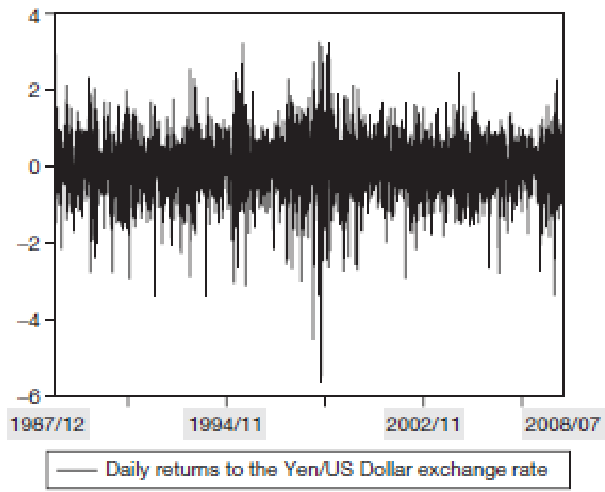
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.187	0.187	19.217	0.000
		2	0.109	0.077	25.738	0.000
		3	0.153	0.125	38.715	0.000
		4	0.138	0.088	49.233	0.000
		5	0.120	0.067	57.270	0.000
		6	0.071	0.011	60.066	0.000
		7	0.085	0.037	64.116	0.000
		8	0.067	0.012	66.581	0.000
		9	0.078	0.036	69.985	0.000
		10	0.111	0.068	76.830	0.000
		11	0.109	0.058	83.446	0.000
		12	0.098	0.042	88.885	0.000



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.589	0.589	190.89	0.000
		2	0.525	0.273	342.84	0.000
		3	0.510	0.206	486.51	0.000
		4	0.454	0.079	600.44	0.000
		5	0.451	0.111	713.29	0.000
		6	0.388	-0.006	796.94	0.000
		7	0.381	0.053	877.81	0.000
		8	0.364	0.031	951.46	0.000
		9	0.324	-0.003	1010.2	0.000
		10	0.383	0.137	1092.0	0.000
		11	0.328	-0.011	1152.5	0.000
		12	0.304	-0.007	1204.3	0.000

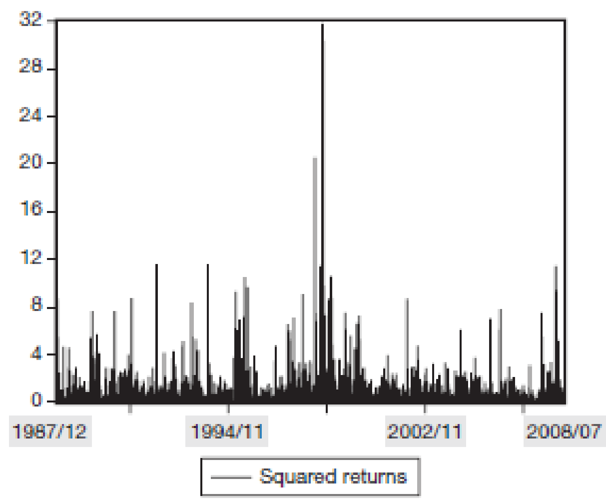
Figure 13.7 Transformations of Weekly Returns to the SP500 Index and Their Autocorrelations

Figure 13.8 Transformations of Daily Returns to Yen/Dollar Exchange Rate and Their Autocorrelations



