Soluções dos exercícios de Primitivas e Cálculo Integral

Exercício 1. Primitivas:

(a) $x^4 + \frac{x^3}{3} - \frac{x^2}{2} + x + c;$

(b) $-\frac{\cos(3x)}{3} - \frac{1}{12}x^6 + \frac{2}{3}\sqrt[2]{x^3} + c;$

(c) $-\frac{1}{x} - 2\sqrt{x} - \frac{1}{15}\sin(5x) + c;$

(d) $\frac{3}{2}\sqrt[3]{x^2} - \frac{1}{3}e^{3x} - 2\arctan(x) - \frac{1}{5}e^{-2x} + c;$

(e) $-\ln|\cos(x)| + \ln|1+x| + c;$

(f) $-\frac{1}{24}(2-4x)^6 + c;$

(g) $-e^{\cos(x)} + c;$

(h) $\frac{1}{3}e^{x^3} - \frac{1}{2}\sin(x^2) + c;$

(i) $\frac{1}{12}(x^3+1)^4 + c;$

(j) $\frac{(1+\ln(x))^6}{6} + c;$

(k) $\frac{\arctan^2 x}{2} + \ln|\arctan x| + c;$

(l) $\ln(1+e^x) + \arctan(e^x) + c;$

(m) $\ln|1+\sin(x)| + \arctan(\sin(x)) + c;$

(n) $\frac{1}{4}\sin^4(x) - \frac{\cos^2(x)}{2} + c;$

(o) $2x - 2\ln|(x+3)(x+7)| + c;$

(p) $\frac{1}{2}\arctan(2x) + \frac{\sqrt{3}}{6}\arctan\left(\frac{2}{\sqrt{3}}x\right) + c;$

(q) $x + \frac{5}{2}\ln(x^2+1) - 3\arctan(x) + c;$

(r) $x - \frac{7}{2}\ln(x^2+9) - \frac{4}{3}\arctan\left(\frac{x}{3}\right) + c;$

(s) $-\frac{1}{5}\sqrt{2-5x^2} - \frac{2}{\sqrt{10}}\arccos\left(\sqrt{\frac{5}{2}}x\right) + c;$

(t) $-\frac{e^{-x^5}}{5} - e^{\frac{1}{x}} + c.$

Exercício 2.

b.

$$\int \cos^2(x)dx = \frac{x}{2} + \frac{\sin(2x)}{4} + c$$

$$\int \sin^2(x)dx = \frac{x}{2} - \frac{\sin(2x)}{4} + c$$

c.

$$\int \cos^4(x)dx = \frac{3}{8}x + \frac{\sin(4x)}{32} + \frac{\sin(2x)}{4} + c$$

$$\int \sin^6(x)dx = -2^{-5}\left(\frac{\sin(6x)}{6} - \frac{3\sin(4x)}{2} + \frac{15\sin(2x)}{2} + 10x\right) + c$$

Exercício 3. Primitivas:

(a) $\dfrac{xe^{2x}}{2} - \dfrac{e^{2x}}{4} + c;$

(b) $-\dfrac{x^2\cos(2x)}{2} + \dfrac{x\sin(2x)}{2} + \dfrac{\cos(2x)}{4} + c;$

(c) $\dfrac{e^{3x}}{3}\left(x^2 + \dfrac{x}{3} + \dfrac{8}{9}\right) + c;$

(d) $x\arctan(3x) - \dfrac{\ln(1+9x^2)}{6} + c;$

(e) $x\ln(x) - x + c;$

(f) $x\arccos(x) - \sqrt{1-x^2} + c;$

(g) $x(\arccos(x) + \arcsin(x)) + c;$

(h) $x(\arctan(x) + \arctan(1/x)) + c;$

(i) $\dfrac{e^x}{5}\left(\sin(2x) - 2\cos(2x)\right) + c;$

(j) $\dfrac{e^{2x}}{5}\left(\sin(x) + 2\cos(x)\right) + c;$

(k) $\dfrac{2}{3}\sqrt{x^3}\ln(x) - \dfrac{4}{9}\sqrt{x^3} + c;$

(l) $x\ln^2(x) - 2x\ln(x) + 2x + c;$

(m) $\dfrac{x^3}{3}\arctan x - \dfrac{x^2}{6} + \dfrac{\ln(1+x^2)}{6} + c;$

(n) $\dfrac{x^2e^{x^2}}{2} - \dfrac{e^{x^2}}{2} + c.$

Exercício 4. Primitivas:

