CHAPTER 3: THEORY OF THE FIRM

Exercise 1

Show that if the production function is homogeneous of degree 1, the marginal rate of technical substitution is independent of the scale of production.

Exercise 2

Solve the long run cost minimization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $a > 0$, $b > 0$.

Exercise 3

Solve the short run cost minimization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $1 > a > 0$, $b > 0$, where x_2 represents the quantity of the fixed input.

Exercise 4

Derive the conditional input demand functions and the cost function for the technologies given by:

a. $f(x)= x_1+ x_2$.
b. $f(x)= \min\{x_1, x_2\}$.
c. $f(x)= (x_1^a+ x_2^a)^{1/a}$, for $a < 1$.

Exercise 5

Let $f(x_1, x_2, x_3, x_4)=\min\{x_1, x_2\}+ \min\{x_3, x_4\}$ and let $g(x_1, x_2, x_3, x_4)=\min\{x_1+x_2, x_3+x_4\}$.

a. Determine the cost functions and the conditional input demands for both production functions.
b. What kind of returns to scale does each of these technologies exhibit?

Exercise 6

Solve the long run profit maximization problem for a production function given by $f(x) = x^a$, $a > 0$.

Exercise 7

Solve the long run profit maximization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $a + b < 1$, $a > 0$, $b > 0$.

Exercise 8

Solve the short run profit maximization problem for a production function given by $f(x_1, x_2) = x_1^a x_2^b$, $1 > a > 0$, $b > 0$, where x_2 represents the quantity of the fixed input.

Exercise 9

Derive the input demand, the output supply, and the profit functions for the following production functions:

a. $f(x_1, x_2) = x_1 + x_2$.
b. $f(x_1, x_2) = \min\{x_1, x_2\}$.

Exercise 10

Let $f(x)=10x-x^2/2$. Determine the input demand, the output supply, and the profit functions.