Sample: 4251 9414
Included observations: 5164

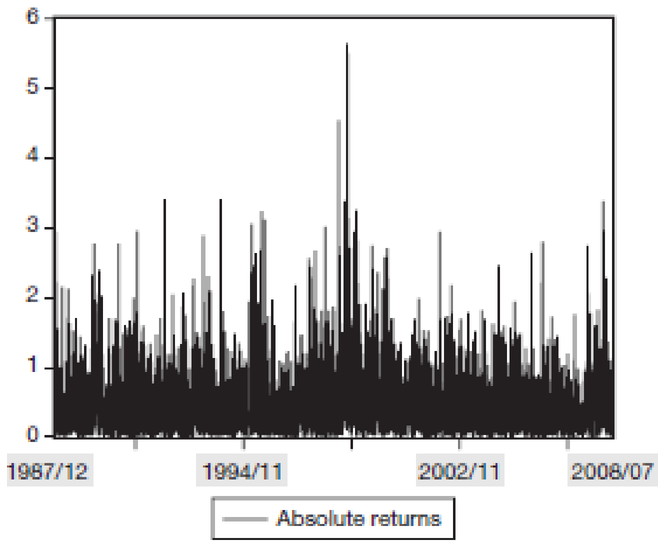
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.020	0.020	2.1653	0.141
		2	0.008	0.008	2.5119	0.285
		3	-0.027	-0.028	6.3895	0.094
		4	-0.001	0.000	6.3949	0.172
		5	0.004	0.005	6.4908	0.261
		6	-0.017	-0.018	7.9488	0.242
		7	0.018	0.019	9.6769	0.208
		8	-0.000	-0.001	9.6778	0.288
		9	0.004	0.003	9.7815	0.368
		10	0.032	0.033	15.050	0.130
		11	-0.008	-0.009	15.373	0.166
		12	0.016	0.015	16.641	0.164



Sample: 4251 9414
Included observations: 5164

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.198	0.198	201.84	0.000
		2	0.113	0.077	268.31	0.000
		3	0.088	0.054	307.91	0.000
		4	0.069	0.037	332.92	0.000
		5	0.103	0.076	387.32	0.000
		6	0.090	0.049	428.94	0.000
		7	0.090	0.049	470.50	0.000
		8	0.094	0.051	515.75	0.000
		9	0.089	0.044	556.38	0.000
		10	0.032	-0.019	561.60	0.000
		11	0.071	0.041	587.55	0.000
		12	0.041	-0.001	596.28	0.000

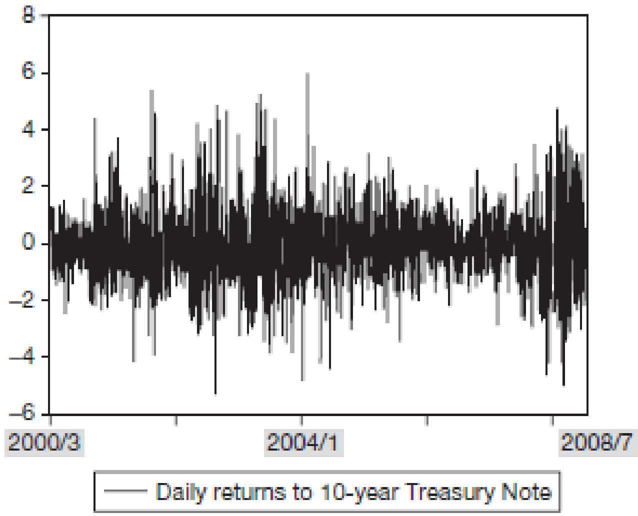
Figure 13.8 Transformations of Daily Returns to Yen/Dollar Exchange Rate and Their Autocorrelations



Sample: 4251 9414
Included observations: 5164

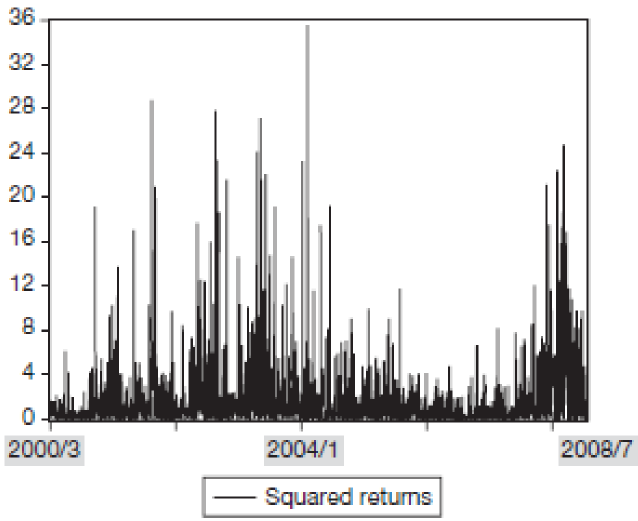
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.122	0.122	76.904	0.000
		2	0.107	0.093	135.78	0.000
		3	0.102	0.081	189.60	0.000
		4	0.104	0.077	245.41	0.000
		5	0.111	0.079	309.65	0.000
		6	0.100	0.061	361.14	0.000
		7	0.104	0.063	417.35	0.000
		8	0.087	0.040	456.12	0.000
		9	0.104	0.058	511.96	0.000
		10	0.067	0.015	535.09	0.000
		11	0.088	0.040	575.23	0.000
		12	0.065	0.013	596.87	0.000

