

1st Part: 70 Marks. All answers shall be given in the space available. During the examination no comments or questions should be asked. Write your name and number on every sheet on the place available. No mobile phones, or any device with *bluetooth* or *wifi*, are allowed at any time.

Name: _____ Number: _____

In the following groups of questions, every right answer has 2.5 marks each, wrong answers have -2.5 each (2.5 penalty mark). Each group of questions will have a mark between 0 (minimum) and 10 (maximum).

Write True (T) or False (F), with an "X" in the appropriate entry.

1. Consider Simple and Compound Interest calculation:

T F

For a positive interest rate, the accumulated value of a principal in simple and compound regimes is always different, no matter the application time.		X
From the quarterly effective (and positive) interest rate were calculated the annual nominal rate and an annual equivalent rate. They are always different, no matter the interest regime considered.		X
Consider Simple Discount. The discount rate is calculated over the accumulated value whereas the interest rate is computed over the initial value.	✓	
In compound interest, if two principals, A and B, are equivalent, and if A is equivalent to a principal C, then whatever the reference time is considered, B is also equivalent to C.	✓	

2. Consider annuities, immediate, deferred, perpetuities. Always, $i > 0$:

T F

For all n and i we have that $s_{\overline{n} i} = s_{\overline{n-1} i} + 1$.		X
A set of consecutive, constant and equally spaced, payments is not an annuity if period payment is monthly.		X
If $a_{\infty i} = 22$, then $i < 5\%$.	✓	
For all i and n , $s_{\overline{n} i} = v^{-n}a_{\overline{n} i}$.	✓	

3. Consider the following financial operations:

T F

Leasing is a financial means but it is not a loan.	✓	
A bond loan issued above the par, and subscribed completely, gets a money inflow higher than the loan value.	✓	
In a loan repayment with equal payments principal amortization is always constant.		X
A coupon zero bond loan is always more beneficial to the issuer because it does not pay interest.		X

4. Consider the following situations:

T F

A €100,000 loan is redeemed five times with constant principal payments. None of the individual total payments is higher than €20,000.	✓	
For rate $i > 0$, we have that $a_{\overline{n} i} > n$.		X
Consider a bond loan issued at the par where redemption is paid with a premium. Investor's yield rate is higher than the coupon rate.	✓	
Shares of a public company pay interest if they are lent at issuance.		X

In the next group of questions, tick \checkmark or write X in the box next to the answer you consider to be correct (only one is). In each group, a correct answer has 5 marks and a wrong answer gets -1.25 marks (penalty 1.25).

5. Mr. Zach sold his car to his friend Zappa for €12,000. Zach received immediately €3,000 and the rest is going to be received through 10 equal installments, at the end of each consecutive quarter, with the first payment to be made 9 months after the transaction, exactly. For $i_T = 1\%$, calculate the value of each constant payment (approximately):

- a) €969.34 ; b) €1,455.50 ; c) €950.24 ; d) None of the others .

$$9000 = x \cdot a_{\overline{10}|1\%} \cdot (1.01)^{-2} \Leftrightarrow x \approx 969.34$$

6. Today is 2019 New Year's day. Zach is supposed to receive the following amounts: €1,500 at the end of the months of January, February and March and, in addition €3,000 at the end of May, July and September (consider all months equal). Alternatively, he has the option to receive a lump sum at the end of December 2019. For $i_A^{(12)} = 6\%$ determine the value of the lump sum.

- a) €13,957.74 ; b) €14,341.20 ; c) €13,250.34 ; d) None of the others .

$$i_M = 0.5\%, \quad i_{1M} = 1.005^2 = 1.0025\% \rightarrow 1500 \cdot a_{\overline{3}|0.5\%} + 3000 \cdot a_{\overline{6}|0.5\%} \cdot (1.005)^4 + 3000 \cdot a_{\overline{3}|0.5\%} \cdot (1.005)^8$$

7. The principal €5,000, accumulated at rate i , semi-annually and simple interest, accumulated for 4 years an amount of €6,281.78. Determine, now in compound interest, the quarterly interest rate that, applied during the same term, accumulates the same amount (approximately).

- a) 1.920% ; b) 1.970% ; c) 1,440% ; d) Nenhuma das outras .

$$5000(1+i_q)^6 = 6281.78 \Leftrightarrow i_q \approx 1.44\%$$

8. "Zach SA" signed today an acquisition contract for an industrial equipment with a payment plan with two installments. Values and due dates are: from today, €6,400 within six months and €7,200 within 18 months, respectively. Compound interest and an annual interest rate of 6% are applied, compute the acquisition value of the equipment.

- a) €12,793.10 ; b) €12,813.64 ; c) €13,600.00 ; d) None of the others .

$$6400(1.06)^{-0.5} + 7200(1.06)^{-1.5} \approx 12,813.64$$

9. Zach did an application of €5,000 in "Treasury Bonds Special Plus" with a maturity of five years, compound interest, and annual interest rates according to the table:

Year	1	2	3	4	5
Interest rate	3.00%	3.00%	5.00%	6.00%	8.00%

Compute the yield rate for this financial application (approximately).

- a) 4.98% ; b) 5.00% ; c) 5.28% ; d) 5.50% .

$$1.03^2(1.05)(1.06)(1.08) = (1+r)^5 \rightarrow \sim 4.98\%$$

10. From a loan, Zach will receive back 12 monthly installments of €200.00 each, where the first is due within three months from today. For monthly effective rate of $i_M = 1\%$, calculate the loan amount (approximately).