(a) $2\ln|x-2| - \ln|x-1| + c;$

(b) $\dfrac{\ln|x-1|}{6} - \dfrac{\ln|x+1|}{2} + \dfrac{\ln|x+2|}{3} + c;$

(c) $\dfrac{\ln(x^2+4)}{2} + c;$

(d) $\dfrac{3}{2}\ln|x+1| - \dfrac{1}{4}\ln(x^2+1) + \dfrac{1}{2}\arctan(x) + c;$

(e) $\dfrac{x^2}{2} + 2\ln|x-1| - \ln|x+1| + c;$

(f) $\dfrac{x^2}{2} + 2x + \ln(x^2+1) + 2\arctan(x) + c.$

Exercício 5. Primitivas :

(a) $\frac{1}{2}\left(\arctan(x) + \frac{x}{1+x^2}\right) + c$;

(b) $\ln|1+\cos(x)| - \ln|\cos(x)| + c$;

(c) $\frac{\ln|1-\sin(x)|}{4} + \frac{1}{2(1-\sin(x))} + \frac{\ln|1+\sin(x)|}{4} + c$;

(d) $2\frac{\sqrt{(e^x+1)^3}}{3} - 2\sqrt{e^x+1} + c$;

(e) $-2\arccos\left(\frac{x}{2}\right) + \frac{x}{2}\sqrt{4-x^2} + c$;

(f) $\frac{2}{3}t^{\frac{3}{2}} - \frac{3}{4}t^{\frac{4}{3}} + \frac{6}{7}t^{\frac{7}{6}} - t + \frac{6}{5}t^{\frac{5}{6}} - \frac{3}{2}t^{\frac{2}{3}} + 2t^{\frac{1}{2}} - 3t^{\frac{1}{3}} + 6t^{\frac{1}{6}} - \ln(t^{\frac{1}{6}}+1) + c.$

Exercício 6. b.

$$\int \sqrt{4+t^2}dt = \frac{(t+\sqrt{t^2+4})^2}{8} - \frac{2}{(t+\sqrt{t^2+4})^2} + 2\ln\left(\frac{t+\sqrt{t^2+4}}{2}\right) + c$$

Exercício 7. Integrais:

(a) -3;

(b) 4;

(c) 0;

(d) 0;

(e) -1;

(f) $\frac{b^2-a^2}{2}$.

Exercício 8. Áreas:

(a) $\frac{36}{\sqrt{2}}$;

(b) $\frac{124}{15} - \pi$;

(c) $\frac{1}{12}$;

(d) $\frac{1}{6}$.

Exercício 9. -

Exercício 10. $\lim_{x\to 0} M_f(x) = f(0)$.

Exercício 11. -

Exercício 12. $I = \dfrac{\ln(3)}{6} - \dfrac{\ln(2)}{3} + \dfrac{\arctan(\sqrt{2})}{3\sqrt{2}}$.

Exercício 13. Soluções:

(a) $-\dfrac{1}{2}$;

(b) 4;

(c) $\dfrac{\pi^2}{8}$;

(d) -1;

(e) Divergente;

(f) $\dfrac{\pi}{2}$.

Um último desafio

Exercício 14. Respostas:
$I_0 = \dfrac{\pi}{2}$

$I_1 = 1$.

$$\int_0^{\frac{\pi}{2}} \sin^n(x)dx = \frac{(2k+1)!}{\prod_{i=0}^{k}(2i+1)^2}, n = 2k+1, k \in \mathbb{N}_0.$$