Figure 13.9 Transformations of Daily Returns to the 10-Year Treasury Notes and Their Autocorrelations



Sample: 10000 12136
Included observations: 1960

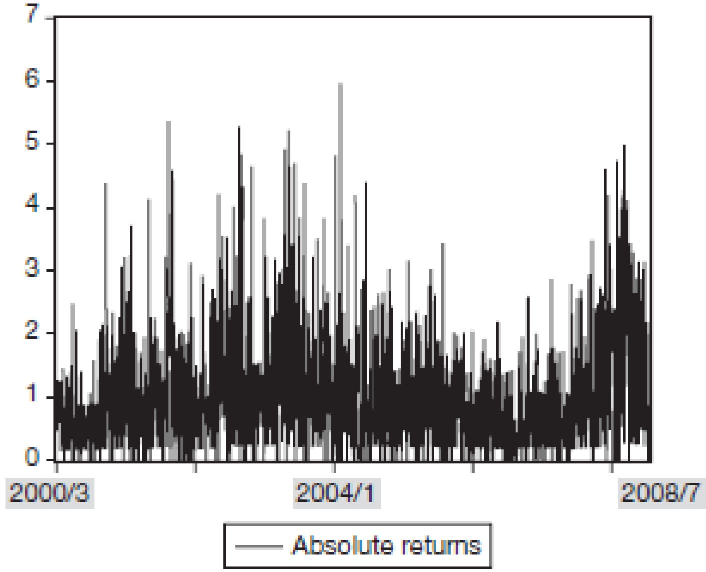
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.004	0.004	0.0262	0.872
		2	-0.062	-0.062	7.5887	0.022
		3	-0.020	-0.020	8.3849	0.039
		4	0.003	0.001	8.3979	0.078
		5	0.040	0.038	11.518	0.042
		6	0.025	0.024	12.710	0.048
		7	0.056	0.061	18.905	0.008
		8	-0.008	-0.004	19.031	0.015
		9	-0.035	-0.027	21.463	0.011
		10	-0.027	-0.028	22.902	0.011
		11	0.013	0.006	23.217	0.016
		12	0.037	0.027	25.850	0.011



Sample: 10000 12136
Included observations: 1960

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.089	0.089	15.563	0.000
		2	0.096	0.089	33.691	0.000
		3	0.067	0.052	42.535	0.000
		4	0.121	0.105	71.317	0.000
		5	0.186	0.163	139.06	0.000
		6	0.109	0.069	162.55	0.000
		7	0.111	0.068	186.85	0.000
		8	0.090	0.045	202.79	0.000
		9	0.126	0.072	234.20	0.000
		10	0.154	0.093	280.80	0.000
		11	0.129	0.067	313.52	0.000
		12	0.128	0.066	345.92	0.000

Figure 13.9 Transformations of Daily Returns to the 10-Year Treasury Notes and Their Autocorrelations



