

**Week 11:**

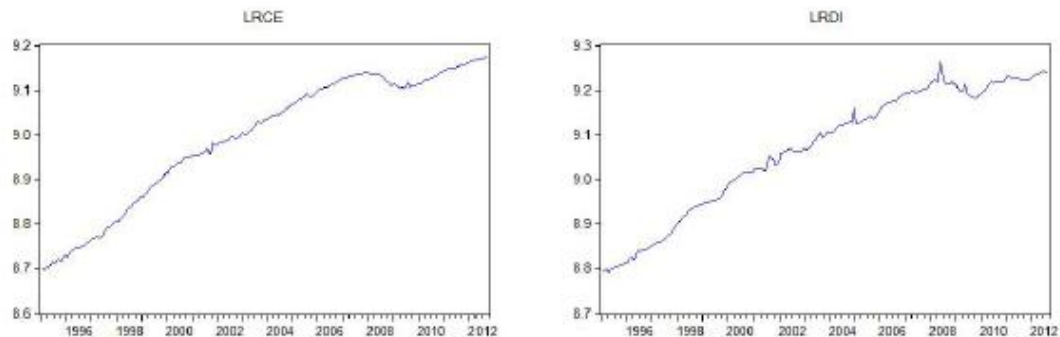
Exercises 8 and 9 (chapter 10 Textbook)

24. Consider the process ARIMA(1,1,0) with  $c = 0$  and  $\phi = 0.9$ . Given  $y_n = 100$  and  $y_{n-1} = 120$  obtain point forecasts for the next two periods.
25. Given that  $X_t$  follows a process ARIMA(0,2,1) with  $c = 0$  and  $\theta = 0.9$  and that the last observed data is  $X_t = 500, X_{t-1} = 490, \hat{\varepsilon}_t = -10$  and  $\hat{\varepsilon}_{t-1} = 2$  obtain point forecasts for the next 4 periods.
26. According to the simple Keynesian model, the relation between consumption,  $C_t$ , and disposable income,  $Y_t$ , can be represented by a linear function:

$$\log(C_t) = \beta_0 + \beta_1 \log(Y_t) + \varepsilon_t \quad (1)$$

Where  $\beta_1$  is the marginal propensity to consume, a quantity of substantial interest and  $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$ :

In the next figures you will find the graphical representation of the series of Consumption Expenditure (left) and Disposable Income (right) in the US, at constant prices and in logs and the estimation EViews output of equation (1).



Dependent Variable: LRCE  
 Method: Least Squares  
 Date: 11/07/12 Time: 14:50  
 Sample: 1995M01 2012M09  
 Included observations: 213

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.341302	0.056125	-6.081114	0.0000
LRDI	1.029216	0.006187	166.3432	0.0000
R-squared	0.992432	Mean dependent var		8.993582
Adjusted R-squared	0.992396	S.D. dependent var		0.144663
S.E. of regression	0.012615	Akaike info criterion		-5.898585
Sum squared resid	0.033576	Schwarz criterion		-5.867024
Log likelihood	630.1994	Hannan-Quinn criter.		-5.885830
F-statistic	27670.05	Durbin-Watson stat		0.353176
Prob(F-statistic)	0.000000			

- Using the available information, what can you conclude from the marginal propensity to consume? Motivate your answer.
- The following figure depicts the result of the ADF test applied to the time series  $\log(C_t)$ . For this time series should you apply the ADF test with a constant and a trend or only the constant term? Justify your answer.
- According to the EViews output, is it possible to conclude that  $\log(C_t)$  has a unit root? Indicate the null and alternative hypotheses, test statistic, significance level, critical value and the test conclusion.

Null Hypothesis: LRCE has a unit root  
 Exogenous: Constant, Linear Trend  
 Lag Length: 1 (Automatic - based on SIC, maxlag=14)

	t-Statistic
Augmented Dickey-Fuller test statistic	-0.616883
Test critical values: 1% level	-4.002142
5% level	-3.431265
10% level	-3.139292

\*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation  
 Dependent Variable: D(LRCE)  
 Method: Least Squares  
 Date: 11/07/12 Time: 16:05  
 Sample (adjusted): 1995M03 2012M09  
 Included observations: 211 after adjustments

Variable	Coefficient	Std. Error	t-Statistic
LRCE(-1)	-0.003970	0.006436	-0.616883
D(LRCE(-1))	-0.228968	0.057366	-3.98880
C	0.039599	0.056317	0.703143
@TREND(1995M01)	-1.04E-05	1.52E-05	-0.683498

R-squared	0.113574	Mean dependent var
Adjusted R-squared	0.100727	S.D. dependent var
S.E. of regression	0.003699	Akaike info criterion
Sum squared resid	0.002832	Schwarz criterion
Log likelihood	384.1650	Hannan-Quinn criter.
F-statistic	8.840660	Durbin-Watson stat
Prob(F-statistic)	0.000015	

**Week 12:**

*Exercises 5 and 6 (chapter 13 Textbook)*

**Week 13:**

*Exercises 5 and 6 (chapter 14 Textbook)*

27. Given the correlograms, presented in Figure 1, of the log returns of a financial series what stylized characteristics can be observed? Define the order of an ARCH model to fit the conditional variance of the series of returns.