- a) €2,228.73 ; b) €2,251.02 ; c) €2,400.00 ; d) None of the others .

$$200 \cdot a_{\overline{12}|1\%} \cdot (1.01)^{-2} \approx 2206.66$$

2nd Part (130/200 marks)

In this group write your calculations in the space below the question and write the final answer in the box provided. Do not forget to present all formulae and intermediate calculations needed.

1. (40 marks)

“Zach PLC” issued a bond loan under the following terms:

- Issuance date: 01/01/2016;
- Face value: €10.00;
- N° of bonds, above the par: 120,000;
- Issue value: €10.20;
- Maturity: 3 years;
- Coupon annual interest rate, compounded semi-annually: 6%;
- 1st redemption, 1 year after issuance;
- Coupon paid semi-annually, 1st payment on 01/07/2016;
- Principal constant payments, annually;
- Redemption premium: €0.20 per bond in the 1st year and €0.30 in the following.

a) Compute the value of the loan.

$Loan: 120\,000(10.00) = €1,200,000.00$, $I_1 = 1200000(0.03) = 36,000$
 $Premium: 12\,000(0.20) = 24,000.00$
 $M. inflow: \underline{1,224,000.00}$

R: just loan: € 1 200 000, 00

b) Fill up the Amortization Schedule, for the **1st year and half**, only:

Time Semester	Initial Balance	Interest	No. of Bonds redeemted	Principal	Premium	Payment	Outstanding Balance
1	1200 000	36 000	—	—	—	36 000	1200 000
2	1200 000	36 000	40 000	400 000	24 000	460 000	800 000
3	800 000	24 000	—	—	—	24 000	800 000

c) Mr. Zappa bought 50 bonds immediately after the payment of the 3rd coupon and kept them until maturity (last redemption). He got an yield rate of 11%, **write the equation** that allows to calculate how much money this investor spent for buying the 50 bonds.

let r : yield rate and $\lambda^* = (1.11)^{1/2} - 1$; and
 $Money\ spent = 10(0.03)50 a_{\overline{3}|\lambda^*} + 50(10.30) \frac{(1.11)^{-1.5}}{(1+\lambda^*)^{-3}}$

2. (50 marks)

A "Zach PLC" is acquiring an automobile through a leasing contract. Contract value is €20,000. The company received the following proposal from *LeasingAuto, Ltd*:

- Quarterly interest rate: 2%;
- Values to be paid:
 - Initial payment, in the contract signing date: 5% of the purchase value;
 - 12 quarterly installments: Immediate and constant, the first to be paid 3 months after the contract signing date;
 - Residual value: 10% of purchase value, to be paid together with the last installment.

a) Calculate the amount of each quarterly installment.

$$\text{Initial Payment: } 0.05(20,000) = 1,000, \quad \frac{i^{(4)}}{4} = 0.02, \quad \text{RV} = 0.1(20,000) = 2,000$$

$$20,000 = 1,000 + R a_{\overline{12}|2\%} + 2000(1.02)^{-12}$$

$$R \approx 1647.51$$

R: $\approx \text{€ } 1,647.51$

b) Calculate the amount yet to be paid one year after the contract date, immediately after the payment of the respective installment.

$$\text{Yet to be paid: } 8 \text{ installments, with } A = R a_{\overline{8}|2\%} \approx 12,068.83$$

R: $\approx \text{€ } 12,068.83$

c) In the meantime "Zach PLC" received another proposal from another *Leasing* company, with a quarterly interest rate of 2% and the following due payments:

- No initial payment is due;
- 11 quarterly installments, immediate and growing in geometric progression with rate 1.2, where the first is due at the end of the first quarter;
- The residual value corresponds to the last installment.

Compute the value of the first quarterly payment of this new proposal.

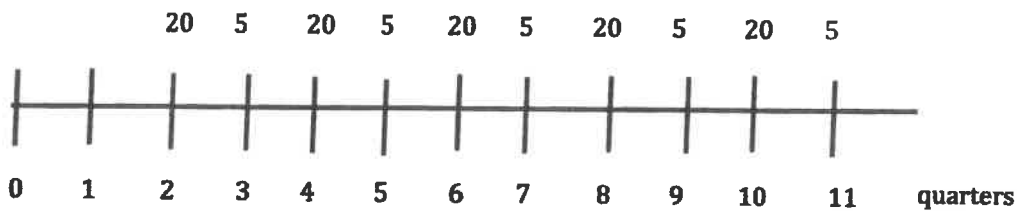
$$20000 = C \frac{1 - 1.2^{11} (1.02)^{-11}}{1.02 - 1.2} \approx 723.51$$

-0.18

R: $\approx \text{€ } 723.51$

3. (40 marks)

Consider the following set of money amounts:



- a) Calculate the present value, using formulae referred to annuities, $a_{\overline{n}|i}$ or $s_{\overline{n}|i}$, for a semi-annual interest rate of 3%.

$$A = 20 a_{\overline{5}|3\%} + 5 a_{\overline{5}|3\%} (1.03)^{-0.5} \approx 114.16$$
$$= a_{\overline{5}|3\%} [20 + 5(1.03)^{-1/2}]$$

R: $\sim \text{€ } 114.16$

- b) Calculate its accumulated value at the end of Quarter 11, for the same interest rate.

$$S = A (1.03)^{5.5} \approx 134.31$$

R: $\sim \text{€ } 134.31$