Sample: 10000 12136
Included observations: 1960

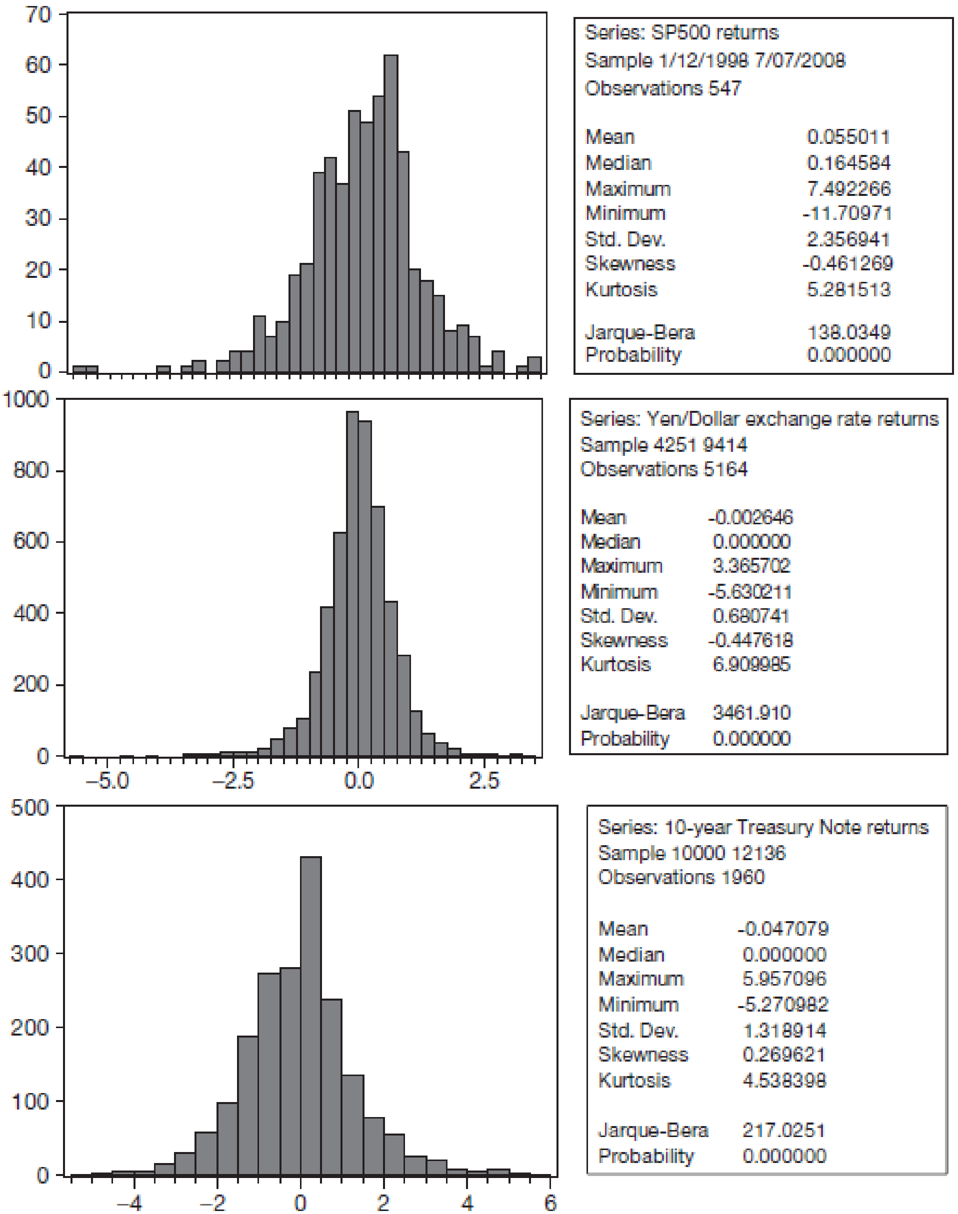
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.107	0.107	22.671	0.000
		2	0.131	0.121	56.344	0.000
		3	0.103	0.079	77.035	0.000
		4	0.128	0.099	109.23	0.000
		5	0.178	0.143	171.22	0.000
		6	0.132	0.082	205.62	0.000
		7	0.118	0.058	232.99	0.000
		8	0.126	0.067	264.32	0.000
		9	0.144	0.080	305.40	0.000
		10	0.177	0.107	367.31	0.000
		11	0.138	0.060	404.70	0.000
		12	0.142	0.063	444.49	0.000

Volatility is time varying and exhibits time dependence.