Correlogram of $(1 - B)\ln(y_t)$					Correlogram of squared $(1 - B)\ln(y_t)$						
Included observations: 952					Included observations: 952						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat		
		1	-0.001	-0.001	0.0015			1	0.011	0.011	0.1093
		2	0.007	0.007	0.0436			2	0.126	0.126	15.376
		3	0.037	0.037	1.3636			3	0.124	0.124	30.202
		4	0.042	0.042	3.0663			4	0.097	0.083	39.244
		5	0.059	0.059	6.4413			5	0.162	0.138	64.519
		6	-0.041	-0.042	8.0183			6	0.057	0.028	67.642
		7	0.027	0.023	8.7098			7	0.060	0.008	71.051
		8	-0.018	-0.024	9.0378			8	0.081	0.033	77.313
		9	-0.022	-0.025	9.5155			9	0.029	-0.011	78.122
		10	-0.018	-0.020	9.8171			10	0.062	0.016	81.842
		11	0.041	0.046	11.401			11	0.048	0.021	84.098
		12	-0.084	-0.085	18.152			12	0.067	0.042	88.450
		13	-0.002	0.005	18.157			13	0.076	0.049	94.102
		14	0.027	0.027	18.843			14	0.046	0.022	96.122
		15	0.052	0.057	21.416			15	0.091	0.056	104.08
		16	-0.030	-0.030	22.262			16	0.002	-0.036	104.08
		17	-0.010	0.001	22.369			17	0.073	0.026	109.27
		18	-0.008	-0.025	22.424			18	0.003	-0.037	109.28
		19	-0.058	-0.059	25.674			19	0.087	0.055	116.63
		20	0.042	0.040	27.426			20	0.077	0.053	122.48

**Figure 1**

28. True or False? Correct the sentence and justify when appropriate.

- Volatility clustering is one of the most prominent features of financial returns. Time series analysis reproduces this stylized fact using the ARMA model with white noise errors.
- An ARCH(2) model is equivalent to an AR(2) model for the squared returns.
- A GARCH(1,1) model is equivalent to a MA(2) model for the squared returns.
- The ARMA-GARCH model only generates forecasts for the variance.
- Usually the final ARMA-GARCH model uses the same ARMA model that was fitted before modelling the volatility.
- The GARCH model is able to describe adequately the dynamic properties of volatility of standard financial time series with less parameters than the ARCH model.

29. Consider the estimation output presented below:

Dependent Variable: D(LOG(PB))				
Method: ML - ARCH (Marquardt) - Normal distribution				
Sample (adjusted): 1/04/2000 4/04/2011				
Included observations: 2935 after adjustments				
Convergence achieved after 10 iterations				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 + C(8)*RESID(-6)^2				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001321	0.000396	3.337769	0.0008
Variance Equation				
C	0.000280	1.41E-05	19.87422	0.0000
RESID(-1)^2	0.089043	0.014307	6.223588	0.0000
RESID(-2)^2	0.068808	0.017353	3.965242	0.0001
RESID(-3)^2	0.082096	0.017453	4.703768	0.0000
RESID(-4)^2	0.100291	0.016906	5.932440	0.0000
RESID(-5)^2	0.092199	0.019404	4.751628	0.0000
RESID(-6)^2	0.054665	0.016496	3.313866	0.0009
R-squared	-0.001094	Mean dependent var		0.000543
Adjusted R-squared	-0.003488	S.D. dependent var		0.023535
S.E. of regression	0.023576	Akaike info criterion		-4.754119
Sum squared resid	1.626930	Schwarz criterion		-4.737807
Log likelihood	6984.670	Hannan-Quinn criter.		-4.748245
Durbin-Watson stat	1.952501			

- Write explicitly the estimated equation.
- Obtain the estimate for the unconditional variance of the error of the series.
- Comment on the correlogram of the standardized squared residuals presented below.

Correlogram of Standardized Residuals Squared						
Sample: 1/04/2000 4/04/2011						
Included observations: 2935						
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.010	0.010	0.3221	0.570
		2	-0.010	-0.010	0.6151	0.735
		3	-0.018	-0.018	1.5714	0.666
		4	-0.030	-0.030	4.2216	0.377
		5	-0.014	-0.014	4.8152	0.439
		6	-0.026	-0.026	6.7396	0.346
		7	0.026	0.025	8.7758	0.269
		8	0.040	0.037	13.388	0.099
		9	0.077	0.076	30.951	0.000
		10	0.031	0.030	33.817	0.000
		11	0.015	0.018	34.440	0.000
		12	0.056	0.062	43.776	0.000
		13	0.014	0.022	44.325	0.000
		14	0.042	0.050	49.509	0.000
		15	0.031	0.038	52.422	0.000
		16	0.049	0.052	59.550	0.000
		17	0.027	0.027	61.649	0.000
		18	-0.012	-0.011	62.080	0.000
		19	0.009	0.008	62.325	0.000
		20	0.009	0.007	62.549	0.000
		21	0.042	0.034	67.804	0.000
		22	0.044	0.037	73.525	0.000
		23	0.013	0.002	74.047	0.000
		24	0.036	0.023	77.784	0.000
		25	0.015	0.006	78.462	0.000
		26	0.045	0.039	84.482	0.000
		27	0.029	0.029	87.061	0.000
		28	0.040	0.036	91.834	0.000
		29	0.030	0.024	94.528	0.000

32. Suppose that the return series of a given stock,  $r_t$ , is well described by the following model:

$$r_t = \varepsilon_t = \sigma_t z_t, \quad z_t \stackrel{iid}{\rightarrow} D(0,1)$$

$$\sigma_t^2 = 1 + 0.4\varepsilon_t^2 + 0.2\varepsilon_{t-1}^2 + 0.3\varepsilon_{t-2}^2$$

- Derive the forecast equations that are used to obtain the forecasts,  $\sigma_{T+s|T}^2$  with origin at time T.
- Suppose that the last two observations for the returns are  $r_{T-1} = 0.03$  and  $r_T = 0.06$ . Using  $\sigma_T^2 = 1$  obtain the forecasts for  $\sigma_{T+1}^2$ ,  $\sigma_{T+2}^2$  and  $\sigma_{T+3}^2$  with origin at time T.