CHAPTER 7: GAME THEORY

Exercise 1

Solve the following games using (iterative) elimination of (weakly) dominated strategies.

1.

	L	*CL*	*CR*	*R*
U	5, 10	0, 11	1, 10	10, 20
MU	4, 0	1, 0	2, 0	20, 1
MD	3, 2	0, 4	4, 3	50, 1
D	2, 93	0, 92	0, 91	100, 90

2.

	L	*C*	*R*
U	3, 3	0, 3	0, 0
M	3, 0	2, 2	0, 2
D	0, 0	2, 0	1, 1

Exercise 2

Consider a second-price sealed-bid auction with two bidders denoted by i = 1, 2, with valuations $v_1 > v_2$. Assume that, in case of a tie, i.e., if both bidders submit the same bid, bidder 1 wins the auction. Formalize this auction as a strategic-form game and find the equilibrium in weakly dominant strategies.

Exercise 3

Players 1 and 2 simultaneously choose a positive integer smaller or equal to *K*. If both players choose the same number, player 2 pays 1€ to player 1; otherwise no payment occurs. Determine the unique Nash equilibrium of this game.

Exercise 4

Determine the set of Nash equilibria of the following games:

1.

	L	*R*
U	0, 1	0, 2
D	2, 2	0, 1

2.

	L	*R*
U	6, 0	0, 6
D	3, 2	6, 0

3.

		M1			*M2*	
		L	*R*		*L*	*R*
	U	1, 1, 1	0, 0, 0	*U*	0, 0, 0	0, 0, 0
	D	0, 0, 0	0, 0, 0	*D*	0, 0, 0	2, 2, 2

Exercise 5

Consider a first-price sealed-bid auction with two bidders denoted by $i = 1, 2$, with valuations $v_1 > v_2$. Assume that, in case of a tie, i.e., if both bidders submit the same bid, bidder 1 wins the auction. Also assume that valuations are common knowledge. Formalize this auction as a strategic-form game and determine the set of Nash equilibria.

Exercise 6

Consider the Cournot model with n firms, which simultaneously choose how much to produce. Let q_i be the quantity produced by firm i and let $Q = q_1 + ... + q_n$ be total quantity produced. Let p be the equilibrium price and assume that the inverse market demand is: $p(Q) = \max\{0, a-Q\}$. Total cost of producing q_i by firm i is $c_i(q_i) = c_i\, q_i$, with $c_i < a$ for all i=1,..., n. All of this is common knowledge.

i. Assume $c_i = c$ for all i=1,..., n. Determine, as a function of n, the quantities produced, the price, and the profits in Nash equilibrium (Cournot equilibrium).

ii. Determine the limits of the functions obtained in i. when n goes to infinity. Explain.

iii. Assume n =2. Determine the Nash equilibrium when $0 < c_i < \frac{a}{2}$ for each firm? What if $c_1 < c_2 < a$, but $2c_2 > a + c_1$?

Exercise 7

Find the Nash equilibrium of a Bertrand duopoly where the two firms in the market have the same cost structure.

Exercise 8

Find all Bayesian-Nash equilibria of the following game with incomplete information:

(a) Nature chooses J_1 and J_2 with 50% probability.
(b) Player 1 observes Nature's choice, but player 2 does not.
(c) Player 1 chooses *C* or *B*; simultaneously, player 2 chooses *E* or *D*.

J_1	*E*	*D*
C	1, 1	0, 0
B	0, 0	0, 0

J_2	*E*	*D*
C	0, 0	0, 0
B	0, 0	2, 2

Exercise 9

Consider a Cournot duopoly with market demand given by P(Q) = a – Q, where Q = $q_1 + q_2$. Firm 1's cost function, given the quantity produced, is $C_1(q_1) = cq_1$ and firm 2's cost function is $C_2(q_2) = c_H q_2$ with probability *a* and $C_2(q_2) = c_L q_2$ with probability

1 – *a*. All of this is common knowledge. However, information is asymmetric: firm 2 knows its cost function, but firm 1 does not.

i. Formulate this situation as game in strategic form.
ii. Compute a Bayesian-Nash equilibrium.