As Mandelbrot (1963) pointed out:
"large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes."

A phenomenon called "volatility clustering"

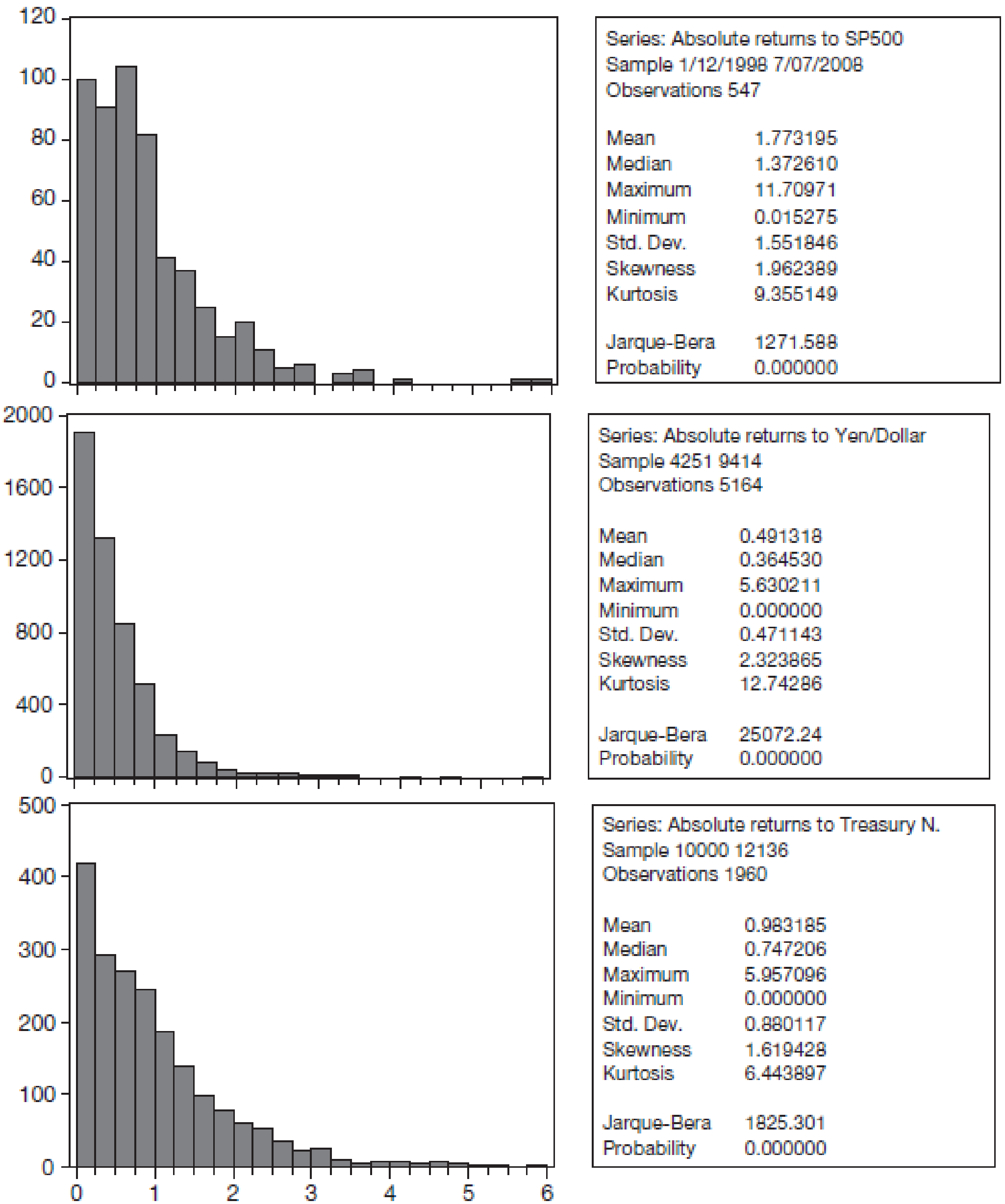
Figure 13.10 Unconditional Histograms of SP500, Exchange Rate, and 10-Year Treasury Returns



Not normally distributed:
skewness (asymmetry) ≠ 0
kurtosis > 3 (heavy tails)
Jarque-Bera test rejects Ho: Normal

Returns are leptokurtic: they have **fat tails**, i.e., extreme events occur with large probability

Figure 13.10 Unconditional Histograms of SP500,Exchange Rate,10-Year Treasury Returns (continued)



NOW MUCH MORE:

Not normally distributed:
skewness (asymmetry) ≠ 0
kurtosis > 3 (heavy tails)
Jarque-Bera test rejects Ho: Normal

Volatility is strongly skewed to the right

Stylized facts of financial time series:

• ***Nonstationarity:*** Stock prices, exchange rates, interest rates, etc., have stochastic trends - unit root behavior;

• ***Returns are uncorrelated:*** white noise behavior;

• ***Returns have time varying-volatility;***

• ***Volatility Clustering:*** large (small) movements tend to be followed by large (small) movements;

• **Returns are non normally distributed** (fat tails) : they are leptokutric with kurtosis coefficient larger than 3;

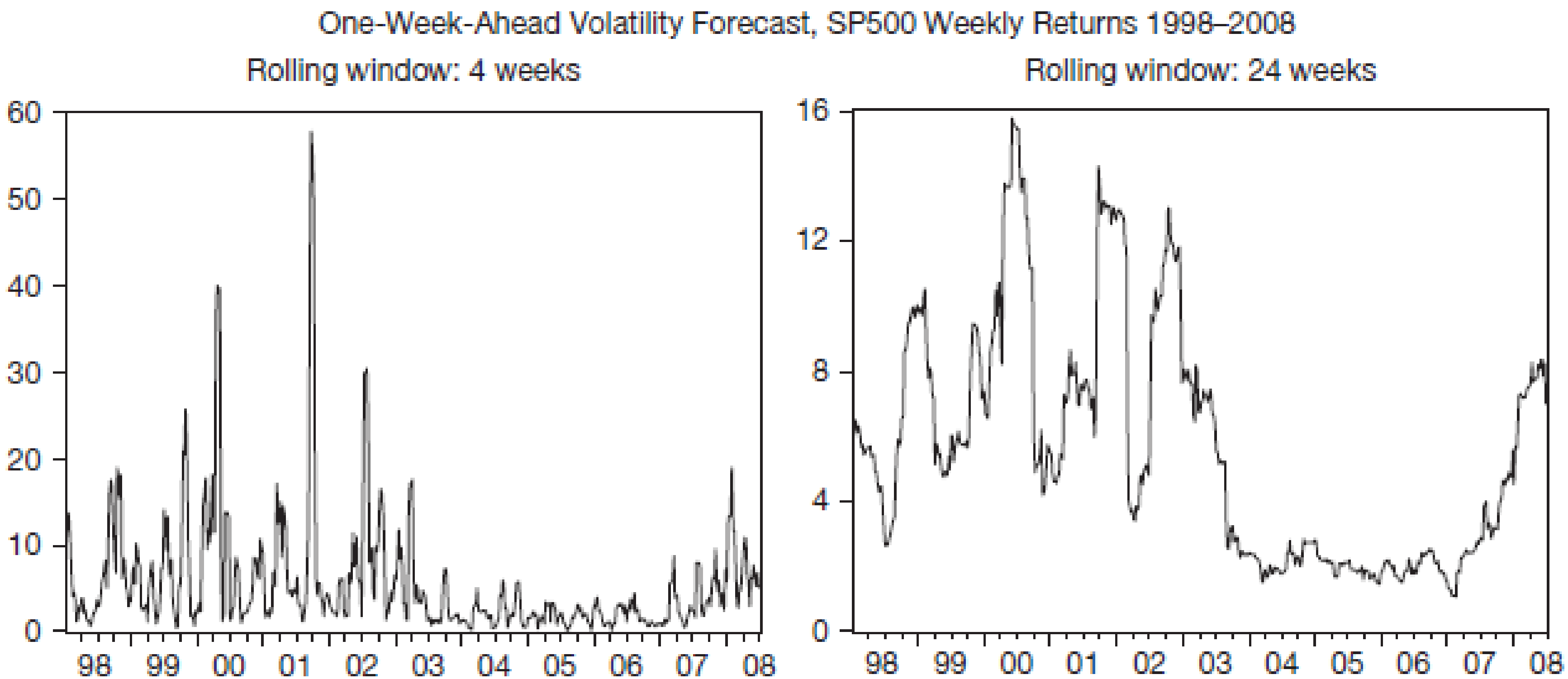
•***Asymmetry and Leverage effect:*** negative price movements (negative shocks) have a higher impact on volatility than positive surprises (positive shocks);