Exercise 10

Consider the Battle of Sexes:

	Bach	*Stravinski*
Bach	3, 1	0, 0
Stravinski	0, 0	1, 3

i. Find all Nash equilibria of this game.
ii. Now assume that this game has incomplete information:

	Bach	*Stravinski*
Bach	$3 + t_1$, 1	0, 0
Stravinski	0, 0	1, $3 + t_2$

Where t_1 and t_2 follow a uniform distribution in [0, x]. Determine the Bayesian-Nash equilibrium in pure strategies and show that as x goes to 0, the Bayesian-Nash equilibrium tends to the mixed strategies equilibrium of the complete information game.

Exercise 11

Two players, 1 and 2, share 1€ using the following procedure: each player *i* chooses a number s_i, $s_i \in [0, 1]$, i =1,2. The choices are simultaneous. If $s_1 + s_2 \leq 1$, each player gets the amount chosen; if $s_1 + s_2 > 1$ both get 0.

i. Determine the set of pure Nash equilibria.

Suppose that player 2, before choosing s_2, observes the number chosen by player 1 and this fact is common knowledge.

ii. Find a few examples of pure Nash equilibria of the modified game.

iii. Determine the set of pure subgame perfect Nash equilibria.

Exercise 12

Player 1 may choose Stop or Continue. If he chooses Stop, the game ends and each player gets 1€. If he chooses Continue, both players simultaneously choose non-negative integers and each player gets the product of the chosen numbers.

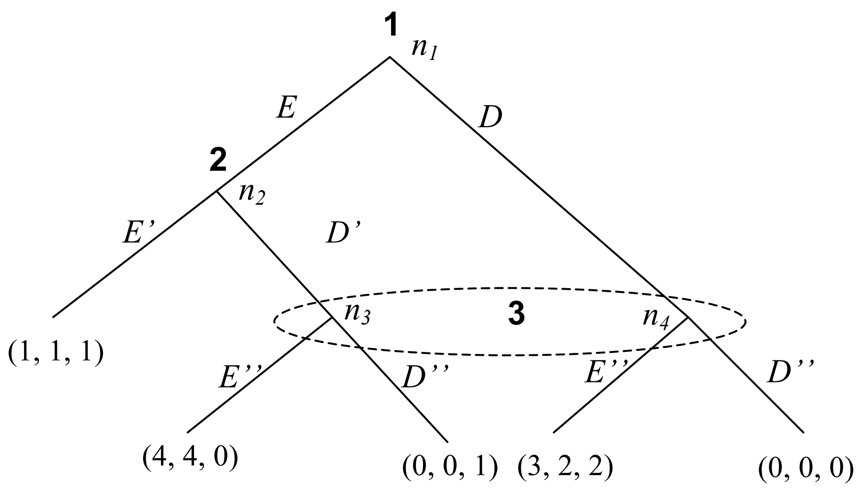
i. Formulate this situation as an extensive-form game with imperfect information.

ii. Determine the set of pure subgame perfect Nash equilibria.

iii. How does this set change if the non-negative integers are at most equal to $M > 1$?

Exercise 13

Consider the following extensive-form game with imperfect information Γ:



This game has two types of Nash equilibria:

Type 1: $x_1(E)=1$, $x_2(E')=1$ and $x_3(E'') \in [0, 1/4]$.

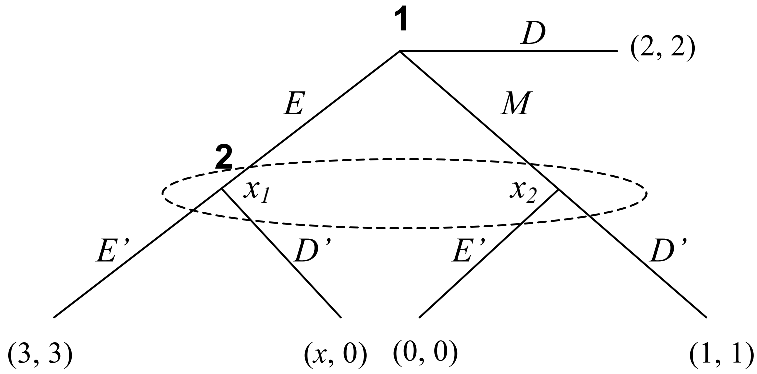
Type 2: $x_1(E)=0$, $x_2(E') \in [1/3, 1]$ and $x_3(E'')=1$.

i. Show that equilibria of Type 1 are perfect Bayesian equilibria of Γ.

ii. Show that no equilibrium of Type 2 is a perfect Bayesian equilibria of Γ.

Exercise 14

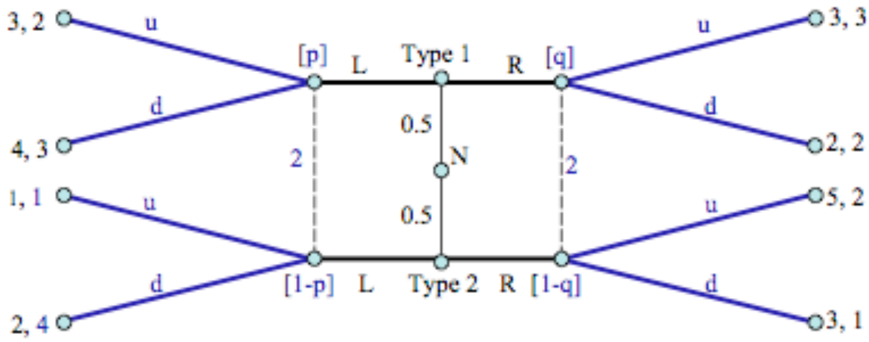
Consider the following extensive-form game with imperfect information Γ:



Show that the strategy $x = (x_1, x_2)$, with $x_1(D)=1$ and $x_2(D')=1$, is a perfect Bayesian equilibrium for $x < 2$.

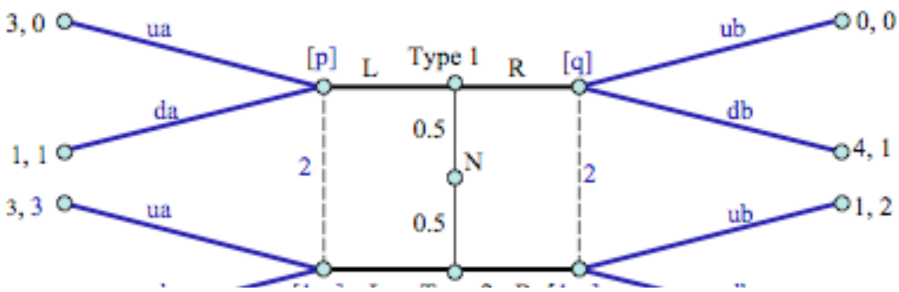
Exercise 15

Check whether Player 1's strategies (L,R), (R,L), (R,R) and (L,L) are part of perfect Bayesian equilibria of the following game:



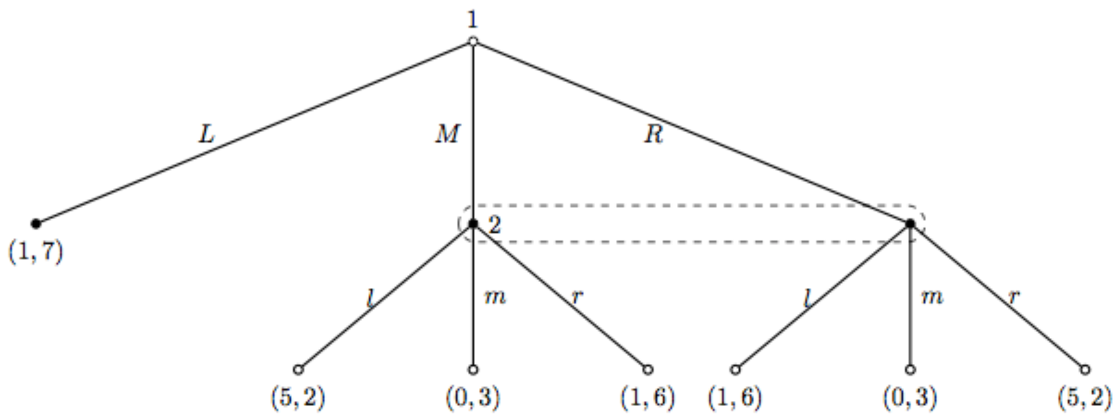
Exercise 16

Show that Player 1's strategy (L,R) is part of a perfect Bayesian equilibrium of the following game:



Exercise 17

Consider the following extensive-form game with imperfect information:



a) Write the game in normal form.
b) Determine the set of pure strategy Nash equilibria of the game.
c) How many subgames does this game have?
d) Determine the set of pure strategy subgame perfect Nash equilibria of the game.
e) Check whether the equilibria found in d) are perfect Bayesian equilibria.