13.5.1 Rolling Window Volatility

$$\hat{\sigma}^2_{t|t-1} = \frac{1}{n}\sum_{i=1}^{n}(r_{t-i} - \mu)^2$$

The estimator can be considered as a *moving average* in which all the components of the moving average have the same weight 1/*n*

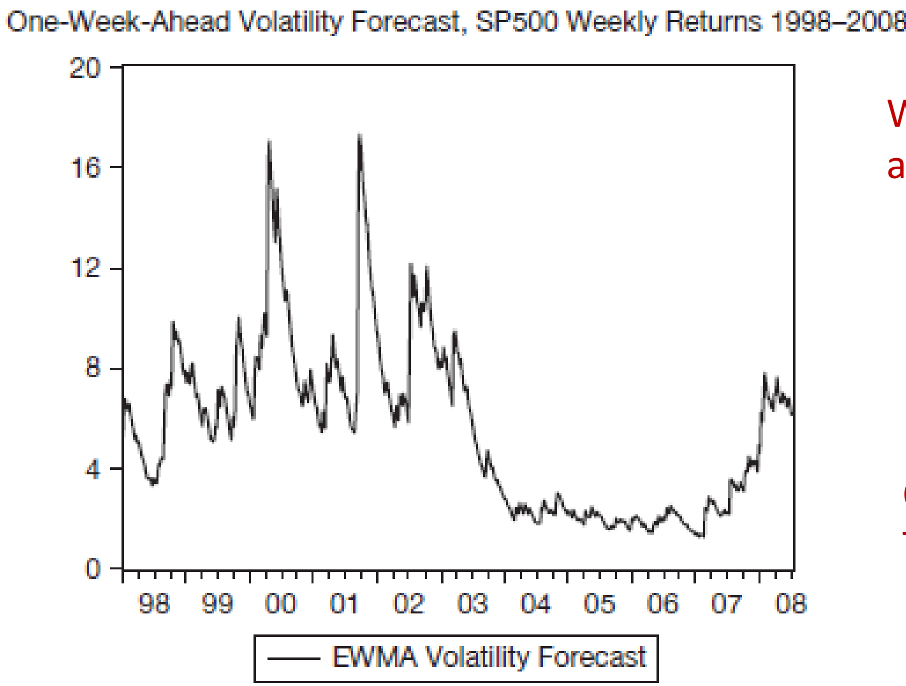
Figure 13.11 Rolling Window Volatility Forecast



13.5.2 Exponentially Weighted Moving Average (EWMA) Volatility

If instead of equally weighting past squared realizations, we assign more weight to the most recent realizations, then we can define another estimator of the conditional variance as:

$$\hat{\sigma}^2_{t\,|t-1} = (1-\lambda)\sum_{i=1}^{t-1}\lambda^{i-1}(r_{t-i}-\mu)^2 \quad \text{for } \lambda \in (0,1)$$



When the sample size is large, this estimator can be approximated by the recursive formula:

$$\hat{\sigma}^2_{t\,|t-1} = \lambda\hat{\sigma}^2_{t-1|t-2} + (1-\lambda)(r_{t-1}-\mu)^2$$

Of course, this is the EWMA:
The **exponential smoothing** method

Figure 13.12 EWMA Volatility Forecast